

3.Theorem: For any integer n , the number $n^2 + n + 1$ is odd.

The n of the first two terms can be factored out. The new expression can be written as $n(n + 1) + 1$.

$n(n + 1)$ is always even, as when n is even $n + 1$ is odd and vice versa and the product of an even and odd integer is always even.

The sum of 1 and an even integer is always odd. Therefore, the theorem is true.