

FUNCTIONS & GRAPHS

1) $f(x) = 3x^2$ and $g(x) = 2x + 1$.

Order is important in this question

$$g \circ f(x) = g(f(x)) = g(3x^2) = 2(3x^2) + 1 = 6x^2 + 1$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(2x+1) = 3(2x+1)^2 \\ &= 3(2x+1)(2x+1) \\ &= 3(4x^2 + 2x + 2x + 1) \\ &= 3(4x^2 + 4x + 1) \\ &= 12x^2 + 12x + 3 \end{aligned}$$

2) a) $P(x) = \frac{1}{2x-6}$

Functions are not defined when the denominator equals zero

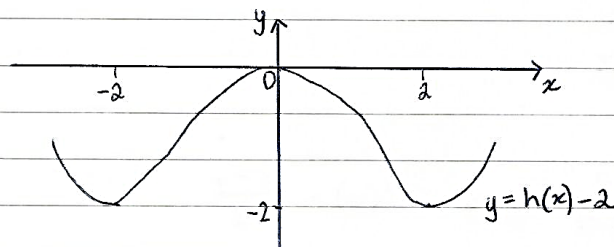
$P(x)$ is defined for all x except $x = \frac{6}{2} = 3$.

b) $Q(x) = \sqrt{x^2 - 9}$

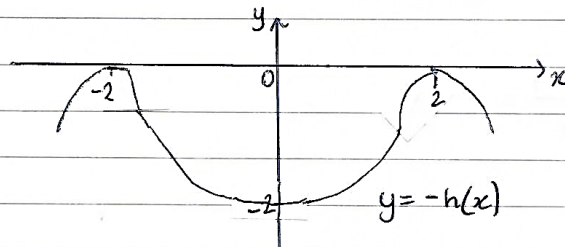
x^2 has to be more than 9 because we cannot take the root of a negative

$Q(x)$ is defined for $x^2 - 9 > 0$ or $x > 3$ and $x < -3$.

3) a)



The graph has moved down by 2 units



The graph has been reflected in the x-axis

b) $h(x)$ is an even function since it is symmetrical in the y-axis.

An odd function shape remains the same after a 180° rotation

4) $f(x) = \frac{2}{x-1}$

a) $f^{-1}(x)$ is the inverse of $f(x)$. To find $f^{-1}(x)$, simply exchange the locations of x and y in the equation and solve for y .

$$f(x) = 3x + 4$$

$$y = 3x + 4$$

Interchange x and y :

$$x = 3y + 4$$

Solve for y :

$$3y = x - 4$$

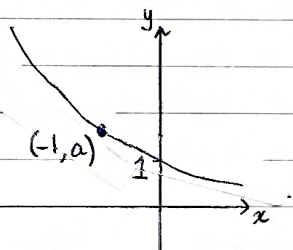
$$y = \frac{x-4}{3}$$

b) $f(x) = \frac{2}{x-1}$ and $f^{-1}(x) = \frac{x-4}{3}$

$$f(-1) = \frac{2}{(-1)-1} = \frac{2}{-2} = -1$$

$$f^{-1}(-1) = \frac{(-1)-4}{3} = -\frac{5}{3}$$

5) a) $y = a^{-x}$

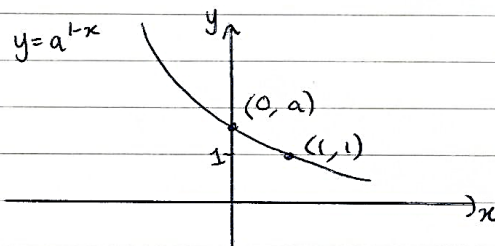


The graph has been reflected in the y-axis. This graph shows exponential decay.

b) $y = a^{1-x}$

If the question asked for $a^{-x} + 1$, then the above graph would be shifted 1 unit up. However, the 1 is inside the exponential. This means that the above graph will be shifted sideways.

When the power is zero, the exponential is 1. So for a^{1-x} , the power is zero when $1-x=0$. That is $x=1$. The basic plot $(0,1)$ has been shifted to the point $(1,1)$, so the graph has been shifted one point to the right.



6) $f(x) = 2x + 3$ and $g(x) = 2x - 3$.

a) i) $f(g(x)) = f(2x-3) = 2(2x-3) + 3$
 $= 4x - 6 + 3$
 $= 4x - 3$

ii) $g(f(x)) = g(2x+3) = 2(2x+3) - 3$
 $= 4x + 6 - 3$
 $= 4x + 3$

$$\begin{aligned}
 b) f(g(x)) \times g(f(x)) &= (4x-3)(4x+3) \\
 &= 16x^2 + 12x - 12x - 9 \\
 &= 16x^2 - 9
 \end{aligned}$$

$$7) f(x) = 2x^2 - 1 \text{ and } g(x) = 6 - x$$

First find $g(f(x))$ and the
substitute $x=3$ to get answer

$$\begin{aligned}
 g(f(x)) &= g(2x^2 - 1) \\
 &= 6 - (2x^2 - 1) \\
 &= 6 - 2x^2 + 1 \\
 &= 7 - 2x^2
 \end{aligned}$$

$$\begin{aligned}
 g(f(3)) &= 7 - 2(3)^2 \\
 &= 7 - 2(9) \\
 &= 7 - 18 \\
 &= -11
 \end{aligned}$$

Hence the correct option is A.