7. Prove that for any natural number n, $2 + 2^2 + 2^3 + ... + 2^n = 2^{n + 1} - 2$

Proof by induction.

If n = 1, $2 = 2^2 - 2$. Base case works.

If n = n + 1, 2, 4, 16, ... + $2^n + 2^{n + 1} = 2^{n + 1} - 2 + 2^{n + 1}$

$2^{n + 1} + 2^{n + 1} - 2 = 2 * 2^{n + 1} - 2 = 2^{n + 1 + 1} - 2 = 2^{n + 2} - 2$

The result is 2... + $2^n + 2^{n + 1} = 2^{n + 1} - 2 + 2^{n + 1} = 2^{n + 2} - 2$

The theorem holds true as n increases, and so the theorem is true.