

Factorization of the Coefficient of x^{10}

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var('x, a1, a2, a3, a4, b1, b2, b3, b4, p1, p2, p3, p4, q1, q2, q3, q4')
w0 = SR.wild(0)
sage_server.MAX_STDOUT_SIZE=250000
sage_server.MAX_OUTPUT=250000
#This defines our variables, sets up our wild card and increases max output for\
the program.
(x, a1, a2, a3, a4, b1, b2, b3, b4, p1, p2, p3, p4, q1, q2, q3, q4)

E=(x-a1*a2*a3*a4)*(x-a1*a2*a3*b4)*(x-a1*a2*b3*a4)*(x-a1*a2*b3*b4)*(x-a1*b2*a3*\
a4)*(x-a1*b2*a3*b4)*(x-a1*b2*b3*a4)*(x-a1*b2*b3*b4)*(x-b1*a2*a3*a4)*(x-b1*a2*\
*a3*b4)*(x-b1*a2*b3*a4)*(x-b1*a2*b3*b4)*(x-b1*b2*a3*a4)*(x-b1*b2*a3*b4)*(x-\
b1*b2*b3*a4)*(x-b1*b2*b3*b4)
F=E.expand()
#Multiplication of the characteristic function, this is not show do to length.

f10=F.coefficient(x,10)
f10
#Gathering of the coefficient for x^10 from our expansion of the characteristic \
function. The general way we work this out is first seperate the coefficient \
by the numerical part of each term then seperate it further into managable \
groups that have clear subsubstitution.
a1^3*a2^4*a3^4*a4^6*b1^3*b2^2*b3^2 + a1^4*a2^3*a3^4*a4^6*b1^2*b2^3*b3^2 +
2*a1^3*a2^3*a3^4*a4^6*b1^3*b2^3*b3^2 + a1^2*a2^3*a3^4*a4^6*b1^4*b2^3*b3^2 +
a1^3*a2^2*a3^4*a4^6*b1^3*b2^4*b3^2 + a1^4*a2^4*a3^3*a4^6*b1^2*b2^2*b3^3 +
2*a1^3*a2^4*a3^3*a4^6*b1^3*b2^2*b3^3 + a1^2*a2^4*a3^3*a4^6*b1^4*b2^2*b3^3 +
2*a1^4*a2^3*a3^3*a4^6*b1^2*b2^3*b3^3 + 4*a1^3*a2^3*a3^3*a4^6*b1^3*b2^3*b3^3 +
2*a1^2*a2^3*a3^3*a4^6*b1^4*b2^3*b3^3 + a1^4*a2^2*a3^3*a4^6*b1^2*b2^4*b3^3 +
2*a1^3*a2^2*a3^3*a4^6*b1^3*b2^4*b3^3 + a1^2*a2^2*a3^3*a4^6*b1^4*b2^4*b3^3 +
a1^3*a2^4*a3^2*a4^6*b1^3*b2^2*b3^4 + a1^4*a2^3*a3^2*a4^6*b1^2*b2^3*b3^4 +
2*a1^3*a2^3*a3^2*a4^6*b1^3*b2^3*b3^4 + a1^2*a2^3*a3^2*a4^6*b1^4*b2^3*b3^4 +
a1^3*a2^2*a3^2*a4^6*b1^3*b2^4*b3^4 + a1^4*a2^4*a3^5*a4^5*b1^2*b2^2*b3*b4 +
2*a1^3*a2^4*a3^5*a4^5*b1^3*b2^2*b3*b4 + a1^2*a2^4*a3^5*a4^5*b1^4*b2^2*b3*b4 +
2*a1^4*a2^3*a3^5*a4^5*b1^2*b2^3*b3*b4 + 4*a1^3*a2^3*a3^5*a4^5*b1^3*b2^3*b3*b4 +
2*a1^2*a2^3*a3^5*a4^5*b1^4*b2^3*b3*b4 + a1^4*a2^2*a3^5*a4^5*b1^2*b2^4*b3*b4 +
2*a1^3*a2^2*a3^5*a4^5*b1^3*b2^4*b3*b4 + a1^2*a2^2*a3^5*a4^5*b1^4*b2^4*b3*b4 +
a1^4*a2^5*a3^4*a4^5*b1^2*b2*b3^2*b4 + 2*a1^3*a2^5*a3^4*a4^5*b1^3*b2*b3^2*b4 +
a1^2*a2^5*a3^4*a4^5*b1^4*b2*b3^2*b4 + a1^5*a2^4*a3^4*a4^5*b1*b2^2*b3^2*b4 +
7*a1^4*a2^4*a3^4*a4^5*b1^2*b2^2*b3^2*b4 + 12*a1^3*a2^4*a3^4*a4^5*b1^3*b2^2*b3^2*b4 +
7*a1^2*a2^4*a3^4*a4^5*b1^4*b2^2*b3^2*b4 + a1*a2^4*a3^4*a4^5*b1^5*b2^2*b3^2*b4 +
2*a1^5*a2^3*a3^4*a4^5*b1*b2^3*b3^2*b4 + 12*a1^4*a2^3*a3^4*a4^5*b1^2*b2^3*b3^2*b4 +
20*a1^3*a2^3*a3^4*a4^5*b1^3*b2^3*b3^2*b4 + 12*a1^2*a2^3*a3^4*a4^5*b1^4*b2^3*b3^2*b4 +
2*a1*a2^3*a3^4*a4^5*b1^5*b2^3*b3^2*b4 + a1^5*a2^2*a3^4*a4^5*b1*b2^4*b3^2*b4 +
7*a1^4*a2^2*a3^4*a4^5*b1^2*b2^4*b3^2*b4 + 12*a1^3*a2^2*a3^4*a4^5*b1^3*b2^4*b3^2*b4 +
7*a1^2*a2^2*a3^4*a4^5*b1^4*b2^4*b3^2*b4 + a1*a2^2*a3^4*a4^5*b1^5*b2^4*b3^2*b4 +
a1^4*a2*a3^4*a4^5*b1^2*b2^5*b3^2*b4 + 2*a1^3*a2*a3^4*a4^5*b1^3*b2^5*b3^2*b4 +
a1^2*a2*a3^4*a4^5*b1^4*b2^5*b3^2*b4 + 2*a1^4*a2^5*a3^3*a4^5*b1^2*b2*b3^3*b4 +
4*a1^3*a2^5*a3^3*a4^5*b1^3*b2*b3^3*b4 + 2*a1^2*a2^5*a3^3*a4^5*b1^4*b2*b3^3*b4 +
2*a1^5*a2^4*a3^3*a4^5*b1*b2^2*b3^3*b4 + 12*a1^4*a2^4*a3^3*a4^5*b1^2*b2^2*b3^3*b4 +
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$$\begin{aligned}
& 20*a_1^3*a_2^4*a_3^3*a_4^5*b_1^3*b_2^2*b_3^3*b_4 + 12*a_1^2*a_2^4*a_3^3*a_4^5*b_1^4*b_2^2*b_3^3*b_4 + \\
& 2*a_1*a_2^4*a_3^3*a_4^5*b_1^5*b_2^2*b_3^3*b_4 + 4*a_1^5*a_2^3*a_3^3*a_4^5*b_1*b_2^3*b_3^3*b_4 + \\
& 20*a_1^4*a_2^3*a_3^3*a_4^5*b_1^2*b_2^3*b_3^3*b_4 + 32*a_1^3*a_2^3*a_3^3*a_4^5*b_1^3*b_2^3*b_3^3*b_4 + \\
& 20*a_1^2*a_2^3*a_3^3*a_4^5*b_1^4*b_2^3*b_3^3*b_4 + 4*a_1*a_2^3*a_3^3*a_4^5*b_1^5*b_2^3*b_3^3*b_4 + \\
& 2*a_1^5*a_2^2*a_3^3*a_4^5*b_1*b_2^4*b_3^3*b_4 + 12*a_1^4*a_2^2*a_3^3*a_4^5*b_1^2*b_2^4*b_3^3*b_4 + \\
& 20*a_1^3*a_2^2*a_3^3*a_4^5*b_1^3*b_2^4*b_3^3*b_4 + 12*a_1^2*a_2^2*a_3^3*a_4^5*b_1^4*b_2^4*b_3^3*b_4 + \\
& 2*a_1*a_2^2*a_3^3*a_4^5*b_1^5*b_2^4*b_3^3*b_4 + 2*a_1^4*a_2*a_3^3*a_4^5*b_1^2*b_2^5*b_3^3*b_4 + \\
& 4*a_1^3*a_2*a_3^3*a_4^5*b_1^3*b_2^5*b_3^3*b_4 + 2*a_1^2*a_2*a_3^3*a_4^5*b_1^4*b_2^5*b_3^3*b_4 + \\
& a_1^4*a_2^5*a_3^2*a_4^5*b_1^2*b_2*b_3^4*b_4 + 2*a_1^3*a_2^5*a_3^2*a_4^5*b_1^3*b_2*b_3^4*b_4 + \\
& a_1^2*a_2^5*a_3^2*a_4^5*b_1^4*b_2*b_3^4*b_4 + a_1^5*a_2^4*a_3^2*a_4^5*b_1*b_2^2*b_3^4*b_4 + \\
& 7*a_1^4*a_2^4*a_3^2*a_4^5*b_1^2*b_2^2*b_3^4*b_4 + 12*a_1^3*a_2^4*a_3^2*a_4^5*b_1^3*b_2^2*b_3^4*b_4 + \\
& 7*a_1^2*a_2^4*a_3^2*a_4^5*b_1^4*b_2^2*b_3^4*b_4 + a_1*a_2^4*a_3^2*a_4^5*b_1^5*b_2^2*b_3^4*b_4 + \\
& 2*a_1^5*a_2^3*a_3^2*a_4^5*b_1*b_2^3*b_3^4*b_4 + 12*a_1^4*a_2^3*a_3^2*a_4^5*b_1^2*b_2^3*b_3^4*b_4 + \\
& 20*a_1^3*a_2^3*a_3^2*a_4^5*b_1^3*b_2^3*b_3^4*b_4 + 12*a_1^2*a_2^3*a_3^2*a_4^5*b_1^4*b_2^3*b_3^4*b_4 + \\
& 2*a_1*a_2^3*a_3^2*a_4^5*b_1^5*b_2^3*b_3^4*b_4 + a_1^5*a_2^2*a_3^2*a_4^5*b_1*b_2^4*b_3^4*b_4 + \\
& 7*a_1^4*a_2^2*a_3^2*a_4^5*b_1^2*b_2^4*b_3^4*b_4 + 12*a_1^3*a_2^2*a_3^2*a_4^5*b_1^3*b_2^4*b_3^4*b_4 + \\
& 7*a_1^2*a_2^2*a_3^2*a_4^5*b_1^4*b_2^4*b_3^4*b_4 + a_1*a_2^2*a_3^2*a_4^5*b_1^5*b_2^4*b_3^4*b_4 + \\
& a_1^4*a_2*a_3^2*a_4^5*b_1^2*b_2^5*b_3^4*b_4 + 2*a_1^3*a_2*a_3^2*a_4^5*b_1^3*b_2^5*b_3^4*b_4 + \\
& a_1^2*a_2*a_3^2*a_4^5*b_1^4*b_2^5*b_3^4*b_4 + a_1^4*a_2^4*a_3*a_4^5*b_1^2*b_2^2*b_3^5*b_4 + \\
& 2*a_1^3*a_2^4*a_3*a_4^5*b_1^3*b_2^2*b_3^5*b_4 + a_1^2*a_2^4*a_3*a_4^5*b_1^4*b_2^2*b_3^5*b_4 + \\
& 2*a_1^4*a_2^3*a_3*a_4^5*b_1^2*b_2^3*b_3^5*b_4 + 4*a_1^3*a_2^3*a_3*a_4^5*b_1^3*b_2^3*b_3^5*b_4 + \\
& 2*a_1^2*a_2^3*a_3*a_4^5*b_1^4*b_2^3*b_3^5*b_4 + a_1^4*a_2^2*a_3*a_4^5*b_1^2*b_2^4*b_3^5*b_4 + \\
& 2*a_1^3*a_2^2*a_3*a_4^5*b_1^3*b_2^4*b_3^5*b_4 + a_1^2*a_2^2*a_3*a_4^5*b_1^4*b_2^4*b_3^5*b_4 + \\
& a_1^3*a_2^4*a_3^6*a_4^4*b_1^3*b_2^2*b_4^2 + a_1^4*a_2^3*a_3^6*a_4^4*b_1^2*b_2^3*b_4^2 + \\
& 2*a_1^3*a_2^3*a_3^6*a_4^4*b_1^3*b_2^3*b_4^2 + a_1^2*a_2^3*a_3^6*a_4^4*b_1^4*b_2^3*b_4^2 + \\
& a_1^3*a_2^2*a_3^6*a_4^4*b_1^3*b_2^4*b_4^2 + a_1^4*a_2^5*a_3^5*a_4^4*b_1^2*b_2*b_3*b_4^2 + \\
& 2*a_1^3*a_2^5*a_3^5*a_4^4*b_1^3*b_2*b_3*b_4^2 + a_1^2*a_2^5*a_3^5*a_4^4*b_1^4*b_2*b_3*b_4^2 + \\
& a_1^5*a_2^4*a_3^5*a_4^4*b_1*b_2^2*b_3*b_4^2 + 7*a_1^4*a_2^4*a_3^5*a_4^4*b_1^2*b_2^2*b_3*b_4^2 + \\
& 12*a_1^3*a_2^4*a_3^5*a_4^4*b_1^3*b_2^2*b_3*b_4^2 + 7*a_1^2*a_2^4*a_3^5*a_4^4*b_1^4*b_2^2*b_3*b_4^2 + \\
& a_1*a_2^4*a_3^5*a_4^4*b_1^5*b_2^2*b_3*b_4^2 + 2*a_1^5*a_2^3*a_3^5*a_4^4*b_1*b_2^3*b_3*b_4^2 + \\
& 12*a_1^4*a_2^3*a_3^5*a_4^4*b_1^2*b_2^3*b_3*b_4^2 + 20*a_1^3*a_2^3*a_3^5*a_4^4*b_1^3*b_2^3*b_3*b_4^2 + \\
& 12*a_1^2*a_2^3*a_3^5*a_4^4*b_1^4*b_2^3*b_3*b_4^2 + 2*a_1*a_2^3*a_3^5*a_4^4*b_1^5*b_2^3*b_3*b_4^2 + \\
& a_1^5*a_2^2*a_3^5*a_4^4*b_1*b_2^4*b_3*b_4^2 + 7*a_1^4*a_2^2*a_3^5*a_4^4*b_1^2*b_2^4*b_3*b_4^2 + \\
& 12*a_1^3*a_2^2*a_3^5*a_4^4*b_1^3*b_2^4*b_3*b_4^2 + 7*a_1^2*a_2^2*a_3^5*a_4^4*b_1^4*b_2^4*b_3*b_4^2 + \\
& a_1*a_2^2*a_3^5*a_4^4*b_1^5*b_2^4*b_3*b_4^2 + a_1^4*a_2*a_3^5*a_4^4*b_1^2*b_2^5*b_3*b_4^2 + \\
& 2*a_1^3*a_2*a_3^5*a_4^4*b_1^3*b_2^5*b_3*b_4^2 + a_1^2*a_2*a_3^5*a_4^4*b_1^4*b_2^5*b_3*b_4^2 + \\
& a_1^3*a_2^6*a_3^4*a_4^4*b_1^3*b_3^2*b_4^2 + a_1^5*a_2^5*a_3^4*a_4^4*b_1^2*b_2*b_3^2*b_4^2 + \\
& 7*a_1^4*a_2^5*a_3^4*a_4^4*b_1^3*b_2*b_3^2*b_4^2 + 12*a_1^3*a_2^5*a_3^4*a_4^4*b_1^4*b_2*b_3^2*b_4^2 + \\
& 7*a_1^2*a_2^5*a_3^4*a_4^4*b_1^5*b_2*b_3^2*b_4^2 + a_1*a_2^5*a_3^4*a_4^4*b_1^6*b_2*b_3^2*b_4^2 + \\
& 7*a_1^5*a_2^4*a_3^4*a_4^4*b_1*b_2^2*b_3^2*b_4^2 + 28*a_1^4*a_2^4*a_3^4*a_4^4*b_1^2*b_2^2*b_3^2*b_4^2 + \\
& 46*a_1^3*a_2^4*a_3^4*a_4^4*b_1^3*b_2^2*b_3^2*b_4^2 + 28*a_1^2*a_2^4*a_3^4*a_4^4*b_1^4*b_2^2*b_3^2*b_4^2 + \\
& 7*a_1*a_2^4*a_3^4*a_4^4*b_1^5*b_2^2*b_3^2*b_4^2 + a_1^6*a_2^3*a_3^4*a_4^4*b_2^3*b_3^2*b_4^2 + \\
& 12*a_1^5*a_2^3*a_3^4*a_4^4*b_1^2*b_2^3*b_3^2*b_4^2 + 46*a_1^4*a_2^3*a_3^4*a_4^4*b_1^3*b_2^3*b_3^2*b_4^2 + \\
& 70*a_1^3*a_2^3*a_3^4*a_4^4*b_1^4*b_2^3*b_3^2*b_4^2 + 46*a_1^2*a_2^3*a_3^4*a_4^4*b_1^5*b_2^3*b_3^2*b_4^2 + \\
& 12*a_1*a_2^3*a_3^4*a_4^4*b_1^6*b_2^3*b_3^2*b_4^2 + a_2^3*a_3^4*a_4^4*b_1^6*b_2^3*b_3^2*b_4^2 + \\
& 7*a_1^5*a_2^2*a_3^4*a_4^4*b_1^2*b_2^4*b_3^2*b_4^2 + 28*a_1^4*a_2^2*a_3^4*a_4^4*b_1^3*b_2^4*b_3^2*b_4^2 + \\
& 46*a_1^3*a_2^2*a_3^4*a_4^4*b_1^4*b_2^4*b_3^2*b_4^2 + 28*a_1^2*a_2^2*a_3^4*a_4^4*b_1^5*b_2^4*b_3^2*b_4^2 + \\
& 7*a_1*a_2^2*a_3^4*a_4^4*b_1^6*b_2^4*b_3^2*b_4^2 + a_1^5*a_2^2*a_3^4*a_4^4*b_1^2*b_2^5*b_3^2*b_4^2 + \\
& 7*a_1^4*a_2^2*a_3^4*a_4^4*b_1^3*b_2^5*b_3^2*b_4^2 + 12*a_1^3*a_2^2*a_3^4*a_4^4*b_1^4*b_2^5*b_3^2*b_4^2 + \\
& 7*a_1^2*a_2^2*a_3^4*a_4^4*b_1^5*b_2^5*b_3^2*b_4^2 + a_1*a_2^2*a_3^4*a_4^4*b_1^6*b_2^5*b_3^2*b_4^2 + \\
& a_1^3*a_3^4*a_4^4*b_1^3*b_2^6*b_3^2*b_4^2 + a_1^4*a_2^6*a_3^3*a_4^4*b_1^2*b_3^3*b_4^2 + \\
& 2*a_1^3*a_2^6*a_3^3*a_4^4*b_1^3*b_3^3*b_4^2 + a_1^2*a_2^6*a_3^3*a_4^4*b_1^4*b_3^3*b_4^2 + \\
& 2*a_1^5*a_2^5*a_3^3*a_4^4*b_1^2*b_2*b_3^3*b_4^2 + 12*a_1^4*a_2^5*a_3^3*a_4^4*b_1^3*b_2*b_3^3*b_4^2 + \\
& 20*a_1^3*a_2^5*a_3^3*a_4^4*b_1^4*b_2*b_3^3*b_4^2 + 12*a_1^2*a_2^5*a_3^3*a_4^4*b_1^5*b_2*b_3^3*b_4^2 + \\
& 2*a_1*a_2^5*a_3^3*a_4^4*b_1^6*b_2^2*b_3^3*b_4^2 + a_1^6*a_2^4*a_3^3*a_4^4*b_2^2*b_3^3*b_4^2 + \\
& 12*a_1^5*a_2^4*a_3^3*a_4^4*b_1^2*b_2^2*b_3^3*b_4^2 + 46*a_1^4*a_2^4*a_3^3*a_4^4*b_1^3*b_2^2*b_3^3*b_4^2 + \\
& 70*a_1^3*a_2^4*a_3^3*a_4^4*b_1^4*b_2^2*b_3^3*b_4^2 + 46*a_1^2*a_2^4*a_3^3*a_4^4*b_1^5*b_2^2*b_3^3*b_4^2 + \\
& 12*a_1*a_2^4*a_3^3*a_4^4*b_1^6*b_2^2*b_3^3*b_4^2 + a_2^4*a_3^3*a_4^4*b_1^6*b_2^2*b_3^3*b_4^2 + \\
& 2*a_1^6*a_2^3*a_3^3*a_4^4*b_2^3*b_3^3*b_4^2 + 20*a_1^5*a_2^3*a_3^3*a_4^4*b_1^2*b_2^3*b_3^3*b_4^2 + \\
& 70*a_1^4*a_2^3*a_3^3*a_4^4*b_1^3*b_2^3*b_3^3*b_4^2 + 104*a_1^3*a_2^3*a_3^3*a_4^4*b_1^4*b_2^3*b_3^3*b_4^2 + \\
& 70*a_1^2*a_2^3*a_3^3*a_4^4*b_1^5*b_2^3*b_3^3*b_4^2 + 20*a_1*a_2^3*a_3^3*a_4^4*b_1^6*b_2^3*b_3^3*b_4^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*a^2^3*a^3^3*a^4^4*b^1^6*b^2^3*b^3^3*b^4^2 + a^1^6*a^2^2*a^3^3*a^4^4*b^2^4*b^3^3*b^4^2 + \\
& 12*a^1^5*a^2^2*a^3^3*a^4^4*b^1*b^2^4*b^3^3*b^4^2 + 46*a^1^4*a^2^2*a^3^3*a^4^4*b^1^2*b^2^4*b^3^3*b^4^2 + \\
& 70*a^1^3*a^2^2*a^3^3*a^4^4*b^1^3*b^2^4*b^3^3*b^4^2 + 46*a^1^2*a^2^2*a^3^3*a^4^4*b^1^4*b^2^4*b^3^3*b^4^2 + \\
& 12*a^1*a^2^2*a^3^3*a^4^4*b^1^5*b^2^4*b^3^3*b^4^2 + a^2^2*a^3^3*a^4^4*b^1^6*b^2^4*b^3^3*b^4^2 + \\
& 2*a^1^5*a^2*a^3^3*a^4^4*b^1*b^2^5*b^3^3*b^4^2 + 12*a^1^4*a^2*a^3^3*a^4^4*b^1^2*b^2^5*b^3^3*b^4^2 + \\
& 20*a^1^3*a^2*a^3^3*a^4^4*b^1^3*b^2^5*b^3^3*b^4^2 + 12*a^1^2*a^2*a^3^3*a^4^4*b^1^4*b^2^5*b^3^3*b^4^2 + \\
& 2*a^1*a^2*a^3^3*a^4^4*b^1^5*b^2^5*b^3^3*b^4^2 + a^1^4*a^3^3*a^4^4*b^1^2*b^2^6*b^3^3*b^4^2 + \\
& 2*a^1^3*a^3^3*a^4^4*b^1^3*b^2^6*b^3^3*b^4^2 + a^1^2*a^3^3*a^4^4*b^1^4*b^2^6*b^3^3*b^4^2 + \\
& a^1^3*a^2^6*a^3^2*a^4^4*b^1^3*b^3^4*b^4^2 + a^1^5*a^2^5*a^3^2*a^4^4*b^1*b^2*b^3^4*b^4^2 + \\
& 7*a^1^4*a^2^5*a^3^2*a^4^4*b^1^2*b^2*b^3^4*b^4^2 + 12*a^1^3*a^2^5*a^3^2*a^4^4*b^1^3*b^2*b^3^4*b^4^2 + \\
& 7*a^1^2*a^2^5*a^3^2*a^4^4*b^1^4*b^2*b^3^4*b^4^2 + a^1*a^2^5*a^3^2*a^4^4*b^1^5*b^2*b^3^4*b^4^2 + \\
& 7*a^1^5*a^2^4*a^3^2*a^4^4*b^1*b^2^2*b^3^4*b^4^2 + 28*a^1^4*a^2^4*a^3^2*a^4^4*b^1^2*b^2^2*b^3^4*b^4^2 + \\
& 46*a^1^3*a^2^4*a^3^2*a^4^4*b^1^3*b^2^2*b^3^4*b^4^2 + 28*a^1^2*a^2^4*a^3^2*a^4^4*b^1^4*b^2^2*b^3^4*b^4^2 + \\
& 7*a^1*a^2^4*a^3^2*a^4^4*b^1^5*b^2^2*b^3^4*b^4^2 + a^1^6*a^2^3*a^3^2*a^4^4*b^2^3*b^3^4*b^4^2 + \\
& 12*a^1^5*a^2^3*a^3^2*a^4^4*b^1*b^2^3*b^3^4*b^4^2 + 46*a^1^4*a^2^3*a^3^2*a^4^4*b^1^2*b^2^3*b^3^4*b^4^2 + \\
& 70*a^1^3*a^2^3*a^3^2*a^4^4*b^1^3*b^2^3*b^3^4*b^4^2 + 46*a^1^2*a^2^3*a^3^2*a^4^4*b^1^4*b^2^3*b^3^4*b^4^2 + \\
& 12*a^1*a^2^3*a^3^2*a^4^4*b^1^5*b^2^3*b^3^4*b^4^2 + a^2^3*a^3^2*a^4^4*b^1^6*b^2^3*b^3^4*b^4^2 + \\
& 7*a^1^5*a^2^2*a^3^2*a^4^4*b^1*b^2^4*b^3^4*b^4^2 + 28*a^1^4*a^2^2*a^3^2*a^4^4*b^1^2*b^2^4*b^3^4*b^4^2 + \\
& 46*a^1^3*a^2^2*a^3^2*a^4^4*b^1^3*b^2^4*b^3^4*b^4^2 + 28*a^1^2*a^2^2*a^3^2*a^4^4*b^1^4*b^2^4*b^3^4*b^4^2 + \\
& 7*a^1*a^2^2*a^3^2*a^4^4*b^1^5*b^2^4*b^3^4*b^4^2 + a^1^5*a^2^2*a^3^2*a^4^4*b^1^6*b^2^4*b^3^4*b^4^2 + \\
& 7*a^1^4*a^2^2*a^3^2*a^4^4*b^1^2*b^2^5*b^3^4*b^4^2 + 12*a^1^3*a^2^2*a^3^2*a^4^4*b^1^3*b^2^5*b^3^4*b^4^2 + \\
& 7*a^1^2*a^2^2*a^3^2*a^4^4*b^1^4*b^2^5*b^3^4*b^4^2 + a^1*a^2^2*a^3^2*a^4^4*b^1^5*b^2^5*b^3^4*b^4^2 + \\
& a^1^3*a^3^2*a^4^4*b^1^3*b^2^6*b^3^4*b^4^2 + a^1^4*a^2^5*a^3^2*a^4^4*b^1^2*b^2*b^3^5*b^4^2 + \\
& 2*a^1^3*a^2^5*a^3^2*a^4^4*b^1^3*b^2*b^3^5*b^4^2 + a^1^2*a^2^5*a^3^2*a^4^4*b^1^4*b^2*b^3^5*b^4^2 + \\
& a^1^5*a^2^4*a^3^2*a^4^4*b^1*b^2^2*b^3^5*b^4^2 + 7*a^1^4*a^2^4*a^3^2*a^4^4*b^1^2*b^2^2*b^3^5*b^4^2 + \\
& 12*a^1^3*a^2^4*a^3^2*a^4^4*b^1^3*b^2^2*b^3^5*b^4^2 + 7*a^1^2*a^2^4*a^3^2*a^4^4*b^1^4*b^2^2*b^3^5*b^4^2 + \\
& a^1*a^2^4*a^3^2*a^4^4*b^1^5*b^2^2*b^3^5*b^4^2 + 2*a^1^5*a^2^3*a^3^2*a^4^4*b^1*b^2^3*b^3^5*b^4^2 + \\
& 12*a^1^4*a^2^3*a^3^2*a^4^4*b^1^2*b^2^3*b^3^5*b^4^2 + 20*a^1^3*a^2^3*a^3^2*a^4^4*b^1^3*b^2^3*b^3^5*b^4^2 + \\
& 12*a^1^2*a^2^3*a^3^2*a^4^4*b^1^4*b^2^3*b^3^5*b^4^2 + 2*a^1*a^2^3*a^3^2*a^4^4*b^1^5*b^2^3*b^3^5*b^4^2 + \\
& a^1^5*a^2^2*a^3^2*a^4^4*b^1^6*b^2^3*b^3^5*b^4^2 + 7*a^1^4*a^2^2*a^3^2*a^4^4*b^1^2*b^2^4*b^3^5*b^4^2 + \\
& 12*a^1^3*a^2^2*a^3^2*a^4^4*b^1^3*b^2^4*b^3^5*b^4^2 + 7*a^1^2*a^2^2*a^3^2*a^4^4*b^1^4*b^2^4*b^3^5*b^4^2 + \\
& a^1*a^2^2*a^3^2*a^4^4*b^1^5*b^2^4*b^3^5*b^4^2 + a^1^4*a^2^2*a^3^2*a^4^4*b^1^2*b^2^5*b^3^5*b^4^2 + \\
& 2*a^1^3*a^2^2*a^3^2*a^4^4*b^1^3*b^2^5*b^3^5*b^4^2 + a^1^2*a^2^2*a^3^2*a^4^4*b^1^4*b^2^5*b^3^5*b^4^2 + \\
& a^1^3*a^2^4*a^4^4*b^1^3*b^2^2*b^3^6*b^4^2 + a^1^4*a^2^3*a^4^4*b^1^2*b^2^3*b^3^6*b^4^2 + \\
& 2*a^1^3*a^2^3*a^4^4*b^1^3*b^2^3*b^3^6*b^4^2 + a^1^2*a^2^3*a^4^4*b^1^4*b^2^3*b^3^6*b^4^2 + \\
& a^1^3*a^2^2*a^4^4*b^1^3*b^2^4*b^3^6*b^4^2 + a^1^4*a^2^4*a^3^6*a^4^3*b^1^2*b^2^2*b^4^3 + \\
& 2*a^1^3*a^2^4*a^3^6*a^4^3*b^1^3*b^2^2*b^4^3 + a^1^2*a^2^4*a^3^6*a^4^3*b^1^4*b^2^2*b^4^3 + \\
& 2*a^1^4*a^2^3*a^3^6*a^4^3*b^1^2*b^2^3*b^4^3 + 4*a^1^3*a^2^3*a^3^6*a^4^3*b^1^3*b^2^3*b^4^3 + \\
& 2*a^1^2*a^2^3*a^3^6*a^4^3*b^1^4*b^2^3*b^4^3 + a^1^4*a^2^2*a^3^6*a^4^3*b^1^2*b^2^4*b^4^3 + \\
& 2*a^1^3*a^2^2*a^3^6*a^4^3*b^1^3*b^2^4*b^4^3 + a^1^2*a^2^2*a^3^6*a^4^3*b^1^4*b^2^4*b^4^3 + \\
& 2*a^1^4*a^2^5*a^3^5*a^4^3*b^1^2*b^2*b^3*b^4^3 + 4*a^1^3*a^2^5*a^3^5*a^4^3*b^1^3*b^2*b^3*b^4^3 + \\
& 2*a^1^2*a^2^5*a^3^5*a^4^3*b^1^4*b^2*b^3*b^4^3 + 2*a^1^5*a^2^4*a^3^5*a^4^3*b^1*b^2^2*b^3*b^4^3 + \\
& 12*a^1^4*a^2^4*a^3^5*a^4^3*b^1^2*b^2^2*b^3*b^4^3 + 20*a^1^3*a^2^4*a^3^5*a^4^3*b^1^3*b^2^2*b^3*b^4^3 + \\
& 12*a^1^2*a^2^4*a^3^5*a^4^3*b^1^4*b^2^2*b^3*b^4^3 + 2*a^1*a^2^4*a^3^5*a^4^3*b^1^5*b^2^2*b^3*b^4^3 + \\
& 4*a^1^5*a^2^3*a^3^5*a^4^3*b^1*b^2^3*b^3*b^4^3 + 20*a^1^4*a^2^3*a^3^5*a^4^3*b^1^2*b^2^3*b^3*b^4^3 + \\
& 32*a^1^3*a^2^3*a^3^5*a^4^3*b^1^3*b^2^3*b^3*b^4^3 + 20*a^1^2*a^2^3*a^3^5*a^4^3*b^1^4*b^2^3*b^3*b^4^3 + \\
& 4*a^1*a^2^3*a^3^5*a^4^3*b^1^5*b^2^3*b^3*b^4^3 + 2*a^1^5*a^2^2*a^3^5*a^4^3*b^1^6*b^2^3*b^3*b^4^3 + \\
& 12*a^1^4*a^2^2*a^3^5*a^4^3*b^1^2*b^2^4*b^3*b^4^3 + 20*a^1^3*a^2^2*a^3^5*a^4^3*b^1^3*b^2^4*b^3*b^4^3 + \\
& 12*a^1^2*a^2^2*a^3^5*a^4^3*b^1^4*b^2^4*b^3*b^4^3 + 2*a^1*a^2^2*a^3^5*a^4^3*b^1^5*b^2^4*b^3*b^4^3 + \\
& 2*a^1^4*a^2^2*a^3^5*a^4^3*b^1^2*b^2^5*b^3*b^4^3 + 4*a^1^3*a^2^2*a^3^5*a^4^3*b^1^3*b^2^5*b^3*b^4^3 + \\
& 2*a^1^2*a^2^2*a^3^5*a^4^3*b^1^4*b^2^5*b^3*b^4^3 + a^1^4*a^2^6*a^3^4*a^4^3*b^1^2*b^3^2*b^4^3 + \\
& 2*a^1^3*a^2^6*a^3^4*a^4^3*b^1^3*b^3^2*b^4^3 + a^1^2*a^2^6*a^3^4*a^4^3*b^1^4*b^3^2*b^4^3 + \\
& 2*a^1^5*a^2^5*a^3^4*a^4^3*b^1*b^2*b^3^2*b^4^3 + 12*a^1^4*a^2^5*a^3^4*a^4^3*b^1^2*b^2*b^3^2*b^4^3 + \\
& 20*a^1^3*a^2^5*a^3^4*a^4^3*b^1^3*b^2*b^3^2*b^4^3 + 12*a^1^2*a^2^5*a^3^4*a^4^3*b^1^4*b^2*b^3^2*b^4^3 + \\
& 2*a^1*a^2^5*a^3^4*a^4^3*b^1^5*b^2*b^3^2*b^4^3 + a^1^6*a^2^4*a^3^4*a^4^3*b^2^2*b^3^2*b^4^3 + \\
& 12*a^1^5*a^2^4*a^3^4*a^4^3*b^1^2*b^2^2*b^3^2*b^4^3 + 46*a^1^4*a^2^4*a^3^4*a^4^3*b^1^3*b^2^2*b^3^2*b^4^3 + \\
& 70*a^1^3*a^2^4*a^3^4*a^4^3*b^1^4*b^2^2*b^3^2*b^4^3 + 46*a^1^2*a^2^4*a^3^4*a^4^3*b^1^5*b^2^2*b^3^2*b^4^3 + \\
& 12*a^1*a^2^4*a^3^4*a^4^3*b^1^6*b^2^2*b^3^2*b^4^3 + 2*a^1^6*a^2^3*a^3^4*a^4^3*b^1^2*b^3^2*b^4^3 + \\
& 70*a^1^4*a^2^3*a^3^4*a^4^3*b^1^3*b^2^3*b^3^2*b^4^3 + 104*a^1^3*a^2^3*a^3^4*a^4^3*b^1^4*b^2^3*b^3^2*b^4^3 + \\
& 70*a^1^2*a^2^3*a^3^4*a^4^3*b^1^5*b^2^3*b^3^2*b^4^3 + 20*a^1*a^2^3*a^3^4*a^4^3*b^1^6*b^2^3*b^3^2*b^4^3 +
\end{aligned}$$

$$\begin{aligned}
& 2*a^2^3*a^3^4*a^4^3*b^1^6*b^2^3*b^3^2*b^4^3 + a^1^6*a^2^2*a^3^4*a^4^3*b^2^4*b^3^2*b^4^3 + \\
& 12*a^1^5*a^2^2*a^3^4*a^4^3*b^1*b^2^4*b^3^2*b^4^3 + 46*a^1^4*a^2^2*a^3^4*a^4^3*b^1^2*b^2^4*b^3^2*b^4^3 + \\
& 70*a^1^3*a^2^2*a^3^4*a^4^3*b^1^3*b^2^4*b^3^2*b^4^3 + 46*a^1^2*a^2^2*a^3^4*a^4^3*b^1^4*b^2^4*b^3^2*b^4^3 + \\
& 12*a^1*a^2^2*a^3^4*a^4^3*b^1^5*b^2^4*b^3^2*b^4^3 + a^2^2*a^3^4*a^4^3*b^1^6*b^2^4*b^3^2*b^4^3 + \\
& 2*a^1^5*a^2*a^3^4*a^4^3*b^1*b^2^5*b^3^2*b^4^3 + 12*a^1^4*a^2*a^3^4*a^4^3*b^1^2*b^2^5*b^3^2*b^4^3 + \\
& 20*a^1^3*a^2*a^3^4*a^4^3*b^1^3*b^2^5*b^3^2*b^4^3 + 12*a^1^2*a^2*a^3^4*a^4^3*b^1^4*b^2^5*b^3^2*b^4^3 + \\
& 2*a^1*a^2*a^3^4*a^4^3*b^1^5*b^2^5*b^3^2*b^4^3 + a^1^4*a^3^4*a^4^3*b^1^2*b^2^6*b^3^2*b^4^3 + \\
& 2*a^1^3*a^3^4*a^4^3*b^1^3*b^2^6*b^3^2*b^4^3 + a^1^2*a^3^4*a^4^3*b^1^4*b^2^6*b^3^2*b^4^3 + \\
& 2*a^1^4*a^2^6*a^3^3*a^4^3*b^1^2*b^3^3*b^4^3 + 4*a^1^3*a^2^6*a^3^3*a^4^3*b^1^3*b^3^3*b^4^3 + \\
& 2*a^1^2*a^2^6*a^3^3*a^4^3*b^1^4*b^3^3*b^4^3 + 4*a^1^5*a^2^5*a^3^3*a^4^3*b^1*b^2*b^3^3*b^4^3 + \\
& 20*a^1^4*a^2^5*a^3^3*a^4^3*b^1^2*b^2*b^3^3*b^4^3 + 32*a^1^3*a^2^5*a^3^3*a^4^3*b^1^3*b^2*b^3^3*b^4^3 + \\
& 20*a^1^2*a^2^5*a^3^3*a^4^3*b^1^4*b^2*b^3^3*b^4^3 + 4*a^1*a^2^5*a^3^3*a^4^3*b^1^5*b^2*b^3^3*b^4^3 + \\
& 2*a^1^6*a^2^4*a^3^3*a^4^3*b^2^2*b^3^3*b^4^3 + 20*a^1^5*a^2^4*a^3^3*a^4^3*b^1*b^2^2*b^3^3*b^4^3 + \\
& 70*a^1^4*a^2^4*a^3^3*a^4^3*b^1^2*b^2^2*b^3^3*b^4^3 + 104*a^1^3*a^2^4*a^3^3*a^4^3*b^1^3*b^2^2*b^3^3*b^4^3 + \\
& 70*a^1^2*a^2^4*a^3^3*a^4^3*b^1^4*b^2^2*b^3^3*b^4^3 + 20*a^1*a^2^4*a^3^3*a^4^3*b^1^5*b^2^2*b^3^3*b^4^3 + \\
& 2*a^2^4*a^3^3*a^4^3*b^1^6*b^2^2*b^3^3*b^4^3 + 4*a^1^6*a^2^3*a^3^3*a^4^3*b^2^3*b^3^3*b^4^3 + \\
& 32*a^1^5*a^2^3*a^3^3*a^4^3*b^1*b^2^3*b^3^3*b^4^3 + 104*a^1^4*a^2^3*a^3^3*a^4^3*b^1^2*b^2^3*b^3^3*b^4^3 + \\
& 152*a^1^3*a^2^3*a^3^3*a^4^3*b^1^3*b^2^3*b^3^3*b^4^3 + 104*a^1^2*a^2^3*a^3^3*a^4^3*b^1^4*b^2^3*b^3^3*b^4^3 + \\
& + 32*a^1*a^2^3*a^3^3*a^4^3*b^1^5*b^2^3*b^3^3*b^4^3 + 4*a^2^3*a^3^3*a^4^3*b^1^6*b^2^3*b^3^3*b^4^3 + \\
& 2*a^1^6*a^2^2*a^3^3*a^4^3*b^2^4*b^3^3*b^4^3 + 20*a^1^5*a^2^2*a^3^3*a^4^3*b^1*b^2^4*b^3^3*b^4^3 + \\
& 70*a^1^4*a^2^2*a^3^3*a^4^3*b^1^2*b^2^4*b^3^3*b^4^3 + 104*a^1^3*a^2^2*a^3^3*a^4^3*b^1^3*b^2^4*b^3^3*b^4^3 + \\
& 70*a^1^2*a^2^2*a^3^3*a^4^3*b^1^4*b^2^4*b^3^3*b^4^3 + 20*a^1*a^2^2*a^3^3*a^4^3*b^1^5*b^2^4*b^3^3*b^4^3 + \\
& 2*a^2^2*a^3^3*a^4^3*b^1^6*b^2^4*b^3^3*b^4^3 + 4*a^1^5*a^2*a^3^3*a^4^3*b^1*b^2^5*b^3^3*b^4^3 + \\
& 20*a^1^4*a^2*a^3^3*a^4^3*b^1^2*b^2^5*b^3^3*b^4^3 + 32*a^1^3*a^2*a^3^3*a^4^3*b^1^3*b^2^5*b^3^3*b^4^3 + \\
& 20*a^1^2*a^2*a^3^3*a^4^3*b^1^4*b^2^5*b^3^3*b^4^3 + 4*a^1*a^2*a^3^3*a^4^3*b^1^5*b^2^5*b^3^3*b^4^3 + \\
& 2*a^1^4*a^3^3*a^4^3*b^1^2*b^2^6*b^3^3*b^4^3 + 4*a^1^3*a^3^3*a^4^3*b^1^3*b^2^6*b^3^3*b^4^3 + \\
& 2*a^1^2*a^3^3*a^4^3*b^1^4*b^2^6*b^3^3*b^4^3 + a^1^4*a^2^6*a^3^2*a^4^3*b^1^2*b^3^4*b^4^3 + \\
& 2*a^1^3*a^2^6*a^3^2*a^4^3*b^1^3*b^3^4*b^4^3 + a^1^2*a^2^6*a^3^2*a^4^3*b^1^4*b^3^4*b^4^3 + \\
& 2*a^1^5*a^2^5*a^3^2*a^4^3*b^1*b^2*b^3^4*b^4^3 + 12*a^1^4*a^2^5*a^3^2*a^4^3*b^1^2*b^2*b^3^4*b^4^3 + \\
& 20*a^1^3*a^2^5*a^3^2*a^4^3*b^1^3*b^2*b^3^4*b^4^3 + 12*a^1^2*a^2^5*a^3^2*a^4^3*b^1^4*b^2*b^3^4*b^4^3 + \\
& 2*a^1*a^2^5*a^3^2*a^4^3*b^1^5*b^2*b^3^4*b^4^3 + a^1^6*a^2^4*a^3^2*a^4^3*b^2^2*b^3^4*b^4^3 + \\
& 12*a^1^5*a^2^4*a^3^2*a^4^3*b^1*b^2^2*b^3^4*b^4^3 + 46*a^1^4*a^2^4*a^3^2*a^4^3*b^1^2*b^2^2*b^3^4*b^4^3 + \\
& 70*a^1^3*a^2^4*a^3^2*a^4^3*b^1^3*b^2^2*b^3^4*b^4^3 + 46*a^1^2*a^2^4*a^3^2*a^4^3*b^1^4*b^2^2*b^3^4*b^4^3 + \\
& 12*a^1*a^2^4*a^3^2*a^4^3*b^1^5*b^2^2*b^3^4*b^4^3 + a^2^4*a^3^2*a^4^3*b^1^6*b^2^2*b^3^4*b^4^3 + \\
& 2*a^1^6*a^2^3*a^3^2*a^4^3*b^2^3*b^3^4*b^4^3 + 20*a^1^5*a^2^3*a^3^2*a^4^3*b^1*b^2^3*b^3^4*b^4^3 + \\
& 70*a^1^4*a^2^3*a^3^2*a^4^3*b^1^2*b^2^3*b^3^4*b^4^3 + 104*a^1^3*a^2^3*a^3^2*a^4^3*b^1^3*b^2^3*b^3^4*b^4^3 + \\
& 70*a^1^2*a^2^3*a^3^2*a^4^3*b^1^4*b^2^3*b^3^4*b^4^3 + 20*a^1*a^2^3*a^3^2*a^4^3*b^1^5*b^2^3*b^3^4*b^4^3 + \\
& 2*a^2^3*a^3^2*a^4^3*b^1^6*b^2^3*b^3^4*b^4^3 + a^1^6*a^2^2*a^3^2*a^4^3*b^2^4*b^3^4*b^4^3 + \\
& 12*a^1^5*a^2^2*a^3^2*a^4^3*b^1*b^2^4*b^3^4*b^4^3 + 46*a^1^4*a^2^2*a^3^2*a^4^3*b^1^2*b^2^4*b^3^4*b^4^3 + \\
& 70*a^1^3*a^2^2*a^3^2*a^4^3*b^1^3*b^2^4*b^3^4*b^4^3 + 46*a^1^2*a^2^2*a^3^2*a^4^3*b^1^4*b^2^4*b^3^4*b^4^3 + \\
& 12*a^1*a^2^2*a^3^2*a^4^3*b^1^5*b^2^4*b^3^4*b^4^3 + a^2^2*a^3^2*a^4^3*b^1^6*b^2^4*b^3^4*b^4^3 + \\
& 2*a^1^5*a^2*a^3^2*a^4^3*b^1*b^2^5*b^3^4*b^4^3 + 12*a^1^4*a^2*a^3^2*a^4^3*b^1^2*b^2^5*b^3^4*b^4^3 + \\
& 20*a^1^3*a^2*a^3^2*a^4^3*b^1^3*b^2^5*b^3^4*b^4^3 + 12*a^1^2*a^2*a^3^2*a^4^3*b^1^4*b^2^5*b^3^4*b^4^3 + \\
& 2*a^1*a^2*a^3^2*a^4^3*b^1^5*b^2^5*b^3^4*b^4^3 + a^1^4*a^3^2*a^4^3*b^1^2*b^2^6*b^3^4*b^4^3 + \\
& 2*a^1^3*a^3^2*a^4^3*b^1^3*b^2^6*b^3^4*b^4^3 + a^1^2*a^3^2*a^4^3*b^1^4*b^2^6*b^3^4*b^4^3 + \\
& 2*a^1^4*a^2^5*a^3*a^4^3*b^1^2*b^2*b^3^5*b^4^3 + 4*a^1^3*a^2^5*a^3*a^4^3*b^1^3*b^2*b^3^5*b^4^3 + \\
& 2*a^1^2*a^2^5*a^3*a^4^3*b^1^4*b^2*b^3^5*b^4^3 + 2*a^1^5*a^2^4*a^3*a^4^3*b^1*b^2^2*b^3^5*b^4^3 + \\
& 12*a^1^4*a^2^4*a^3*a^4^3*b^1^2*b^2^2*b^3^5*b^4^3 + 20*a^1^3*a^2^4*a^3*a^4^3*b^1^3*b^2^2*b^3^5*b^4^3 + \\
& 12*a^1^2*a^2^4*a^3*a^4^3*b^1^4*b^2^2*b^3^5*b^4^3 + 2*a^1*a^2^4*a^3*a^4^3*b^1^5*b^2^2*b^3^5*b^4^3 + \\
& 4*a^1^5*a^2^3*a^3*a^4^3*b^1*b^2^3*b^3^5*b^4^3 + 20*a^1^4*a^2^3*a^3*a^4^3*b^1^2*b^2^3*b^3^5*b^4^3 + \\
& 32*a^1^3*a^2^3*a^3*a^4^3*b^1^3*b^2^3*b^3^5*b^4^3 + 20*a^1^2*a^2^3*a^3*a^4^3*b^1^4*b^2^3*b^3^5*b^4^3 + \\
& 4*a^1*a^2^3*a^3*a^4^3*b^1^5*b^2^3*b^3^5*b^4^3 + 2*a^1^5*a^2^2*a^3*a^4^3*b^1*b^2^4*b^3^5*b^4^3 + \\
& 12*a^1^4*a^2^2*a^3*a^4^3*b^1^2*b^2^4*b^3^5*b^4^3 + 20*a^1^3*a^2^2*a^3*a^4^3*b^1^3*b^2^4*b^3^5*b^4^3 + \\
& 12*a^1^2*a^2^2*a^3*a^4^3*b^1^4*b^2^4*b^3^5*b^4^3 + 2*a^1*a^2^2*a^3*a^4^3*b^1^5*b^2^4*b^3^5*b^4^3 + \\
& 2*a^1^4*a^2*a^3*a^4^3*b^1^2*b^2^5*b^3^5*b^4^3 + 4*a^1^3*a^2*a^3*a^4^3*b^1^3*b^2^5*b^3^5*b^4^3 + \\
& 2*a^1^2*a^2*a^3*a^4^3*b^1^4*b^2^5*b^3^5*b^4^3 + a^1^4*a^2^4*a^4^3*b^1^2*b^2^2*b^3^6*b^4^3 + \\
& 2*a^1^3*a^2^4*a^4^3*b^1^3*b^2^2*b^3^6*b^4^3 + a^1^2*a^2^4*a^4^3*b^1^4*b^2^2*b^3^6*b^4^3 + \\
& 2*a^1^4*a^2^3*a^4^3*b^1^2*b^2^3*b^3^6*b^4^3 + 4*a^1^3*a^2^3*a^4^3*b^1^3*b^2^3*b^3^6*b^4^3 + \\
& 2*a^1^2*a^2^3*a^4^3*b^1^4*b^2^3*b^3^6*b^4^3 + a^1^4*a^2^2*a^4^3*b^1^2*b^2^4*b^3^6*b^4^3 + \\
& 2*a^1^3*a^2^2*a^4^3*b^1^3*b^2^4*b^3^6*b^4^3 + a^1^2*a^2^2*a^4^3*b^1^4*b^2^4*b^3^6*b^4^3 + \\
& a^1^3*a^2^4*a^3^6*a^4^2*b^1^3*b^2^2*b^4^4 + a^1^4*a^2^3*a^3^6*a^4^2*b^1^2*b^2^3*b^4^4 +
\end{aligned}$$

$$\begin{aligned}
& a_1^5 a_2^4 a_3^4 a_4^2 b_1^2 b_2^2 b_3^5 b_4^4 + 7 a_1^4 a_2^4 a_3^4 a_4^2 b_1^2 b_2^2 b_3^5 b_4^4 + \\
& 12 a_1^3 a_2^4 a_3^4 a_4^2 b_1^3 b_2^2 b_3^5 b_4^4 + 7 a_1^2 a_2^4 a_3^4 a_4^2 b_1^4 b_2^2 b_3^5 b_4^4 + \\
& a_1 a_2^4 a_3^4 a_4^2 b_1^5 b_2^2 b_3^5 b_4^4 + 2 a_1^5 a_2^3 a_3^4 a_4^2 b_1^2 b_2^3 b_3^5 b_4^4 + \\
& 12 a_1^4 a_2^3 a_3^4 a_4^2 b_1^3 b_2^3 b_3^5 b_4^4 + 20 a_1^3 a_2^3 a_3^4 a_4^2 b_1^3 b_2^3 b_3^5 b_4^4 + \\
& 12 a_1^2 a_2^3 a_3^4 a_4^2 b_1^4 b_2^3 b_3^5 b_4^4 + 2 a_1 a_2^3 a_3^4 a_4^2 b_1^5 b_2^3 b_3^5 b_4^4 + \\
& a_1^5 a_2^2 a_3^4 a_4^2 b_1^2 b_2^4 b_3^5 b_4^4 + 7 a_1^4 a_2^2 a_3^4 a_4^2 b_1^2 b_2^4 b_3^5 b_4^4 + \\
& 12 a_1^3 a_2^2 a_3^4 a_4^2 b_1^3 b_2^4 b_3^5 b_4^4 + 7 a_1^2 a_2^2 a_3^4 a_4^2 b_1^4 b_2^4 b_3^5 b_4^4 + \\
& a_1 a_2^2 a_3^4 a_4^2 b_1^5 b_2^4 b_3^5 b_4^4 + a_1^4 a_2 a_3^4 a_4^2 b_1^2 b_2^5 b_3^5 b_4^4 + \\
& 2 a_1^3 a_2 a_3^4 a_4^2 b_1^3 b_2^5 b_3^5 b_4^4 + a_1^2 a_2 a_3^4 a_4^2 b_1^4 b_2^5 b_3^5 b_4^4 + \\
& a_1^3 a_2^4 a_4^2 b_1^3 b_2^2 b_3^6 b_4^4 + a_1^4 a_2^3 a_4^2 b_1^2 b_2^3 b_3^6 b_4^4 + \\
& 2 a_1^3 a_2^3 a_4^2 b_1^3 b_2^3 b_3^6 b_4^4 + a_1^2 a_2^3 a_4^2 b_1^4 b_2^3 b_3^6 b_4^4 + \\
& a_1^3 a_2^2 a_4^2 b_1^3 b_2^4 b_3^6 b_4^4 + a_1^4 a_2^4 a_3^5 a_4 b_1^2 b_2^2 b_3 b_4^5 + \\
& 2 a_1^3 a_2^4 a_3^5 a_4 b_1^3 b_2^2 b_3 b_4^5 + a_1^2 a_2^4 a_3^5 a_4 b_1^4 b_2^2 b_3 b_4^5 + \\
& 2 a_1^4 a_2^3 a_3^5 a_4 b_1^2 b_2^3 b_3 b_4^5 + 4 a_1^3 a_2^3 a_3^5 a_4 b_1^3 b_2^3 b_3 b_4^5 + \\
& 2 a_1^2 a_2^3 a_3^5 a_4 b_1^4 b_2^3 b_3 b_4^5 + a_1^4 a_2^2 a_3^5 a_4 b_1^2 b_2^4 b_3 b_4^5 + \\
& 2 a_1^3 a_2^2 a_3^5 a_4 b_1^3 b_2^4 b_3 b_4^5 + a_1^2 a_2^2 a_3^5 a_4 b_1^4 b_2^4 b_3 b_4^5 + \\
& a_1^4 a_2^5 a_3^4 a_4 b_1^2 b_2 b_3^2 b_4^5 + 2 a_1^3 a_2^5 a_3^4 a_4 b_1^3 b_2 b_3^2 b_4^5 + \\
& a_1^2 a_2^5 a_3^4 a_4 b_1^4 b_2 b_3^2 b_4^5 + a_1^5 a_2^4 a_3^4 a_4 b_1 b_2^2 b_3^2 b_4^5 + \\
& 7 a_1^4 a_2^4 a_3^4 a_4 b_1^2 b_2^2 b_3^2 b_4^5 + 12 a_1^3 a_2^4 a_3^4 a_4 b_1^3 b_2^2 b_3^2 b_4^5 + \\
& 7 a_1^2 a_2^4 a_3^4 a_4 b_1^4 b_2^2 b_3^2 b_4^5 + a_1 a_2^4 a_3^4 a_4 b_1^5 b_2^2 b_3^2 b_4^5 + \\
& 2 a_1^5 a_2^3 a_3^4 a_4 b_1 b_2^3 b_3^2 b_4^5 + 12 a_1^4 a_2^3 a_3^4 a_4 b_1^2 b_2^3 b_3^2 b_4^5 + \\
& 20 a_1^3 a_2^3 a_3^4 a_4 b_1^3 b_2^3 b_3^2 b_4^5 + 12 a_1^2 a_2^3 a_3^4 a_4 b_1^4 b_2^3 b_3^2 b_4^5 + \\
& 2 a_1 a_2^3 a_3^4 a_4 b_1^5 b_2^3 b_3^2 b_4^5 + a_1^5 a_2^2 a_3^4 a_4 b_1 b_2^4 b_3^2 b_4^5 + \\
& 7 a_1^4 a_2^2 a_3^4 a_4 b_1^2 b_2^4 b_3^2 b_4^5 + 12 a_1^3 a_2^2 a_3^4 a_4 b_1^3 b_2^4 b_3^2 b_4^5 + \\
& 7 a_1^2 a_2^2 a_3^4 a_4 b_1^4 b_2^4 b_3^2 b_4^5 + a_1 a_2^2 a_3^4 a_4 b_1^5 b_2^4 b_3^2 b_4^5 + \\
& a_1^4 a_2 a_3^4 a_4 b_1^2 b_2^5 b_3^2 b_4^5 + 2 a_1^3 a_2 a_3^4 a_4 b_1^3 b_2^5 b_3^2 b_4^5 + \\
& a_1^2 a_2 a_3^4 a_4 b_1^4 b_2^5 b_3^2 b_4^5 + 2 a_1^4 a_2^5 a_3^3 a_4 b_1^2 b_2 b_3^3 b_4^5 + \\
& 4 a_1^3 a_2^5 a_3^3 a_4 b_1^3 b_2 b_3^3 b_4^5 + 2 a_1^2 a_2^5 a_3^3 a_4 b_1^4 b_2 b_3^3 b_4^5 + \\
& 2 a_1^5 a_2^4 a_3^3 a_4 b_1 b_2^2 b_3^3 b_4^5 + 12 a_1^4 a_2^4 a_3^3 a_4 b_1^2 b_2^2 b_3^3 b_4^5 + \\
& 20 a_1^3 a_2^4 a_3^3 a_4 b_1^3 b_2^2 b_3^3 b_4^5 + 12 a_1^2 a_2^4 a_3^3 a_4 b_1^4 b_2^2 b_3^3 b_4^5 + \\
& 2 a_1 a_2^4 a_3^3 a_4 b_1^5 b_2^2 b_3^3 b_4^5 + 4 a_1^5 a_2^3 a_3^3 a_4 b_1 b_2^3 b_3^3 b_4^5 + \\
& 20 a_1^4 a_2^3 a_3^3 a_4 b_1^2 b_2^3 b_3^3 b_4^5 + 32 a_1^3 a_2^3 a_3^3 a_4 b_1^3 b_2^3 b_3^3 b_4^5 + \\
& 20 a_1^2 a_2^3 a_3^3 a_4 b_1^4 b_2^3 b_3^3 b_4^5 + 4 a_1 a_2^3 a_3^3 a_4 b_1^5 b_2^3 b_3^3 b_4^5 + \\
& 2 a_1^5 a_2^2 a_3^3 a_4 b_1 b_2^4 b_3^3 b_4^5 + 12 a_1^4 a_2^2 a_3^3 a_4 b_1^2 b_2^4 b_3^3 b_4^5 + \\
& 20 a_1^3 a_2^2 a_3^3 a_4 b_1^3 b_2^4 b_3^3 b_4^5 + 12 a_1^2 a_2^2 a_3^3 a_4 b_1^4 b_2^4 b_3^3 b_4^5 + \\
& 2 a_1 a_2^2 a_3^3 a_4 b_1^5 b_2^4 b_3^3 b_4^5 + 2 a_1^4 a_2 a_3^3 a_4 b_1^2 b_2^5 b_3^3 b_4^5 + \\
& 4 a_1^3 a_2 a_3^3 a_4 b_1^3 b_2^5 b_3^3 b_4^5 + 2 a_1^2 a_2 a_3^3 a_4 b_1^4 b_2^5 b_3^3 b_4^5 + \\
& a_1^4 a_2^5 a_3^2 a_4 b_1^2 b_2 b_3^4 b_4^5 + 2 a_1^3 a_2^5 a_3^2 a_4 b_1^3 b_2 b_3^4 b_4^5 + \\
& a_1^2 a_2^5 a_3^2 a_4 b_1^4 b_2 b_3^4 b_4^5 + a_1^5 a_2^4 a_3^2 a_4 b_1 b_2^2 b_3^4 b_4^5 + \\
& 7 a_1^4 a_2^4 a_3^2 a_4 b_1^2 b_2^2 b_3^4 b_4^5 + 12 a_1^3 a_2^4 a_3^2 a_4 b_1^3 b_2^2 b_3^4 b_4^5 + \\
& 7 a_1^2 a_2^4 a_3^2 a_4 b_1^4 b_2^2 b_3^4 b_4^5 + a_1 a_2^4 a_3^2 a_4 b_1^5 b_2^2 b_3^4 b_4^5 + \\
& 2 a_1^5 a_2^3 a_3^2 a_4 b_1 b_2^3 b_3^4 b_4^5 + 12 a_1^4 a_2^3 a_3^2 a_4 b_1^2 b_2^3 b_3^4 b_4^5 + \\
& 20 a_1^3 a_2^3 a_3^2 a_4 b_1^3 b_2^3 b_3^4 b_4^5 + 12 a_1^2 a_2^3 a_3^2 a_4 b_1^4 b_2^3 b_3^4 b_4^5 + \\
& 2 a_1 a_2^3 a_3^2 a_4 b_1^5 b_2^3 b_3^4 b_4^5 + a_1^5 a_2^2 a_3^2 a_4 b_1 b_2^4 b_3^4 b_4^5 + \\
& 7 a_1^4 a_2^2 a_3^2 a_4 b_1^2 b_2^4 b_3^4 b_4^5 + 12 a_1^3 a_2^2 a_3^2 a_4 b_1^3 b_2^4 b_3^4 b_4^5 + \\
& 7 a_1^2 a_2^2 a_3^2 a_4 b_1^4 b_2^4 b_3^4 b_4^5 + a_1 a_2^2 a_3^2 a_4 b_1^5 b_2^4 b_3^4 b_4^5 + \\
& a_1^4 a_2 a_3^2 a_4 b_1^2 b_2^5 b_3^4 b_4^5 + 2 a_1^3 a_2 a_3^2 a_4 b_1^3 b_2^5 b_3^4 b_4^5 + \\
& a_1^2 a_2 a_3^2 a_4 b_1^4 b_2^5 b_3^4 b_4^5 + a_1^4 a_2^4 a_3 a_4 b_1^2 b_2^2 b_3^5 b_4^5 + \\
& 2 a_1^3 a_2^4 a_3 a_4 b_1^3 b_2^2 b_3^5 b_4^5 + a_1^2 a_2^4 a_3 a_4 b_1^4 b_2^2 b_3^5 b_4^5 + \\
& 2 a_1^4 a_2^3 a_3 a_4 b_1^2 b_2^3 b_3^5 b_4^5 + 4 a_1^3 a_2^3 a_3 a_4 b_1^3 b_2^3 b_3^5 b_4^5 + \\
& 2 a_1^2 a_2^3 a_3 a_4 b_1^4 b_2^3 b_3^5 b_4^5 + a_1^4 a_2^2 a_3 a_4 b_1^2 b_2^4 b_3^5 b_4^5 + \\
& 2 a_1^3 a_2^2 a_3 a_4 b_1^3 b_2^4 b_3^5 b_4^5 + a_1^2 a_2^2 a_3 a_4 b_1^4 b_2^4 b_3^5 b_4^5 + \\
& a_1^3 a_2^4 a_3^4 b_1^3 b_2^2 b_3^2 b_4^6 + a_1^4 a_2^3 a_3^4 b_1^2 b_2^3 b_3^2 b_4^6 + \\
& 2 a_1^3 a_2^3 a_3^4 b_1^3 b_2^3 b_3^2 b_4^6 + a_1^2 a_2^3 a_3^4 b_1^4 b_2^3 b_3^2 b_4^6 + \\
& 2 a_1^4 a_2^2 a_3^3 b_1^3 b_2^2 b_3^3 b_4^6 + a_1^2 a_2^4 a_3^3 b_1^4 b_2^2 b_3^3 b_4^6 + \\
& 2 a_1^3 a_2^4 a_3^3 b_1^2 b_2^3 b_3^3 b_4^6 + a_1^4 a_2^3 a_3^3 b_1^3 b_2^3 b_3^3 b_4^6 + \\
& 2 a_1^2 a_2^3 a_3^3 b_1^4 b_2^3 b_3^3 b_4^6 + 4 a_1^3 a_2^3 a_3^3 b_1^3 b_2^3 b_3^3 b_4^6 + \\
& 2 a_1^2 a_2^3 a_3^3 b_1^4 b_2^3 b_3^3 b_4^6 + a_1^4 a_2^2 a_3^3 b_1^2 b_2^4 b_3^3 b_4^6 + \\
& 2 a_1^3 a_2^2 a_3^3 b_1^3 b_2^4 b_3^3 b_4^6 + a_1^2 a_2^2 a_3^3 b_1^4 b_2^4 b_3^3 b_4^6 + \\
& a_1^3 a_2^4 a_3^2 b_1^3 b_2^2 b_3^4 b_4^6 + a_1^4 a_2^3 a_3^2 b_1^2 b_2^3 b_3^4 b_4^6 + \\
& 2 a_1^3 a_2^3 a_3^2 b_1^3 b_2^3 b_3^4 b_4^6 + a_1^2 a_2^3 a_3^2 b_1^4 b_2^3 b_3^4 b_4^6 +
\end{aligned}$$

$2*a1^6*a2^3*a3^3*a4^2*b2^3*b3^3*b4^4 + 2*a2^3*a3^3*a4^2*b1^6*b2^3*b3^3*b4^4 +$
 $2*a1^5*a2*a3^3*a4^2*b1*b2^5*b3^3*b4^4 + 2*a1*a2*a3^3*a4^2*b1^5*b2^5*b3^3*b4^4 +$
 $2*a1^3*a3^3*a4^2*b1^3*b2^6*b3^3*b4^4 + 2*a1^3*a2^5*a3*a4^2*b1^3*b2*b3^5*b4^4 +$
 $2*a1^5*a2^3*a3^3*a4^2*b1*b2^3*b3^5*b4^4 + 2*a1*a2^3*a3^3*a4^2*b1^5*b2^3*b3^5*b4^4 +$
 $2*a1^3*a2*a3^3*a4^2*b1^3*b2^3*b3^6*b4^4 + 2*a1^3*a2^4*a3^5*a4*b1^3*b2^2*b3*b4^5 +$
 $2*a1^4*a2^3*a3^5*a4*b1^2*b2^3*b3*b4^5 + 2*a1^2*a2^3*a3^5*a4*b1^4*b2^3*b3*b4^5 +$
 $2*a1^3*a2^5*a3^4*a4*b1^3*b2*b3^2*b4^5 + 2*a1^5*a2^3*a3^4*a4*b1*b2^3*b3^2*b4^5 +$
 $2*a1^3*a2^3*a3^4*a4*b1^5*b2^3*b3^2*b4^5 + 2*a1^3*a2^3*a3^4*a4*b1^3*b2^5*b3^2*b4^5 +$
 $2*a1^4*a2^5*a3^3*a4*b1^2*b2*b3^3*b4^5 + 2*a1^2*a2^5*a3^3*a4*b1^4*b2*b3^3*b4^5 +$
 $2*a1^5*a2^4*a3^3*a4*b1*b2^2*b3^3*b4^5 + 2*a1*a2^4*a3^3*a4*b1^5*b2^2*b3^3*b4^5 +$
 $2*a1^3*a2^2*a3^3*a4*b1^3*b2^4*b3^3*b4^5 + 2*a1^5*a2^2*a3^3*a4*b1^5*b2^4*b3^3*b4^5 +$
 $2*a1^4*a2^3*a3^3*a4*b1^2*b2^5*b3^3*b4^5 + 2*a1^2*a2^3*a3^3*a4*b1^4*b2^5*b3^3*b4^5 +$
 $2*a1^3*a2^5*a3^2*a4*b1^3*b2*b3^4*b4^5 + 2*a1^5*a2^3*a3^2*a4*b1^5*b2^3*b3^4*b4^5 +$
 $2*a1^4*a2^3*a3^2*a4*b1^3*b2^5*b3^4*b4^5 + 2*a1^2*a2^3*a3^2*a4*b1^4*b2^5*b3^4*b4^5 +$
 $2*a1^3*a2^4*a3^2*a4*b1^3*b2^4*b3^4*b4^5 + 2*a1^5*a2^2*a3^2*a4*b1^5*b2^4*b3^4*b4^5 +$
 $2*a1^4*a2^3*a3^2*a4*b1^2*b2^3*b3^5*b4^5 + 2*a1^2*a2^3*a3^2*a4*b1^4*b2^3*b3^5*b4^5 +$
 $2*a1^3*a2^3*a3^4*b1^3*b2^3*b3^2*b4^6 + 2*a1^3*a2^4*a3^3*b1^3*b2^2*b3^3*b4^6 +$
 $2*a1^4*a2^3*a3^3*b1^2*b2^3*b3^3*b4^6 + 2*a1^2*a2^3*a3^3*b1^4*b2^3*b3^3*b4^6 +$
 $2*a1^3*a2^2*a3^3*b1^3*b2^4*b3^3*b4^6 + 2*a1^3*a2^3*a3^2*b1^3*b2^3*b3^4*b4^6$

```

c2123=sum(c2.find(a1^3*a2^3*a3^4*w0))+sum(c2.find(a1^3*a2^3*b3^4*w0))
c2123
c2123.factor()
s2123=2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2
s2123

```

#This is the sum of the terms with $a1^3$, $a2^3$, and either $a3^4$ or $b3^4$, its \ factorization, and its substitution.

$2*a1^3*a2^3*a3^4*a4^6*b1^3*b2^3*b3^2 + 2*a1^3*a2^3*a3^2*a4^6*b1^3*b2^3*b3^4 +$
 $2*a1^3*a2^3*a3^4*b1^3*b2^3*b3^2*b4^6 + 2*a1^3*a2^3*a3^2*b1^3*b2^3*b3^4*b4^6$
 $2*(a4^4 - a4^2*b4^2 + b4^4)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^3*a2^3*a3^2*b1^3*b2^3*b3^2$
 $2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2$

```

c2124=sum(c2.find(a1^3*a2^3*a4^4*w0))+sum(c2.find(a1^3*a2^3*b4^4*w0))
c2124
c2124.factor()
s2124=2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q4^2
s2124

```

#This is the sum of the terms with $a1^3$, $a2^3$, and either $a4^4$ or $b4^4$, its \ factorization, and its substitution.

$2*a1^3*a2^3*a3^6*a4^4*b1^3*b2^3*b4^2 + 2*a1^3*a2^3*a4^4*b1^3*b2^3*b3^6*b4^2 +$
 $2*a1^3*a2^3*a3^6*a4^2*b1^3*b2^3*b4^4 + 2*a1^3*a2^3*a4^2*b1^3*b2^3*b3^6*b4^4$
 $2*(a3^4 - a3^2*b3^2 + b3^4)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^3*a2^3*a4^2*b1^3*b2^3*b4^2$
 $2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q4^2$

```

c2132=sum(c2.find(a1^3*a3^3*a2^4*w0))+sum(c2.find(a1^3*a3^3*b2^4*w0))
c2132
c2132.factor()
s2132=2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3
s2132

```

#This is the sum of the terms with $a1^3$, $a3^3$, and either $a2^4$ or $b2^4$, its \ factorization, and its substitution.

$2*a1^3*a2^4*a3^3*a4^6*b1^3*b2^2*b3^3 + 2*a1^3*a2^2*a3^3*a4^6*b1^3*b2^4*b3^3 +$
 $2*a1^3*a2^4*a3^3*b1^3*b2^2*b3^3*b4^6 + 2*a1^3*a2^2*a3^3*b1^3*b2^4*b3^3*b4^6$
 $2*(a4^4 - a4^2*b4^2 + b4^4)*(a2^2 + b2^2)*(a4^2 + b4^2)*a1^3*a2^2*a3^3*b1^3*b2^2*b3^3$
 $2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3$

```

c2134=sum(c2.find(a1^3*a3^3*a4^4*w0))+sum(c2.find(a1^3*a3^3*b4^4*w0))
c2134
c2134.factor()

```



```

s2134=2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q3^3*q4^2
s2134
#This is the sum of the terms with a1^3, a3^3, and either a4^4 or b4^4, its \
factorization, and its substitution.
2*a1^3*a2^6*a3^3*a4^4*b1^3*b3^3*b4^2 + 2*a1^3*a3^3*a4^4*b1^3*b2^6*b3^3*b4^2 +
2*a1^3*a2^6*a3^3*a4^2*b1^3*b3^3*b4^4 + 2*a1^3*a3^3*a4^2*b1^3*b2^6*b3^3*b4^4
2*(a2^4 - a2^2*b2^2 + b2^4)*(a2^2 + b2^2)*(a4^2 + b4^2)*a1^3*a3^3*a4^2*b1^3*b3^3*b4^2
2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q3^3*q4^2

c2142=sum(c2.find(a1^3*a4^3*a2^4*w0))+sum(c2.find(a1^3*a4^3*b2^4*w0))
c2142
c2142.factor()
s2142=2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q4^3
s2142
#This is the sum of the terms with a1^3, a4^3, and either a2^4 or b2^4, its \
factorization, and its substitution.
2*a1^3*a2^4*a3^6*a4^3*b1^3*b2^2*b4^3 + 2*a1^3*a2^2*a3^6*a4^3*b1^3*b2^4*b4^3 +
2*a1^3*a2^4*a4^3*b1^3*b2^2*b3^6*b4^3 + 2*a1^3*a2^2*a4^3*b1^3*b2^4*b3^6*b4^3
2*(a3^4 - a3^2*b3^2 + b3^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^3*a2^2*a4^3*b1^3*b2^2*b4^3
2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q4^3

c2143=sum(c2.find(a1^3*a4^3*a3^4*w0))+sum(c2.find(a1^3*a4^3*b3^4*w0))
c2143
c2143.factor()
s2143=2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q3^2*q4^3
s2143
#This is the sum of the terms with a1^3, a4^3, and either a3^4 or b3^4, its \
factorization, and its substitution.
2*a1^3*a2^6*a3^4*a4^3*b1^3*b3^2*b4^3 + 2*a1^3*a3^4*a4^3*b1^3*b2^6*b3^2*b4^3 +
2*a1^3*a2^6*a3^2*a4^3*b1^3*b3^4*b4^3 + 2*a1^3*a3^2*a4^3*b1^3*b2^6*b3^4*b4^3
2*(a2^4 - a2^2*b2^2 + b2^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^3*a3^2*a4^3*b1^3*b3^2*b4^3
2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q3^2*q4^3

c2231=sum(c2.find(a2^3*a3^3*a1^4*w0))+sum(c2.find(a2^3*a3^3*b1^4*w0))
c2231
c2231.factor()
s2231=2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3
s2231
#This is the sum of the terms with a2^3, a3^3, and either a1^4 or b1^4, its \
factorization, and its substitution.
2*a1^4*a2^3*a3^3*a4^6*b1^2*b2^3*b3^3 + 2*a1^2*a2^3*a3^3*a4^6*b1^4*b2^3*b3^3 +
2*a1^4*a2^3*a3^3*b1^2*b2^3*b3^3*b4^6 + 2*a1^2*a2^3*a3^3*b1^4*b2^3*b3^3*b4^6
2*(a4^4 - a4^2*b4^2 + b4^4)*(a1^2 + b1^2)*(a4^2 + b4^2)*a1^2*a2^3*a3^3*b1^2*b2^3*b3^3
2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3

c2234=sum(c2.find(a2^3*a3^3*a4^4*w0))+sum(c2.find(a2^3*a3^3*b4^4*w0))
c2234
c2234.factor()
s2234=2*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q2^3*q3^3*q4^2
s2234
#This is the sum of the terms with a2^3, a3^3, and either a4^4 or b4^4, its \
factorization, and its substitution.
2*a1^6*a2^3*a3^3*a4^4*b2^3*b3^3*b4^2 + 2*a2^3*a3^3*a4^4*b1^6*b2^3*b3^3*b4^2 +
2*a1^6*a2^3*a3^3*a4^2*b2^3*b3^3*b4^4 + 2*a2^3*a3^3*a4^2*b1^6*b2^3*b3^3*b4^4
2*(a1^4 - a1^2*b1^2 + b1^4)*(a1^2 + b1^2)*(a4^2 + b4^2)*a2^3*a3^3*a4^2*b2^3*b3^3*b4^2
2*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q2^3*q3^3*q4^2

c2241=sum(c2.find(a2^3*a4^3*a1^4*w0))+sum(c2.find(a2^3*a4^3*b1^4*w0))
c2241
c2241.factor()
s2241=2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^3*q4^3

```

s2241

#This is the sum of the terms with a^2^3 , a^4^3 , and either a^1^4 or b^4^1 , its \ factorization , and its substitution .

$$2*a^1^4*a^2^3*a^3^6*a^4^3*b^1^2*b^2^3*b^4^3 + 2*a^1^2*a^2^3*a^3^6*a^4^3*b^1^4*b^2^3*b^4^3 + 2*a^1^4*a^2^3*a^4^3*b^1^2*b^2^3*b^3^6*b^4^3 + 2*a^1^2*a^2^3*a^4^3*b^1^4*b^2^3*b^3^6*b^4^3$$
$$2*(a^3^4 - a^3^2*b^3^2 + b^3^4)*(a^1^2 + b^1^2)*(a^3^2 + b^3^2)*a^1^2*a^2^3*a^4^3*b^1^2*b^2^3*b^4^3$$
$$2*((p^3^2 + 2*q^3)^2 - 3*q^3^2)*(p^1^2 + 2*q^1)*(p^3^2 + 2*q^3)*q^1^2*q^2^3*q^4^3$$

$$c2243=\text{sum}(c2.\text{find}(a^2^3*a^4^3*a^3^4*w0))+\text{sum}(c2.\text{find}(a^2^3*a^4^3*b^3^4*w0))$$

c2243

c2243.factor()

$$s2243=2*((p^1^2 + 2*q^1)^2 - 3*q^1^2)*(p^1^2 + 2*q^1)*(p^3^2 + 2*q^3)*q^2^3*q^3^2*q^4^3$$

s2243

#This is the sum of the terms with a^2^3 , a^4^3 , and either a^3^4 or b^3^4 , its \ factorization , and its substitution .

$$2*a^1^6*a^2^3*a^3^4*a^4^3*b^2^3*b^3^2*b^4^3 + 2*a^2^3*a^3^4*a^4^3*b^1^6*b^2^3*b^3^2*b^4^3 + 2*a^1^6*a^2^3*a^3^2*a^4^3*b^2^3*b^3^4*b^4^3 + 2*a^2^3*a^3^2*a^4^3*b^1^6*b^2^3*b^3^4*b^4^3$$
$$2*(a^1^4 - a^1^2*b^1^2 + b^1^4)*(a^1^2 + b^1^2)*(a^3^2 + b^3^2)*a^2^3*a^3^2*a^4^3*b^2^3*b^3^2*b^4^3$$
$$2*((p^1^2 + 2*q^1)^2 - 3*q^1^2)*(p^1^2 + 2*q^1)*(p^3^2 + 2*q^3)*q^2^3*q^3^2*q^4^3$$

$$c2341=\text{sum}(c2.\text{find}(a^3^3*a^4^3*a^1^4*w0))+\text{sum}(c2.\text{find}(a^3^3*a^4^3*b^1^4*w0))$$

c2341

c2341.factor()

$$s2341=2*((p^2^2 + 2*q^2)^2 - 3*q^2^2)*(p^1^2 + 2*q^1)*(p^2^2 + 2*q^2)*q^1^2*q^3^3*q^4^3$$

s2341

#This is the sum of the terms with a^3^3 , a^4^3 , and either a^1^4 or b^1^4 , its \ factorization , and its substitution .

$$2*a^1^4*a^2^6*a^3^3*a^4^3*b^1^2*b^3^3*b^4^3 + 2*a^1^2*a^2^6*a^3^3*a^4^3*b^1^4*b^3^3*b^4^3 + 2*a^1^4*a^3^3*a^4^3*b^1^2*b^2^6*b^3^3*b^4^3 + 2*a^1^2*a^3^3*a^4^3*b^1^4*b^2^6*b^3^3*b^4^3$$
$$2*(a^2^4 - a^2^2*b^2^2 + b^2^4)*(a^1^2 + b^1^2)*(a^2^2 + b^2^2)*a^1^2*a^3^3*a^4^3*b^1^2*b^3^3*b^4^3$$
$$2*((p^2^2 + 2*q^2)^2 - 3*q^2^2)*(p^1^2 + 2*q^1)*(p^2^2 + 2*q^2)*q^1^2*q^3^3*q^4^3$$

$$c2342=\text{sum}(c2.\text{find}(a^3^3*a^4^3*a^2^4*w0))+\text{sum}(c2.\text{find}(a^3^3*a^4^3*b^2^4*w0))$$

c2342

c2342.factor()

$$s2342=2*((p^1^2 + 2*q^1)^2 - 3*q^1^2)*(p^1^2 + 2*q^1)*(p^2^2 + 2*q^2)*q^2^2*q^3^3*q^4^3$$

s2342

#This is the sum of the terms with a^3^3 , a^4^3 , and either a^1^4 or b^1^4 , its \ factorization , and its substitution .

$$2*a^1^6*a^2^4*a^3^3*a^4^3*b^2^2*b^3^3*b^4^3 + 2*a^2^4*a^3^3*a^4^3*b^1^6*b^2^2*b^3^3*b^4^3 + 2*a^1^6*a^2^2*a^3^3*a^4^3*b^2^4*b^3^3*b^4^3 + 2*a^2^2*a^3^3*a^4^3*b^1^6*b^2^4*b^3^3*b^4^3$$
$$2*(a^1^4 - a^1^2*b^1^2 + b^1^4)*(a^1^2 + b^1^2)*(a^2^2 + b^2^2)*a^2^2*a^3^3*a^4^3*b^2^2*b^3^3*b^4^3$$
$$2*((p^1^2 + 2*q^1)^2 - 3*q^1^2)*(p^1^2 + 2*q^1)*(p^2^2 + 2*q^2)*q^2^2*q^3^3*q^4^3$$

$$c02=c2 - c2123 - c2124 - c2132 - c2134 - c2142 - c2143 - c2231 - c2234 - c2241 - c2243 - c2341 - c2342$$

c02

#This gets all the terms from c2 that have not been assigned to a group. The \ groups we form will have one of the four a variables to the third power and \ either a second a variable or its corresponding b variable to the forth \ power .

$$2*a^1^3*a^2^4*a^3^5*a^4^5*b^1^3*b^2^2*b^3*b^4 + 2*a^1^4*a^2^3*a^3^5*a^4^5*b^1^2*b^2^3*b^3*b^4 + 2*a^1^2*a^2^3*a^3^5*a^4^5*b^1^4*b^2^3*b^3*b^4 + 2*a^1^3*a^2^2*a^3^5*a^4^5*b^1^3*b^2^4*b^3*b^4 + 2*a^1^3*a^2^5*a^3^4*a^4^5*b^1^3*b^2*b^3^2*b^4 + 2*a^1^5*a^2^3*a^3^4*a^4^5*b^1*b^2^3*b^3^2*b^4 + 2*a^1*a^2^3*a^3^4*a^4^5*b^1^5*b^2^3*b^3^2*b^4 + 2*a^1^3*a^2*a^3^4*a^4^5*b^1^3*b^2^5*b^3^2*b^4 + 2*a^1^4*a^2^5*a^3^3*a^4^5*b^1^2*b^2*b^3^3*b^4 + 2*a^1^2*a^2^5*a^3^3*a^4^5*b^1^4*b^2*b^3^3*b^4 + 2*a^1^5*a^2^4*a^3^3*a^4^5*b^1*b^2^2*b^3^3*b^4 + 2*a^1*a^2^4*a^3^3*a^4^5*b^1^5*b^2^2*b^3^3*b^4 + 2*a^1^5*a^2^2*a^3^3*a^4^5*b^1*b^2^4*b^3^3*b^4 + 2*a^1*a^2^2*a^3^3*a^4^5*b^1^5*b^2^4*b^3^3*b^4 + 2*a^1^4*a^2*a^3^3*a^4^5*b^1^2*b^2^5*b^3^3*b^4 + 2*a^1^2*a^2*a^3^3*a^4^5*b^1^4*b^2^5*b^3^3*b^4 + 2*a^1^3*a^2^5*a^3^2*a^4^5*b^1^3*b^2*b^3^4*b^4 + 2*a^1^5*a^2^3*a^3^2*a^4^5*b^1*b^2^3*b^3^4*b^4 + 2*a^1*a^2^3*a^3^2*a^4^5*b^1^5*b^2^3*b^3^4*b^4 + 2*a^1^3*a^2^2*a^3^2*a^4^5*b^1^3*b^2^5*b^3^4*b^4 + 2*a^1^5*a^2^4*a^3^2*a^4^5*b^1^3*b^2^2*b^3^5*b^4 + 2*a^1^4*a^2^3*a^3^2*a^4^5*b^1^2*b^2^3*b^3^5*b^4 +$$

$$\begin{aligned}
& 2*a1^3*a2^5*a3^4*a4^5*b1^3*b2^2*b3^2*b4 + 2*a1^3*a2^5*a3^4*a4^5*b1^3*b2^5*b3^2*b4 + \\
& 2*a1^3*a2^5*a3^2*a4^5*b1^3*b2^2*b3^4*b4 + 2*a1^3*a2^5*a3^2*a4^5*b1^3*b2^5*b3^4*b4 + \\
& 2*a1^3*a2^5*a3^4*a4^5*b1^3*b2^2*b3^2*b4^5 + 2*a1^3*a2^5*a3^4*a4^5*b1^3*b2^5*b3^2*b4^5 + \\
& 2*a1^3*a2^5*a3^2*a4^5*b1^3*b2^2*b3^4*b4^5 + 2*a1^3*a2^5*a3^2*a4^5*b1^3*b2^5*b3^4*b4^5 \\
& 2*(a2^4 + b2^4)*(a4^4 + b4^4)*(a3^2 + b3^2)*a1^3*a2^5*a3^2*a4^5*b1^3*b2^2*b3^2*b4 \\
& -2*((p^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4
\end{aligned}$$

$$c0214=\text{sum}(c02.\text{find}(a1^3*a4^4*w0))+\text{sum}(c02.\text{find}(a1^3*b4^4*w0))$$

c0214

c0214.factor()

$$s0214=-2*((p^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1\backslash$$

$$^3*q2^2*q3^2*q4^2$$

s0214

#This is the sum of the terms with a1^3 and either a4^4 or b4^4, its \ factorization, and its substitution.

$$\begin{aligned}
& 2*a1^3*a2^5*a3^5*a4^4*b1^3*b2^2*b3^2*b4^2 + 2*a1^3*a2^5*a3^5*a4^4*b1^3*b2^5*b3^2*b4^2 + \\
& 2*a1^3*a2^5*a3^5*a4^4*b1^3*b2^2*b3^5*b4^2 + 2*a1^3*a2^5*a3^5*a4^4*b1^3*b2^5*b3^5*b4^2 + \\
& 2*a1^3*a2^5*a3^5*a4^2*b1^3*b2^2*b3^2*b4^4 + 2*a1^3*a2^5*a3^5*a4^2*b1^3*b2^5*b3^2*b4^4 + \\
& 2*a1^3*a2^5*a3^5*a4^2*b1^3*b2^2*b3^5*b4^4 + 2*a1^3*a2^5*a3^5*a4^2*b1^3*b2^5*b3^5*b4^4 \\
& 2*(a2^4 + b2^4)*(a3^4 + b3^4)*(a4^2 + b4^2)*a1^3*a2^5*a3^5*a4^2*b1^3*b2^2*b3^2*b4^2 \\
& -2*((p^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4^2
\end{aligned}$$

$$c0221=\text{sum}(c02.\text{find}(a2^3*a1^4*w0))+\text{sum}(c02.\text{find}(a2^3*b1^4*w0))$$

c0221

c0221.factor()

$$s0221=-2*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1\backslash$$

$$^2*q2^3*q3^2*q4$$

s0221

#This is the sum of the terms with a2^3 and either a1^4 or b1^4, its \ factorization, and its substitution.

$$\begin{aligned}
& 2*a1^4*a2^3*a3^5*a4^5*b1^2*b2^3*b3^2*b4 + 2*a1^4*a2^3*a3^5*a4^5*b1^4*b2^3*b3^2*b4 + \\
& 2*a1^4*a2^3*a3^5*a4^5*b1^2*b2^3*b3^5*b4 + 2*a1^4*a2^3*a3^5*a4^5*b1^4*b2^3*b3^5*b4 + \\
& 2*a1^4*a2^3*a3^5*a4^2*b1^2*b2^3*b3^2*b4^5 + 2*a1^4*a2^3*a3^5*a4^2*b1^4*b2^3*b3^2*b4^5 + \\
& 2*a1^4*a2^3*a3^5*a4^2*b1^2*b2^3*b3^5*b4^5 + 2*a1^4*a2^3*a3^5*a4^2*b1^4*b2^3*b3^5*b4^5 \\
& 2*(a3^4 + b3^4)*(a4^4 + b4^4)*(a1^2 + b1^2)*a1^4*a2^3*a3^5*a4^2*b1^2*b2^3*b3^2*b4 \\
& -2*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^2*q4
\end{aligned}$$

$$c0223=\text{sum}(c02.\text{find}(a2^3*a3^4*w0))+\text{sum}(c02.\text{find}(a2^3*b3^4*w0))$$

c0223

c0223.factor()

$$s0223=-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1\backslash$$

$$*q2^3*q3^2*q4$$

s0223

#This is the sum of the terms with a2^3 and either a3^4 or b3^4, its \ factorization, and its substitution.

$$\begin{aligned}
& 2*a1^5*a2^3*a3^4*a4^5*b1^2*b2^3*b3^2*b4 + 2*a1^5*a2^3*a3^4*a4^5*b1^5*b2^3*b3^2*b4 + \\
& 2*a1^5*a2^3*a3^2*a4^5*b1^2*b2^3*b3^4*b4 + 2*a1^5*a2^3*a3^2*a4^5*b1^5*b2^3*b3^4*b4 + \\
& 2*a1^5*a2^3*a3^4*a4^5*b1^2*b2^3*b3^2*b4^5 + 2*a1^5*a2^3*a3^4*a4^5*b1^5*b2^3*b3^2*b4^5 + \\
& 2*a1^5*a2^3*a3^2*a4^5*b1^2*b2^3*b3^4*b4^5 + 2*a1^5*a2^3*a3^2*a4^5*b1^5*b2^3*b3^4*b4^5 \\
& 2*(a1^4 + b1^4)*(a4^4 + b4^4)*(a3^2 + b3^2)*a1^5*a2^3*a3^2*a4^5*b1^2*b2^3*b3^2*b4 \\
& -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1*q2^3*q3^2*q4
\end{aligned}$$

$$c0224=\text{sum}(c02.\text{find}(a2^3*a4^4*w0))+\text{sum}(c02.\text{find}(a2^3*b4^4*w0))$$

c0224

c0224.factor()

$$s0224=-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1\backslash$$

$$*q2^3*q3^2*q4^2$$

s0224

#This is the sum of the terms with a2^3 and either a4^4 or b4^4, its \ factorization, and its substitution.

$$2*a1^5*a2^3*a3^5*a4^4*b1^2*b2^3*b3^2*b4^2 + 2*a1^5*a2^3*a3^5*a4^4*b1^5*b2^3*b3^2*b4^2 +$$

$$\begin{aligned}
& 2*a1^5*a2^3*a3^4*b1^2*b2^3*b3^5*b4^2 + 2*a1*a2^3*a3^4*b1^5*b2^3*b3^5*b4^2 + \\
& 2*a1^5*a2^3*a3^5*a4^2*b1^2*b2^3*b3^5*b4^4 + 2*a1*a2^3*a3^5*a4^2*b1^5*b2^3*b3^5*b4^4 + \\
& 2*a1^5*a2^3*a3^4^2*b1^2*b2^3*b3^5*b4^4 + 2*a1*a2^3*a3^4^2*b1^5*b2^3*b3^5*b4^4 \\
& 2*(a1^4 + b1^4)*(a3^4 + b3^4)*(a4^2 + b4^2)*a1^2*a2^3*a3^4^2*b1^2*b2^3*b3^5*b4^2 \\
& -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1^2*q2^3*q3^5*q4^2
\end{aligned}$$

$$c0231 = \text{sum}(c02.\text{find}(a3^3*a1^4*w0)) + \text{sum}(c02.\text{find}(a3^3*b1^4*w0))$$

c0231

c0231.factor()

$$s0231 = -2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^5*q4^2$$

s0231

#This is the sum of the terms with a3^3 and either a1^4 or b1^4, its factorization, and its substitution.

$$\begin{aligned}
& 2*a1^4*a2^5*a3^3*a4^5*b1^2*b2^3*b3^5*b4 + 2*a1^2*a2^5*a3^3*a4^5*b1^4*b2^3*b3^5*b4 + \\
& 2*a1^4*a2^3*a3^3*a4^5*b1^2*b2^5*b3^3*b4 + 2*a1^2*a2^3*a3^3*a4^5*b1^4*b2^5*b3^3*b4 + \\
& 2*a1^4*a2^5*a3^3*a4^5*b1^2*b2^3*b3^5*b4^5 + 2*a1^2*a2^5*a3^3*a4^5*b1^4*b2^3*b3^5*b4^5 + \\
& 2*a1^4*a2^3*a3^3*a4^5*b1^2*b2^5*b3^3*b4^5 + 2*a1^2*a2^3*a3^3*a4^5*b1^4*b2^5*b3^3*b4^5 \\
& 2*(a2^4 + b2^4)*(a4^4 + b4^4)*(a1^2 + b1^2)*a1^2*a2^5*a3^3*a4^5*b1^2*b2^3*b3^5*b4 \\
& -2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^5*q4
\end{aligned}$$

$$c0232 = \text{sum}(c02.\text{find}(a3^3*a2^4*w0)) + \text{sum}(c02.\text{find}(a3^3*b2^4*w0))$$

c0232

c0232.factor()

$$s0232 = -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*q1^2*q2^2*q3^5*q4$$

s0232

#This is the sum of the terms with a3^3 and either a2^4 or b2^4, its factorization, and its substitution.

$$\begin{aligned}
& 2*a1^5*a2^4*a3^3*a4^5*b1^2*b2^3*b3^5*b4 + 2*a1*a2^4*a3^3*a4^5*b1^5*b2^3*b3^5*b4 + \\
& 2*a1^5*a2^2*a3^3*a4^5*b1^2*b2^4*b3^3*b4 + 2*a1*a2^2*a3^3*a4^5*b1^5*b2^4*b3^3*b4 + \\
& 2*a1^5*a2^4*a3^3*a4^5*b1^2*b2^3*b3^5*b4^5 + 2*a1*a2^4*a3^3*a4^5*b1^5*b2^3*b3^5*b4^5 + \\
& 2*a1^5*a2^2*a3^3*a4^5*b1^2*b2^4*b3^3*b4^5 + 2*a1*a2^2*a3^3*a4^5*b1^5*b2^4*b3^3*b4^5 \\
& 2*(a1^4 + b1^4)*(a4^4 + b4^4)*(a2^2 + b2^2)*a1^2*a2^4*a3^3*a4^5*b1^2*b2^3*b3^5*b4 \\
& -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*q1^2*q2^2*q3^5*q4
\end{aligned}$$

$$c0234 = \text{sum}(c02.\text{find}(a3^3*a4^4*w0)) + \text{sum}(c02.\text{find}(a3^3*b4^4*w0))$$

c0234

c0234.factor()

$$s0234 = -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 + 2*q4)*q1^2*q2^3*q3^5*q4^2$$

s0234

#This is the sum of the terms with a3^3 and either a2^4 or b2^4, its factorization, and its substitution.

$$\begin{aligned}
& 2*a1^5*a2^5*a3^3*a4^4*b1^2*b2^3*b3^5*b4^2 + 2*a1*a2^5*a3^3*a4^4*b1^5*b2^3*b3^5*b4^2 + \\
& 2*a1^5*a2^3*a3^3*a4^4*b1^2*b2^5*b3^3*b4^2 + 2*a1*a2^3*a3^3*a4^4*b1^5*b2^5*b3^3*b4^2 + \\
& 2*a1^5*a2^5*a3^3*a4^2*b1^2*b2^3*b3^5*b4^4 + 2*a1*a2^5*a3^3*a4^2*b1^5*b2^3*b3^5*b4^4 + \\
& 2*a1^5*a2^3*a3^3*a4^2*b1^2*b2^5*b3^3*b4^4 + 2*a1*a2^3*a3^3*a4^2*b1^5*b2^5*b3^3*b4^4 \\
& 2*(a1^4 + b1^4)*(a2^4 + b2^4)*(a4^2 + b4^2)*a1^2*a2^5*a3^3*a4^4*b1^2*b2^3*b3^5*b4^2 \\
& -2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 + 2*q4)*q1^2*q2^3*q3^5*q4^2
\end{aligned}$$

$$c0241 = \text{sum}(c02.\text{find}(a4^3*a1^4*w0)) + \text{sum}(c02.\text{find}(a4^3*b1^4*w0))$$

c0241

c0241.factor()

$$s0241 = -2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^5*q4^3$$

s0241

#This is the sum of the terms with a4^3 and either a1^4 or b1^4, its factorization, and its substitution.

$$\begin{aligned}
& 2*a1^4*a2^5*a3^5*a4^3*b1^2*b2^3*b3^5*b4^3 + 2*a1^2*a2^5*a3^5*a4^3*b1^4*b2^3*b3^5*b4^3 + \\
& 2*a1^4*a2^3*a3^5*a4^3*b1^2*b2^5*b3^3*b4^3 + 2*a1^2*a2^3*a3^5*a4^3*b1^4*b2^5*b3^3*b4^3 +
\end{aligned}$$

```

2*a1^4*a2^5*a3*a4^3*b1^2*b2*b3^5*b4^3 + 2*a1^2*a2^5*a3*a4^3*b1^4*b2*b3^5*b4^3 +
2*a1^4*a2*a3*a4^3*b1^2*b2^5*b3^5*b4^3 + 2*a1^2*a2*a3*a4^3*b1^4*b2^5*b3^5*b4^3
2*(a2^4 + b2^4)*(a3^4 + b3^4)*(a1^2 + b1^2)*a1^2*a2*a3*a4^3*b1^2*b2*b3*b4^3
-2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*q1^2*q2^3*q4^3

c0242=sum(c02.find(a4^3*a2^4*w0))+sum(c02.find(a4^3*b2^4*w0))
c0242
c0242.factor()
s0242=-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*q1\
*q2^2*q3*q4^3
s0242
#This is the sum of the terms with a4^3 and either a2^4 or b2^4, its \
factorization, and its substitution.
2*a1^5*a2^4*a3^5*a4^3*b1*b2^2*b3*b4^3 + 2*a1*a2^4*a3^5*a4^3*b1^5*b2^2*b3*b4^3 +
2*a1^5*a2^2*a3^5*a4^3*b1*b2^4*b3*b4^3 + 2*a1*a2^2*a3^5*a4^3*b1^5*b2^4*b3*b4^3 +
2*a1^5*a2^4*a3*a4^3*b1*b2^2*b3^5*b4^3 + 2*a1*a2^4*a3*a4^3*b1^5*b2^2*b3^5*b4^3 +
2*a1^5*a2^2*a3*a4^3*b1*b2^4*b3^5*b4^3 + 2*a1*a2^2*a3*a4^3*b1^5*b2^4*b3^5*b4^3
2*(a1^4 + b1^4)*(a3^4 + b3^4)*(a2^2 + b2^2)*a1*a2^2*a3*a4^3*b1*b2^2*b3*b4^3
-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*q1*q2^2*q3*q4^3

c0243=sum(c02.find(a4^3*a3^4*w0))+sum(c02.find(a4^3*b3^4*w0))
c0243
c0243.factor()
s0243=-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*q1\
*q2*q3^2*q4^3
s0243
#This is the sum of the terms with a4^3 and either a3^4 or b3^4, its \
factorization, and its substitution.
2*a1^5*a2^5*a3^4*a4^3*b1*b2*b3^2*b4^3 + 2*a1*a2^5*a3^4*a4^3*b1^5*b2*b3^2*b4^3 +
2*a1^5*a2*a3^4*a4^3*b1*b2^5*b3^2*b4^3 + 2*a1*a2*a3^4*a4^3*b1^5*b2^5*b3^2*b4^3 +
2*a1^5*a2^5*a3^2*a4^3*b1*b2*b3^4*b4^3 + 2*a1*a2^5*a3^2*a4^3*b1^5*b2*b3^4*b4^3 +
2*a1^5*a2*a3^2*a4^3*b1*b2^5*b3^4*b4^3 + 2*a1*a2*a3^2*a4^3*b1^5*b2^5*b3^4*b4^3
2*(a1^4 + b1^4)*(a2^4 + b2^4)*(a3^2 + b3^2)*a1*a2*a3^2*a4^3*b1*b2*b3^2*b4^3
-2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*q1*q2^3*q4^3

bool(c2==c0212+c0213+c0214+c0221+c0223+c0224+c0231+c0232+c0234+c0241+c0242+\
c0243+(-c2123-c2124-c2132-c2134-c2142-c2143-c2231-c2234-c2241-c2243-c2341-\
c2342))
#This is a check to make sure as we seperated the groups we used every term in \
c2 once and only once.
True

l2=s0212+s0213+s0214+s0221+s0223+s0224+s0231+s0232+s0234+s0241+s0242+s0243+(-\
s2123-s2124-s2132-s2134-s2142-s2143-s2231-s2234-s2241-s2243-s2341-s2342)
l2
l02=l2(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3=-1*\
a3*b3, q4=-1*a4*b4)
bool(l02==c2)
#This line brings all the group substitutions we created for c2 back into one \
expression and checks to make sure that when back substitute we end up with \
c2.
2*((p4^2 + 2*q4)^2 - 3*q4^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2 + 2*((p4^2 +
2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3 + 2*((p4^2 + 2*q4)^2 -
3*q4^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3 + 2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p3^2 +
2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q4^2 + 2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p4^2 +
2*q4)*q1^3*q3^3*q4^2 + 2*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p4^2 +
2*q4)*q2^3*q3^3*q4^2 + 2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p2^2 + 2*q2)*(p3^2 +
2*q3)*q1^3*q2^2*q4^3 + 2*((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p3^2 +
2*q3)*q1^2*q2^3*q4^3 + 2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p3^2 +
2*q3)*q1^3*q3^2*q4^3 + 2*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p3^2 +
2*q3)*q2^3*q3^2*q4^3 + 2*((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 +

```

```

2*q2)*q1^2*q3^3*q4^3 + 2*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 +
2*q2)*q2^2*q3^3*q4^3 - 2*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 +
2*q2)*q1^3*q2^2*q3^3*q4 - 2*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 +
2*q1)*q1^2*q2^3*q3^3*q4 - 2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 +
2*q3)*q1^3*q2^2*q3^2*q4 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 +
2*q3)*q1*q2^3*q3^2*q4 - 2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 +
2*q1)*q1^2*q2^2*q3^3*q4 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 +
2*q2)*q1*q2^2*q3^3*q4 - 2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 +
2*q4)*q1^3*q2^2*q3^3*q4^2 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 +
2*q4)*q1*q2^3*q3^3*q4^2 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 +
2*q4)*q1*q2^2*q3^3*q4^2 - 2*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 +
2*q1)*q1^2*q2^2*q3^3*q4^3 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 +
2*q2)*q1*q2^2*q3^3*q4^3 - 2*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 +
2*q3)*q1*q2^2*q3^2*q4^3
True

```

```

c4=sum(f10.find(4*w0))
c4
#This finds all our terms in f10 that have a 4 in them and then adds them \
together. The first set of groups we form will have one of the four a \
variables or its corresponding b variable to the sixth power.

```

```

4*a1^3*a2^3*a3^3*a4^6*b1^3*b2^3*b3^3 + 4*a1^3*a2^3*a3^5*a4^5*b1^3*b2^3*b3*b4 +
4*a1^3*a2^5*a3^3*a4^5*b1^3*b2^3*b3^3*b4 + 4*a1^5*a2^3*a3^3*a4^5*b1*b2^3*b3^3*b4 +
4*a1^3*a2^3*a3^3*a4^5*b1^5*b2^3*b3^3*b4 + 4*a1^3*a2^3*a3^3*a4^5*b1^3*b2^5*b3^3*b4 +
4*a1^3*a2^3*a3^3*a4^5*b1^3*b2^3*b3^5*b4 + 4*a1^3*a2^3*a3^6*a4^3*b1^3*b2^3*b4^3 +
4*a1^3*a2^5*a3^5*a4^3*b1^3*b2^3*b3*b4^3 + 4*a1^5*a2^3*a3^5*a4^3*b1*b2^3*b3*b4^3 +
4*a1^3*a2^3*a3^5*a4^3*b1^5*b2^3*b3*b4^3 + 4*a1^3*a2^3*a3^5*a4^3*b1^3*b2^5*b3*b4^3 +
4*a1^3*a2^6*a3^3*a4^3*b1^3*b3^3*b4^3 + 4*a1^5*a2^5*a3^3*a4^3*b1*b2*b3^3*b4^3 +
4*a1^3*a2^5*a3^3*a4^3*b1^5*b2^3*b3^3*b4^3 + 4*a1^6*a2^3*a3^3*a4^3*b2^3*b3^3*b4^3 +
4*a2^3*a3^3*a4^3*b1^6*b2^3*b3^3*b4^3 + 4*a1^5*a2^2*a3^3*a4^3*b1*b2^5*b3^3*b4^3 +
4*a1^3*a2^3*a3^3*a4^3*b1^5*b2^5*b3^3*b4^3 + 4*a1^3*a3^3*a4^3*b1^3*b2^6*b3^3*b4^3 +
4*a1^3*a2^5*a3^3*a4^3*b1^3*b2^3*b3^5*b4^3 + 4*a1^5*a2^3*a3^3*a4^3*b1*b2^3*b3^5*b4^3 +
4*a1^3*a2^3*a3^3*a4^3*b1^5*b2^3*b3^5*b4^3 + 4*a1^3*a2^3*a3^3*a4^3*b1^3*b2^5*b3^5*b4^3 +
4*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^6*b4^3 + 4*a1^3*a2^3*a3^5*a4^3*b1^3*b2^3*b3*b4^5 +
4*a1^3*a2^5*a3^3*a4^3*b1^3*b2^3*b3^3*b4^5 + 4*a1^5*a2^3*a3^3*a4^3*b1*b2^3*b3^3*b4^5 +
4*a1^3*a2^3*a3^3*a4^3*b1^5*b2^3*b3^3*b4^5 + 4*a1^3*a2^3*a3^3*a4^3*b1^3*b2^5*b3^3*b4^5 +
4*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^5*b4^5 + 4*a1^3*a2^3*a3^3*b1^3*b2^3*b3^3*b4^6

```

```

c41=sum(c4.find(a1^6*w0))+sum(c4.find(b1^6*w0))
c41
c41.factor()
s41=-4*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*q2^3*q3^3*q4^3
s41
#This is the sum of the terms with a1^6 or b1^6, its factorization, and its \
substitution.

```

```

4*a1^6*a2^3*a3^3*a4^3*b2^3*b3^3*b4^3 + 4*a2^3*a3^3*a4^3*b1^6*b2^3*b3^3*b4^3
4*(a1^4 - a1^2*b1^2 + b1^4)*(a1^2 + b1^2)*a2^3*a3^3*a4^3*b2^3*b3^3*b4^3
-4*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*q2^3*q3^3*q4^3

```

```

c42=sum(c4.find(a2^6*w0))+sum(c4.find(b2^6*w0))
c42
c42.factor()
s42=-4*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*q1^3*q3^3*q4^3
s42
#This is the sum of the terms with a2^6 or b2^6, its factorization, and its \
substitution.

```

```

4*a1^3*a2^6*a3^3*a4^3*b1^3*b3^3*b4^3 + 4*a1^3*a3^3*a4^3*b1^3*b2^6*b3^3*b4^3
4*(a2^4 - a2^2*b2^2 + b2^4)*(a2^2 + b2^2)*a1^3*a3^3*a4^3*b1^3*b3^3*b4^3
-4*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*q1^3*q3^3*q4^3

```



```

c43=sum(c4.find(a3^6*w0))+sum(c4.find(b3^6*w0))
c43
c43.factor()
s43=-4*((p3^2 + 2*q3)^2 - 3*q3^2)*(p3^2 + 2*q3)*q1^3*q2^3*q4^3
s43
#This is the sum of the terms with a3^6 or b3^6, its factorization, and its \
substitution.

```

```

4*a1^3*a2^3*a3^6*a4^3*b1^3*b2^3*b4^3 + 4*a1^3*a2^3*a4^3*b1^3*b2^3*b3^6*b4^3
4*(a3^4 - a3^2*b3^2 + b3^4)*(a3^2 + b3^2)*a1^3*a2^3*a4^3*b1^3*b2^3*b4^3
-4*((p3^2 + 2*q3)^2 - 3*q3^2)*(p3^2 + 2*q3)*q1^3*q2^3*q4^3

```

```

c44=sum(c4.find(a4^6*w0))+sum(c4.find(b4^6*w0))
c44
c44.factor()
s44=-4*((p4^2 + 2*q4)^2 - 3*q4^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3^3
s44
#This is the sum of the terms with a4^6 or b4^6, its factorization, and its \
substitution.

```

```

4*a1^3*a2^3*a3^3*a4^6*b1^3*b2^3*b3^3 + 4*a1^3*a2^3*a3^3*b1^3*b2^3*b3^3*b4^6
4*(a4^4 - a4^2*b4^2 + b4^4)*(a4^2 + b4^2)*a1^3*a2^3*a3^3*b1^3*b2^3*b3^3
-4*((p4^2 + 2*q4)^2 - 3*q4^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3^3

```

```

c04=c4 - c41 - c42 - c43 - c44
c04
#This gets all the terms from c4 that have not been assigned to a group. The \
groups we form will have two of the four a variables to the third power.

```

```

4*a1^3*a2^3*a3^5*a4^5*b1^3*b2^3*b3*b4 + 4*a1^3*a2^5*a3^3*a4^5*b1^3*b2*b3^3*b4 +
4*a1^5*a2^3*a3^3*a4^5*b1*b2^3*b3^3*b4 + 4*a1*a2^3*a3^3*a4^5*b1^5*b2^3*b3^3*b4 +
4*a1^3*a2*a3^3*a4^5*b1^3*b2^5*b3^3*b4 + 4*a1^3*a2^3*a3*a4^5*b1^3*b2^3*b3^5*b4 +
4*a1^3*a2^5*a3^5*a4^3*b1^3*b2*b3*b4^3 + 4*a1^5*a2^3*a3^5*a4^3*b1*b2^3*b3*b4^3 +
4*a1*a2^3*a3^5*a4^3*b1^5*b2^3*b3*b4^3 + 4*a1^3*a2*a3^5*a4^3*b1^3*b2^5*b3*b4^3 +
4*a1^5*a2^5*a3^3*a4^3*b1*b2*b3^3*b4^3 + 4*a1*a2^5*a3^3*a4^3*b1^5*b2*b3^3*b4^3 +
4*a1^3*a2^5*a3^3*a4^3*b1*b2^5*b3^3*b4^3 + 4*a1*a2^5*a3^3*a4^3*b1^3*b2^5*b3^3*b4^3 +
4*a1^5*a2^3*a3^3*a4^3*b1^5*b2^3*b3^5*b4^3 + 4*a1^3*a2^3*a3^3*a4^3*b1^3*b2^5*b3^5*b4^3 +
4*a1^3*a2^3*a3^5*a4*b1^3*b2^3*b3*b4^5 + 4*a1^3*a2^5*a3^3*a4*b1^3*b2*b3^3*b4^5 +
4*a1^5*a2^3*a3^3*a4*b1*b2^3*b3^3*b4^5 + 4*a1*a2^3*a3^3*a4*b1^5*b2^3*b3^3*b4^5 +
4*a1^3*a2*a3^3*a4*b1^3*b2^5*b3^3*b4^5 + 4*a1^5*a2^3*a3^3*a4*b1^3*b2^3*b3^5*b4^5

```

```

c0412=sum(c04.find(a1^3*a2^3*w0))
c0412
c0412.factor()
s0412=4*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3*q4
s0412
#This is the sum of the terms with a1^3 and a2^3, its factorization, and its \
substitution.

```

```

4*a1^3*a2^3*a3^5*a4^5*b1^3*b2^3*b3*b4 + 4*a1^3*a2^3*a3^3*a4^5*b1^3*b2^3*b3^5*b4 +
4*a1^3*a2^3*a3^5*a4*b1^3*b2^3*b3*b4^5 + 4*a1^3*a2^3*a3^3*a4*b1^3*b2^3*b3^5*b4^5
4*(a3^4 + b3^4)*(a4^4 + b4^4)*a1^3*a2^3*a3^3*a4*b1^3*b2^3*b3*b4
4*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3*q4

```

```

c0413=sum(c04.find(a1^3*a3^3*w0))
c0413
c0413.factor()
s0413=4*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4
s0413
#This is the sum of the terms with a1^3 and a2^3, its factorization, and its \
substitution.

```

```

4*a1^3*a2^5*a3^3*a4^5*b1^3*b2*b3^3*b4 + 4*a1^3*a2^3*a3^3*a4^5*b1^3*b2^5*b3^3*b4 +
4*a1^3*a2^5*a3^3*a4*b1^3*b2*b3^3*b4^5 + 4*a1^3*a2^3*a3^3*a4*b1^3*b2^5*b3^3*b4^5
4*(a2^4 + b2^4)*(a4^4 + b4^4)*a1^3*a2^3*a3^3*a4*b1^3*b2*b3^3*b4

```



```
4*((p2^2 + 2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2*q3^3*q4
```

```
c0414=sum(c04.find(a1^3*a4^3*w0))
```

```
c0414
```

```
c0414.factor()
```

```
s0414=4*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*q1^3*q2*q3*q4^3  
s0414
```

```
#This is the sum of the terms with a1^3 and a4^3, its factorization, and its \  
substitution.
```

```
4*a1^3*a2^5*a3^5*a4^3*b1^3*b2^3*b3^3*b4^3 + 4*a1^3*a2^5*a3^5*a4^3*b1^3*b2^5*b3^3*b4^3 +  
4*a1^3*a2^5*a3^5*a4^3*b1^3*b2^3*b3^5*b4^3 + 4*a1^3*a2^5*a3^5*a4^3*b1^3*b2^5*b3^5*b4^3  
4*(a2^4 + b2^4)*(a3^4 + b3^4)*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^3*b4^3  
4*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*q1^3*q2*q3^3*q4^3
```

```
c0423=sum(c04.find(a2^3*a3^3*w0))
```

```
c0423
```

```
c0423.factor()
```

```
s0423=4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1*q2^3*q3^3*q4  
s0423
```

```
#This is the sum of the terms with a2^3 and a3^3, its factorization, and its \  
substitution.
```

```
4*a1^5*a2^3*a3^3*a4^5*b1*b2^3*b3^3*b4 + 4*a1*a2^3*a3^3*a4^5*b1^5*b2^3*b3^3*b4 +  
4*a1^5*a2^3*a3^3*a4^5*b1*b2^3*b3^3*b4^5 + 4*a1*a2^3*a3^3*a4^5*b1^5*b2^3*b3^3*b4^5  
4*(a1^4 + b1^4)*(a4^4 + b4^4)*a1*a2^3*a3^3*a4^3*b1^2^3*b3^3*b4  
4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1*q2^3*q3^3*q4
```

```
c0424=sum(c04.find(a2^3*a4^3*w0))
```

```
c0424
```

```
c0424.factor()
```

```
s0424=4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*q1*q2^3*q3^3*q4^3  
s0424
```

```
#This is the sum of the terms with a2^3 and a4^3, its factorization, and its \  
substitution.
```

```
4*a1^5*a2^3*a3^5*a4^3*b1*b2^3*b3^3*b4^3 + 4*a1*a2^3*a3^5*a4^3*b1^5*b2^3*b3^3*b4^3 +  
4*a1^5*a2^3*a3^5*a4^3*b1*b2^3*b3^5*b4^3 + 4*a1*a2^3*a3^5*a4^3*b1^5*b2^3*b3^5*b4^3  
4*(a1^4 + b1^4)*(a3^4 + b3^4)*a1*a2^3*a3^3*a4^3*b1^2^3*b3^3*b4^3  
4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*q1*q2^3*q3^3*q4^3
```

```
c0434=sum(c04.find(a3^3*a4^3*w0))
```

```
c0434
```

```
c0434.factor()
```

```
s0434=4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*q1*q2*q3^3*q4^3  
s0434
```

```
#This is the sum of the terms with a2^3 and a4^3, its factorization, and its \  
substitution.
```

```
4*a1^5*a2^5*a3^3*a4^3*b1*b2^3*b3^3*b4^3 + 4*a1*a2^5*a3^3*a4^3*b1^5*b2^3*b3^3*b4^3 +  
4*a1^5*a2^5*a3^3*a4^3*b1*b2^5*b3^3*b4^3 + 4*a1*a2^5*a3^3*a4^3*b1^5*b2^5*b3^3*b4^3  
4*(a1^4 + b1^4)*(a2^4 + b2^4)*a1*a2^3*a3^3*a4^3*b1^2^3*b3^3*b4^3  
4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*q1*q2*q3^3*q4^3
```

```
bool(c0412+c0413+c0414+c0423+c0424+c0434+c41+c42+c43+c44==c4)
```

```
#This is a check to make sure as we seperated the groups we used every term in \  
c4 once and only once.
```

```
True
```

```
l4=s0412+s0413+s0414+s0423+s0424+s0434+s41+s42+s43+s44
```

```
l4
```

```
l04=l4(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3=-1*\  
a3*b3, q4=-1*a4*b4)
```

```
bool(l04==c4)
```

```
#This line brings all the group substitutions we created for c4 back into one \  

```

expression and checks to make sure that when back substitute we end up with $c4$.

```
-4*((p4^2 + 2*q4)^2 - 3*q4^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3^3 - 4*((p3^2 + 2*q3)^2 -
3*q3^2)*(p3^2 + 2*q3)*q1^3*q2^3*q4^3 - 4*((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 +
2*q2)*q1^3*q3^3*q4^3 - 4*((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*q2^3*q3^3*q4^3 +
4*((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4 + 4*((p2^2 +
2*q2)^2 - 2*q2^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4 + 4*((p1^2 + 2*q1)^2 -
2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*q1*q2^3*q3^3*q4 + 4*((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 +
2*q3)^2 - 2*q3^2)*q1^3*q2^3*q3^3*q4^3 + 4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 -
2*q3^2)*q1*q2^3*q3^3*q4^3 + 4*((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 -
2*q2^2)*q1*q2^3*q3^3*q4^3
True
```

```
c7=sum(f10.find(7*w0))
```

```
c7
```

#This finds all our terms in f10 that have a 7 in them and then adds them together. The groups we form will have one of the four a variables or its corresponding b variable to the fifth power.

```
7*a1^4*a2^4*a3^4*a4^5*b1^2*b2^2*b3^2*b4 + 7*a1^2*a2^4*a3^4*a4^5*b1^4*b2^2*b3^2*b4 +
7*a1^4*a2^2*a3^4*a4^5*b1^2*b2^4*b3^2*b4 + 7*a1^2*a2^2*a3^4*a4^5*b1^4*b2^4*b3^2*b4 +
7*a1^4*a2^4*a3^2*a4^5*b1^2*b2^2*b3^4*b4 + 7*a1^2*a2^4*a3^2*a4^5*b1^4*b2^2*b3^4*b4 +
7*a1^4*a2^2*a3^2*a4^5*b1^2*b2^4*b3^4*b4 + 7*a1^2*a2^2*a3^2*a4^5*b1^4*b2^4*b3^4*b4 +
7*a1^4*a2^4*a3^5*a4^4*b1^2*b2^2*b3^*b4^2 + 7*a1^2*a2^4*a3^5*a4^4*b1^4*b2^2*b3^*b4^2 +
7*a1^4*a2^2*a3^5*a4^4*b1^2*b2^4*b3^*b4^2 + 7*a1^2*a2^2*a3^5*a4^4*b1^4*b2^4*b3^*b4^2 +
7*a1^4*a2^5*a3^4*a4^4*b1^2*b2^2*b3^2*b4^2 + 7*a1^2*a2^5*a3^4*a4^4*b1^4*b2^2*b3^2*b4^2 +
7*a1^4*a2^2*a3^5*a4^4*b1^2*b2^4*b3^2*b4^2 + 7*a1^2*a2^2*a3^5*a4^4*b1^4*b2^4*b3^2*b4^2 +
7*a1^4*a2^5*a3^4*a4^4*b1^2*b2^5*b3^2*b4^2 + 7*a1^2*a2^5*a3^4*a4^4*b1^4*b2^5*b3^2*b4^2 +
7*a1^4*a2^2*a3^4*a4^4*b1^2*b2^4*b3^2*b4^2 + 7*a1^2*a2^2*a3^4*a4^4*b1^4*b2^5*b3^2*b4^2 +
7*a1^4*a2^5*a3^2*a4^4*b1^2*b2^5*b3^2*b4^2 + 7*a1^2*a2^5*a3^2*a4^4*b1^4*b2^5*b3^2*b4^2 +
7*a1^4*a2^4*a3^4*a4^4*b1^2*b2^5*b3^2*b4^2 + 7*a1^2*a2^4*a3^4*a4^4*b1^4*b2^5*b3^2*b4^2 +
7*a1^4*a2^2*a3^2*a4^4*b1^2*b2^5*b3^4*b4^2 + 7*a1^2*a2^2*a3^2*a4^4*b1^4*b2^5*b3^4*b4^2 +
7*a1^4*a2^5*a3^2*a4^4*b1^2*b2^4*b3^5*b4^2 + 7*a1^2*a2^5*a3^2*a4^4*b1^4*b2^4*b3^5*b4^2 +
7*a1^4*a2^2*a3^5*a4^4*b1^2*b2^2*b3^*b4^4 + 7*a1^2*a2^2*a3^5*a4^4*b1^4*b2^2*b3^*b4^4 +
7*a1^4*a2^5*a3^5*a4^2*b1^2*b2^2*b3^*b4^4 + 7*a1^2*a2^5*a3^5*a4^2*b1^4*b2^2*b3^*b4^4 +
7*a1^4*a2^2*a3^4*a4^2*b1^2*b2^2*b3^2*b4^4 + 7*a1^2*a2^2*a3^4*a4^2*b1^4*b2^2*b3^2*b4^4 +
7*a1^4*a2^5*a3^4*a4^2*b1^2*b2^2*b3^2*b4^4 + 7*a1^2*a2^5*a3^4*a4^2*b1^4*b2^2*b3^2*b4^4 +
7*a1^4*a2^2*a3^2*a4^2*b1^2*b2^4*b3^2*b4^4 + 7*a1^2*a2^2*a3^2*a4^2*b1^4*b2^4*b3^2*b4^4 +
7*a1^4*a2^5*a3^2*a4^2*b1^2*b2^4*b3^2*b4^4 + 7*a1^2*a2^5*a3^2*a4^2*b1^4*b2^4*b3^2*b4^4 +
7*a1^4*a2^2*a3^5*a4^2*b1^2*b2^2*b3^4*b4^4 + 7*a1^2*a2^2*a3^5*a4^2*b1^4*b2^2*b3^4*b4^4 +
7*a1^4*a2^5*a3^5*a4^2*b1^2*b2^2*b3^4*b4^4 + 7*a1^2*a2^5*a3^5*a4^2*b1^4*b2^2*b3^4*b4^4 +
7*a1^4*a2^2*a3^4*a4^2*b1^2*b2^5*b3^2*b4^4 + 7*a1^2*a2^2*a3^4*a4^2*b1^4*b2^5*b3^2*b4^4 +
7*a1^4*a2^5*a3^4*a4^2*b1^2*b2^5*b3^2*b4^4 + 7*a1^2*a2^5*a3^4*a4^2*b1^4*b2^5*b3^2*b4^4 +
7*a1^4*a2^2*a3^2*a4^2*b1^2*b2^5*b3^4*b4^4 + 7*a1^2*a2^2*a3^2*a4^2*b1^4*b2^5*b3^4*b4^4 +
7*a1^4*a2^5*a3^2*a4^2*b1^2*b2^4*b3^5*b4^4 + 7*a1^2*a2^5*a3^2*a4^2*b1^4*b2^4*b3^5*b4^4 +
7*a1^4*a2^2*a3^5*a4^2*b1^2*b2^4*b3^5*b4^4 + 7*a1^2*a2^2*a3^5*a4^2*b1^4*b2^4*b3^5*b4^4 +
7*a1^4*a2^5*a3^4*a4*b1^2*b2^2*b3^2*b4^5 + 7*a1^2*a2^5*a3^4*a4*b1^4*b2^2*b3^2*b4^5 +
7*a1^4*a2^2*a3^4*a4*b1^2*b2^4*b3^2*b4^5 + 7*a1^2*a2^2*a3^4*a4*b1^4*b2^4*b3^2*b4^5 +
7*a1^4*a2^5*a3^2*a4*b1^2*b2^2*b3^4*b4^5 + 7*a1^2*a2^5*a3^2*a4*b1^4*b2^2*b3^4*b4^5 +
7*a1^4*a2^2*a3^2*a4*b1^2*b2^4*b3^4*b4^5 + 7*a1^2*a2^2*a3^2*a4*b1^4*b2^4*b3^4*b4^5
```

```
c71=sum(c7.find(a1^5*w0))+sum(c7.find(b1^5*w0))
```

```
c71
```

```
c71.factor()
```

```
s71=-7*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1*\
q2^2*q3^2*q4^2
```

```
s71
```

#This is the sum of the terms with $a1^5$ or $b1^5$, its factorization, and its substitution.

```
7*a1^5*a2^4*a3^4*a4^4*b1*b2^2*b3^2*b4^2 + 7*a1*a2^4*a3^4*a4^4*b1^5*b2^2*b3^2*b4^2 +
```

$$\begin{aligned}
&7*a1^5*a2^2*a3^4*a4^4*b1*b2^4*b3^2*b4^2 + 7*a1*a2^2*a3^4*a4^4*b1^5*b2^4*b3^2*b4^2 + \\
&7*a1^5*a2^4*a3^2*a4^4*b1*b2^2*b3^4*b4^2 + 7*a1*a2^4*a3^2*a4^4*b1^5*b2^2*b3^4*b4^2 + \\
&7*a1^5*a2^2*a3^2*a4^4*b1*b2^4*b3^4*b4^2 + 7*a1*a2^2*a3^2*a4^4*b1^5*b2^4*b3^4*b4^2 + \\
&7*a1^5*a2^4*a3^4*a4^2*b1*b2^2*b3^2*b4^4 + 7*a1*a2^4*a3^4*a4^2*b1^5*b2^2*b3^2*b4^4 + \\
&7*a1^5*a2^2*a3^4*a4^2*b1*b2^4*b3^2*b4^4 + 7*a1*a2^2*a3^4*a4^2*b1^5*b2^4*b3^2*b4^4 + \\
&7*a1^5*a2^4*a3^2*a4^2*b1*b2^2*b3^4*b4^4 + 7*a1*a2^4*a3^2*a4^2*b1^5*b2^2*b3^4*b4^4 + \\
&7*a1^5*a2^2*a3^2*a4^2*b1*b2^4*b3^4*b4^4 + 7*a1*a2^2*a3^2*a4^2*b1^5*b2^4*b3^4*b4^4 \\
&7*(a1^4 + b1^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*(a4^2 + \\
&b4^2)*a1*a2^2*a3^2*a4^2*b1*b2^2*b3^2*b4^2 \\
&-7*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1*q2^2*q3^2*q4^2
\end{aligned}$$

$$c72=\text{sum}(c7.\text{find}(a2^5*w0))+\text{sum}(c7.\text{find}(b2^5*w0))$$

c72

c72.factor()

$$s72=-7*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1\ \backslash$$

$$^2*q2^2*q3^2*q4^2$$

s72

#This is the sum of the terms with a2^5 or b2^5, its factorization, and its \ substitution.

$$\begin{aligned}
&7*a1^4*a2^5*a3^4*a4^4*b1^2*b2*b3^2*b4^2 + 7*a1^2*a2^5*a3^4*a4^4*b1^4*b2*b3^2*b4^2 + \\
&7*a1^4*a2^2*a3^4*a4^4*b1^2*b2^5*b3^2*b4^2 + 7*a1^2*a2^2*a3^4*a4^4*b1^4*b2^5*b3^2*b4^2 + \\
&7*a1^4*a2^5*a3^2*a4^4*b1^2*b2*b3^4*b4^2 + 7*a1^2*a2^5*a3^2*a4^4*b1^4*b2*b3^4*b4^2 + \\
&7*a1^4*a2^2*a3^2*a4^4*b1^2*b2^5*b3^4*b4^2 + 7*a1^2*a2^2*a3^2*a4^4*b1^4*b2^5*b3^4*b4^2 + \\
&7*a1^4*a2^5*a3^4*a4^2*b1^2*b2*b3^2*b4^4 + 7*a1^2*a2^5*a3^4*a4^2*b1^4*b2*b3^2*b4^4 + \\
&7*a1^4*a2^2*a3^4*a4^2*b1^2*b2^5*b3^2*b4^4 + 7*a1^2*a2^2*a3^4*a4^2*b1^4*b2^5*b3^2*b4^4 + \\
&7*a1^4*a2^5*a3^2*a4^2*b1^2*b2*b3^4*b4^4 + 7*a1^2*a2^5*a3^2*a4^2*b1^4*b2*b3^4*b4^4 + \\
&7*a1^4*a2^2*a3^2*a4^2*b1^2*b2^5*b3^4*b4^4 + 7*a1^2*a2^2*a3^2*a4^2*b1^4*b2^5*b3^4*b4^4 \\
&7*(a2^4 + b2^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*(a4^2 + \\
&b4^2)*a1^2*a2^2*a3^2*a4^2*b1^2*b2*b3^2*b4^2 \\
&-7*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4^2
\end{aligned}$$

$$c73=\text{sum}(c7.\text{find}(a3^5*w0))+\text{sum}(c7.\text{find}(b3^5*w0))$$

c73

c73.factor()

$$s73=-7*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1\ \backslash$$

$$^2*q2^2*q3^2*q4^2$$

s73

#This is the sum of the terms with a3^5 or b3^5, its factorization, and its \ substitution.

$$\begin{aligned}
&7*a1^4*a2^4*a3^5*a4^4*b1^2*b2^2*b3*b4^2 + 7*a1^2*a2^4*a3^5*a4^4*b1^4*b2^2*b3*b4^2 + \\
&7*a1^4*a2^2*a3^5*a4^4*b1^2*b2^4*b3*b4^2 + 7*a1^2*a2^2*a3^5*a4^4*b1^4*b2^4*b3*b4^2 + \\
&7*a1^4*a2^4*a3^2*a4^4*b1^2*b2^2*b3^5*b4^2 + 7*a1^2*a2^4*a3^2*a4^4*b1^4*b2^2*b3^5*b4^2 + \\
&7*a1^4*a2^2*a3^2*a4^4*b1^2*b2^5*b3^5*b4^2 + 7*a1^2*a2^2*a3^2*a4^4*b1^4*b2^5*b3^5*b4^2 + \\
&7*a1^4*a2^5*a3^4*a4^2*b1^2*b2*b3^2*b4^4 + 7*a1^2*a2^5*a3^4*a4^2*b1^4*b2*b3^2*b4^4 + \\
&7*a1^4*a2^2*a3^4*a4^2*b1^2*b2^5*b3^2*b4^4 + 7*a1^2*a2^2*a3^4*a4^2*b1^4*b2^5*b3^2*b4^4 + \\
&7*a1^4*a2^5*a3^2*a4^2*b1^2*b2*b3^4*b4^4 + 7*a1^2*a2^5*a3^2*a4^2*b1^4*b2*b3^4*b4^4 + \\
&7*a1^4*a2^2*a3^2*a4^2*b1^2*b2^5*b3^4*b4^4 + 7*a1^2*a2^2*a3^2*a4^2*b1^4*b2^5*b3^4*b4^4 \\
&7*(a3^4 + b3^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a4^2 + \\
&b4^2)*a1^2*a2^2*a3^2*a4^2*b1^2*b2^2*b3^2*b4^2 \\
&-7*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4^2
\end{aligned}$$

$$c74=\text{sum}(c7.\text{find}(a4^5*w0))+\text{sum}(c7.\text{find}(b4^5*w0))$$

c74

c74.factor()

$$s74=-7*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1\ \backslash$$

$$^2*q2^2*q3^2*q4$$

s74

#This is the sum of the terms with a4^5 or b4^5, its factorization, and its \ substitution.

$$\begin{aligned}
&7*a1^4*a2^4*a3^4*a4^5*b1^2*b2^2*b3^2*b4 + 7*a1^2*a2^4*a3^4*a4^5*b1^4*b2^2*b3^2*b4 + \\
&7*a1^4*a2^2*a3^4*a4^5*b1^2*b2^4*b3^2*b4 + 7*a1^2*a2^2*a3^4*a4^5*b1^4*b2^4*b3^2*b4 +
\end{aligned}$$


```

#This is the sum of the terms with a1^3 and either a4^5 or b4^5, its \
factorization , and its substitution.
12*a1^3*a2^4*a3^4*a4^5*b1^3*b2^2*b3^2*b4 + 12*a1^3*a2^2*a3^4*a4^5*b1^3*b2^4*b3^2*b4 +
12*a1^3*a2^4*a3^2*a4^5*b1^3*b2^2*b3^4*b4 + 12*a1^3*a2^2*a3^2*a4^5*b1^3*b2^4*b3^4*b4 +
12*a1^3*a2^4*a3^4*a4*b1^3*b2^2*b3^2*b4^5 + 12*a1^3*a2^2*a3^4*a4*b1^3*b2^4*b3^2*b4^5 +
12*a1^3*a2^4*a3^2*a4*b1^3*b2^2*b3^4*b4^5 + 12*a1^3*a2^2*a3^2*a4*b1^3*b2^4*b3^4*b4^5
12*(a4^4 + b4^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^3*a2^2*a3^2*a4*b1^3*b2^2*b3^2*b4
12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4

c12021=sum(c12.find(a2^3*a1^5*w0))+sum(c12.find(a2^3*b1^5*w0))
c12021
c12021.factor()
s12021=12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1*q2^3*q3^2*\
q4^2
s12021

#This is the sum of the terms with a2^3 and either a1^5 or b1^5, its \
factorization , and its substitution.
12*a1^5*a2^3*a3^4*a4^4*b1*b2^3*b3^2*b4^2 + 12*a1^5*a2^3*a3^4*a4^4*b1^5*b2^3*b3^2*b4^2 +
12*a1^5*a2^3*a3^2*a4^4*b1*b2^3*b3^4*b4^2 + 12*a1^5*a2^3*a3^2*a4^4*b1^5*b2^3*b3^4*b4^2 +
12*a1^5*a2^3*a3^4*a4^2*b1*b2^3*b3^2*b4^4 + 12*a1^5*a2^3*a3^4*a4^2*b1^5*b2^3*b3^2*b4^4 +
12*a1^5*a2^3*a3^2*a4^2*b1*b2^3*b3^4*b4^4 + 12*a1^5*a2^3*a3^2*a4^2*b1^5*b2^3*b3^4*b4^4
12*(a1^4 + b1^4)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^5*a2^3*a3^2*a4^2*b1*b2^3*b3^2*b4^2
12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2*q4^2

c12023=sum(c12.find(a2^3*a3^5*w0))+sum(c12.find(a2^3*b3^5*w0))
c12023
c12023.factor()
s12023=12*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3*\
q4^2
s12023

#This is the sum of the terms with a2^3 and either a3^5 or b3^5, its \
factorization , and its substitution.
12*a1^4*a2^3*a3^5*a4^4*b1^2*b2^3*b3*b4^2 + 12*a1^2*a2^3*a3^5*a4^4*b1^4*b2^3*b3*b4^2 +
12*a1^4*a2^3*a3^5*a4^4*b1^2*b2^3*b3^5*b4^2 + 12*a1^2*a2^3*a3^5*a4^4*b1^4*b2^3*b3^5*b4^2 +
12*a1^4*a2^3*a3^5*a4^2*b1^2*b2^3*b3*b4^4 + 12*a1^2*a2^3*a3^5*a4^2*b1^4*b2^3*b3*b4^4 +
12*a1^4*a2^3*a3^5*a4^2*b1^2*b2^3*b3^5*b4^4 + 12*a1^2*a2^3*a3^5*a4^2*b1^4*b2^3*b3^5*b4^4
12*(a3^4 + b3^4)*(a1^2 + b1^2)*(a4^2 + b4^2)*a1^2*a2^3*a3^5*a4^2*b1^2*b2^3*b3*b4^2
12*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3*q4^2

c12024=sum(c12.find(a2^3*a4^5*w0))+sum(c12.find(a2^3*b4^5*w0))
c12024
c12024.factor()
s12024=12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^3*q3*\
^2*q4
s12024

#This is the sum of the terms with a2^3 and either a4^5 or b4^5, its \
factorization , and its substitution.
12*a1^4*a2^3*a3^4*a4^5*b1^2*b2^3*b3^2*b4 + 12*a1^2*a2^3*a3^4*a4^5*b1^4*b2^3*b3^2*b4 +
12*a1^4*a2^3*a3^2*a4^5*b1^2*b2^3*b3^4*b4 + 12*a1^2*a2^3*a3^2*a4^5*b1^4*b2^3*b3^4*b4 +
12*a1^4*a2^3*a3^4*a4*b1^2*b2^3*b3^2*b4^5 + 12*a1^2*a2^3*a3^4*a4*b1^4*b2^3*b3^2*b4^5 +
12*a1^4*a2^3*a3^2*a4*b1^2*b2^3*b3^4*b4^5 + 12*a1^2*a2^3*a3^2*a4*b1^4*b2^3*b3^4*b4^5
12*(a4^4 + b4^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*a1^2*a2^3*a3^2*a4*b1^2*b2^3*b3^2*b4
12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^3*q3^2*q4

c12031=sum(c12.find(a3^3*a1^5*w0))+sum(c12.find(a3^3*b1^5*w0))
c12031
c12031.factor()
s12031=12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1*q2^2*q3^3*\
q4^2
s12031

```

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#This is the sum of the terms with a3^3 and either a1^5 or b1^5, its \
factorization , and its substitution.
12*a1^5*a2^4*a3^3*a4^4*b1*b2^2*b3^3*b4^2 + 12*a1*a2^4*a3^3*a4^4*b1^5*b2^2*b3^3*b4^2 +
12*a1^5*a2^2*a3^3*a4^4*b1*b2^4*b3^3*b4^2 + 12*a1*a2^2*a3^3*a4^4*b1^5*b2^4*b3^3*b4^2 +
12*a1^5*a2^4*a3^3*a4^2*b1*b2^2*b3^3*b4^4 + 12*a1*a2^4*a3^3*a4^2*b1^5*b2^2*b3^3*b4^4 +
12*a1^5*a2^2*a3^3*a4^2*b1*b2^4*b3^3*b4^4 + 12*a1*a2^2*a3^3*a4^2*b1^5*b2^4*b3^3*b4^4
12*(a1^4 + b1^4)*(a2^2 + b2^2)*(a4^2 + b4^2)*a1*a2^2*a3^3*a4^2*b1*b2^2*b3^3*b4^2
12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1*q2^2*q3^3*q4^2

c12032=sum(c12.find(a3^3*a2^5*w0))+sum(c12.find(a3^3*b2^5*w0))
c12032
c12032.factor()
s12032=12*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2*q3^3\
q4^2
s12032
#This is the sum of the terms with a3^3 and either a2^5 or b2^5, its \
factorization , and its substitution.
12*a1^4*a2^5*a3^3*a4^4*b1^2*b2*b3^3*b4^2 + 12*a1^2*a2^5*a3^3*a4^4*b1^4*b2*b3^3*b4^2 +
12*a1^4*a2^2*a3^3*a4^4*b1^2*b2^5*b3^3*b4^2 + 12*a1^2*a2^2*a3^3*a4^4*b1^4*b2^5*b3^3*b4^2 +
12*a1^4*a2^5*a3^3*a4^2*b1^2*b2*b3^3*b4^4 + 12*a1^2*a2^5*a3^3*a4^2*b1^4*b2*b3^3*b4^4 +
12*a1^4*a2^2*a3^3*a4^2*b1^2*b2^5*b3^3*b4^4 + 12*a1^2*a2^2*a3^3*a4^2*b1^4*b2^5*b3^3*b4^4
12*(a2^4 + b2^4)*(a1^2 + b1^2)*(a4^2 + b4^2)*a1^2*a2*a3^3*a4^2*b1^2*b2*b3^3*b4^2
12*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2*q3^3*q4^2

c12034=sum(c12.find(a3^3*a4^5*w0))+sum(c12.find(a3^3*b4^5*w0))
c12034
c12034.factor()
s12034=12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3\
^3*q4
s12034
#This is the sum of the terms with a3^3 and either a4^5 or b4^5, its \
factorization , and its substitution.
12*a1^4*a2^4*a3^3*a4^5*b1^2*b2^2*b3^3*b4 + 12*a1^2*a2^4*a3^3*a4^5*b1^4*b2^2*b3^3*b4 +
12*a1^4*a2^2*a3^3*a4^5*b1^2*b2^4*b3^3*b4 + 12*a1^2*a2^2*a3^3*a4^5*b1^4*b2^4*b3^3*b4 +
12*a1^4*a2^4*a3^3*a4*b1^2*b2^2*b3^3*b4^5 + 12*a1^2*a2^4*a3^3*a4*b1^4*b2^2*b3^3*b4^5 +
12*a1^4*a2^2*a3^3*a4*b1^2*b2^4*b3^3*b4^5 + 12*a1^2*a2^2*a3^3*a4*b1^4*b2^4*b3^3*b4^5
12*(a4^4 + b4^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*a1^2*a2^2*a3^3*a4*b1^2*b2^2*b3^3*b4
12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^3*q4

c12041=sum(c12.find(a4^3*a1^5*w0))+sum(c12.find(a4^3*b1^5*w0))
c12041
c12041.factor()
s12041=12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1*q2^2*q3^2*\
q4^3
s12041
#This is the sum of the terms with a4^3 and either a1^5 or b1^5, its \
factorization , and its substitution.
12*a1^5*a2^4*a3^4*a4^3*b1*b2^2*b3^2*b4^3 + 12*a1*a2^4*a3^4*a4^3*b1^5*b2^2*b3^2*b4^3 +
12*a1^5*a2^2*a3^4*a4^3*b1*b2^4*b3^2*b4^3 + 12*a1*a2^2*a3^4*a4^3*b1^5*b2^4*b3^2*b4^3 +
12*a1^5*a2^4*a3^2*a4^3*b1*b2^2*b3^4*b4^3 + 12*a1*a2^4*a3^2*a4^3*b1^5*b2^2*b3^4*b4^3 +
12*a1^5*a2^2*a3^2*a4^3*b1*b2^4*b3^4*b4^3 + 12*a1*a2^2*a3^2*a4^3*b1^5*b2^4*b3^4*b4^3
12*(a1^4 + b1^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1*a2^2*a3^2*a4^3*b1*b2^2*b3^2*b4^3
12*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1*q2^2*q3^2*q4^3

c12042=sum(c12.find(a4^3*a2^5*w0))+sum(c12.find(a4^3*b2^5*w0))
c12042
c12042.factor()
s12042=12*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2*q3^2*\
q4^3
s12042

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#This is the sum of the terms with a4^3 and either a2^5 or b2^5, its \
factorization, and its substitution.
12*a1^4*a2^5*a3^4*a4^3*b1^2*b2*b3^2*b4^3 + 12*a1^2*a2^5*a3^4*a4^3*b1^4*b2*b3^2*b4^3 +
12*a1^4*a2^5*a3^4*a4^3*b1^2*b2^5*b3^2*b4^3 + 12*a1^2*a2^5*a3^4*a4^3*b1^4*b2^5*b3^2*b4^3 +
12*a1^4*a2^5*a3^2*a4^3*b1^2*b2*b3^4*b4^3 + 12*a1^2*a2^5*a3^2*a4^3*b1^4*b2*b3^4*b4^3 +
12*a1^4*a2^5*a3^2*a4^3*b1^2*b2^5*b3^4*b4^3 + 12*a1^2*a2^5*a3^2*a4^3*b1^4*b2^5*b3^4*b4^3
12*(a2^4 + b2^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*a1^2*a2^5*a3^2*a4^3*b1^2*b2*b3^2*b4^3
12*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4^3

c12043=sum(c12.find(a4^3*a3^5*w0))+sum(c12.find(a4^3*b3^5*w0))
c12043
c12043.factor()
s12043=12*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^2\
q4^3
s12043
#This is the sum of the terms with a4^3 and either a2^5 or b2^5, its \
factorization, and its substitution.
12*a1^4*a2^4*a3^5*a4^3*b1^2*b2^2*b3*b4^3 + 12*a1^2*a2^4*a3^5*a4^3*b1^4*b2^2*b3*b4^3 +
12*a1^4*a2^2*a3^5*a4^3*b1^2*b2^4*b3*b4^3 + 12*a1^2*a2^2*a3^5*a4^3*b1^4*b2^4*b3*b4^3 +
12*a1^4*a2^4*a3^4*a4^3*b1^2*b2^2*b3^5*b4^3 + 12*a1^2*a2^4*a3^4*a4^3*b1^4*b2^2*b3^5*b4^3 +
12*a1^4*a2^2*a3^4*a4^3*b1^2*b2^4*b3^5*b4^3 + 12*a1^2*a2^2*a3^4*a4^3*b1^4*b2^4*b3^5*b4^3
12*(a3^4 + b3^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*a1^2*a2^2*a3^4*a4^3*b1^2*b2^2*b3*b4^3
12*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^2*q4^3

bool(c12==c12012+c12013+c12014+c12021+c12023+c12024+c12031+c12032+c12034+c12041\
+c12042+c12043)
#This is a check to make sure as we seperated the groups we used every term in \
c12 once and only once.
True

l12=s12012+s12013+s12014+s12021+s12023+s12024+s12031+s12032+s12034+s12041+\
s12042+s12043
l12
l012=l12(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
bool(l012==c12)
#This line brings all the group substitutions we created for c12 back into one \
expression and checks to make sure that when back substitute we end up with \
c12.
12*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4 + 12*((p4^2 +
2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4 + 12*((p4^2 + 2*q4)^2 -
2*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^2*q4 + 12*((p3^2 + 2*q3)^2 -
2*q3^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4 + 12*((p3^2 + 2*q3)^2 -
2*q3^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4 + 12*((p2^2 + 2*q2)^2 -
2*q2^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4 + 12*((p2^2 + 2*q2)^2 -
2*q2^2)*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4 + 12*((p1^2 + 2*q1)^2 -
2*q1^2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4 + 12*((p1^2 + 2*q1)^2 -
2*q1^2)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4 + 12*((p3^2 + 2*q3)^2 -
2*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^2*q4 + 12*((p2^2 + 2*q2)^2 -
2*q2^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4 + 12*((p1^2 + 2*q1)^2 -
2*q1^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4
True

c20=sum(f10.find(20*w0))
c20
#This finds all our terms in f10 that have a 20 in them and then adds them \
together. The groups we form will have two of the four a variables to the \
third power and either a second a variable or its corresponding b variable \
to the forth power.
20*a1^3*a2^3*a3^4*a4^5*b1^3*b2^3*b3^2*b4 + 20*a1^3*a2^4*a3^3*a4^5*b1^3*b2^2*b3^3*b4 +

```


$20*a1^4*a2^3*a3^3*a4^5*b1^2*b2^3*b3^3*b4 + 20*a1^2*a2^3*a3^3*a4^5*b1^4*b2^3*b3^3*b4 + 20*a1^3*a2^2*a3^3*a4^5*b1^3*b2^4*b3^3*b4 + 20*a1^3*a2^3*a3^5*a4^4*b1^3*b2^3*b3^3*b4^2 + 20*a1^3*a2^5*a3^3*a4^4*b1^3*b2^3*b3^3*b4^2 + 20*a1^5*a2^3*a3^3*a4^4*b1*b2^3*b3^3*b4^2 + 20*a1^2*a2^3*a3^3*a4^4*b1^5*b2^3*b3^3*b4^2 + 20*a1^3*a2^2*a3^3*a4^4*b1^3*b2^3*b3^5*b4^2 + 20*a1^3*a2^4*a3^5*a4^3*b1^3*b2^2*b3^3*b4^3 + 20*a1^4*a2^3*a3^5*a4^3*b1^2*b2^3*b3^3*b4^3 + 20*a1^2*a2^3*a3^5*a4^3*b1^4*b2^3*b3^3*b4^3 + 20*a1^3*a2^2*a3^5*a4^3*b1^3*b2^4*b3^3*b4^3 + 20*a1^5*a2^3*a3^4*a4^3*b1^3*b2^3*b3^2*b4^3 + 20*a1^3*a2^3*a3^4*a4^3*b1^5*b2^3*b3^2*b4^3 + 20*a1^3*a2^5*a3^4*a4^3*b1^3*b2^3*b3^2*b4^3 + 20*a1^5*a2^3*a3^4*a4^3*b1^3*b2^3*b3^2*b4^3 + 20*a1^2*a2^5*a3^4*a4^3*b1^4*b2^3*b3^2*b4^3 + 20*a1^3*a2^2*a3^4*a4^3*b1^3*b2^5*b3^2*b4^3 + 20*a1^4*a2^5*a3^3*a4^3*b1^2*b2^3*b3^3*b4^3 + 20*a1^2*a2^5*a3^3*a4^3*b1^4*b2^3*b3^3*b4^3 + 20*a1^5*a2^4*a3^3*a4^3*b1*b2^2*b3^3*b4^3 + 20*a1^2*a2^4*a3^3*a4^3*b1^5*b2^2*b3^3*b4^3 + 20*a1^5*a2^2*a3^3*a4^3*b1^3*b2^4*b3^3*b4^3 + 20*a1^2*a2^2*a3^3*a4^3*b1^5*b2^4*b3^3*b4^3 + 20*a1^4*a2^3*a3^3*a4^3*b1^2*b2^5*b3^3*b4^3 + 20*a1^2*a2^3*a3^3*a4^3*b1^4*b2^5*b3^3*b4^3 + 20*a1^3*a2^5*a3^2*a4^3*b1^3*b2^3*b3^4*b4^3 + 20*a1^5*a2^3*a3^2*a4^3*b1^5*b2^3*b3^4*b4^3 + 20*a1^2*a2^4*a3^2*a4^3*b1^3*b2^2*b3^5*b4^3 + 20*a1^4*a2^2*a3^2*a4^3*b1^5*b2^2*b3^5*b4^3 + 20*a1^3*a2^2*a3^2*a4^3*b1^3*b2^3*b3^5*b4^3 + 20*a1^5*a2^3*a3^2*a4^3*b1^3*b2^3*b3^5*b4^3 + 20*a1^2*a2^3*a3^2*a4^3*b1^4*b2^3*b3^5*b4^3 + 20*a1^3*a2^3*a3^5*a4^2*b1^3*b2^3*b3^4*b4^4 + 20*a1^3*a2^5*a3^3*a4^2*b1^3*b2^3*b3^4*b4^4 + 20*a1^5*a2^3*a3^3*a4^2*b1^5*b2^3*b3^4*b4^4 + 20*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^5*b4^4 + 20*a1^3*a2^3*a3^4*a4^2*b1^3*b2^3*b3^2*b4^5 + 20*a1^3*a2^4*a3^3*a4^2*b1^3*b2^2*b3^3*b4^5 + 20*a1^4*a2^3*a3^3*a4^2*b1^2*b2^3*b3^3*b4^5 + 20*a1^2*a2^3*a3^3*a4^2*b1^4*b2^3*b3^3*b4^5 + 20*a1^3*a2^2*a3^3*a4^2*b1^3*b2^4*b3^3*b4^5 + 20*a1^3*a2^3*a3^2*a4^2*b1^3*b2^3*b3^4*b4^5$

```

c20123=sum(c20.find(a1^3*a2^3*a3^4*w0))+sum(c20.find(a1^3*a2^3*b3^4*w0))
c20123
c20123.factor()
s20123=-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4
s20123

```

#This is the sum of the terms with $a1^3$, $a2^3$ and either $a3^4$ or $b3^4$, its factorization, and its substitution.

$20*a1^3*a2^3*a3^4*a4^5*b1^3*b2^3*b3^2*b4 + 20*a1^3*a2^3*a3^2*a4^5*b1^3*b2^3*b3^4*b4 + 20*a1^3*a2^3*a3^4*a4*b1^3*b2^3*b3^2*b4^5 + 20*a1^3*a2^3*a3^2*a4*b1^3*b2^3*b3^4*b4^5$
 $20*(a4^4 + b4^4)*(a3^2 + b3^2)*a1^3*a2^3*a3^2*a4*b1^3*b2^3*b3^2*b4$
 $-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4$

```

c20124=sum(c20.find(a1^3*a2^3*a4^4*w0))+sum(c20.find(a1^3*a2^3*b4^4*w0))
c20124
c20124.factor()
s20124=-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3*q4^2
s20124

```

#This is the sum of the terms with $a1^3$, $a2^3$ and either $a3^4$ or $b3^4$, its factorization, and its substitution.

$20*a1^3*a2^3*a3^5*a4^4*b1^3*b2^3*b3^3*b4^2 + 20*a1^3*a2^3*a3^3*a4^4*b1^3*b2^3*b3^5*b4^2 + 20*a1^3*a2^3*a3^5*a4^2*b1^3*b2^3*b3^3*b4^4 + 20*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^5*b4^4$
 $20*(a3^4 + b3^4)*(a4^2 + b4^2)*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^3*b4^2$
 $-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3*q4^2$

```

c20132=sum(c20.find(a1^3*a3^3*a2^4*w0))+sum(c20.find(a1^3*a3^3*b2^4*w0))
c20132
c20132.factor()
s20132=-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*q1^3*q2^2*q3^3*q4
s20132

```

#This is the sum of the terms with $a1^3$, $a3^3$ and either $a2^4$ or $b2^4$, its factorization, and its substitution.

$20*a1^3*a2^4*a3^3*a4^5*b1^3*b2^2*b3^3*b4 + 20*a1^3*a2^2*a3^3*a4^5*b1^3*b2^4*b3^3*b4 + 20*a1^3*a2^4*a3^3*a4*b1^3*b2^2*b3^3*b4^5 + 20*a1^3*a2^2*a3^3*a4*b1^3*b2^4*b3^3*b4^5$
 $20*(a4^4 + b4^4)*(a2^2 + b2^2)*a1^3*a2^2*a3^3*a4*b1^3*b2^2*b3^3*b4$
 $-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*q1^3*q2^2*q3^3*q4$

```

c20134=sum(c20.find(a1^3*a3^3*a4^4*w0))+sum(c20.find(a1^3*a3^3*b4^4*w0))

```

```

c20134
c20134.factor()
s20134=-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 + 2*q4)*q1^3*q2*q3^3*q4^2
s20134
#This is the sum of the terms with a1^3, a3^3 and either a4^4 or b4^4, its \
factorization, and its substitution.
20*a1^3*a2^5*a3^3*a4^4*b1^3*b2*b3^3*b4^2 + 20*a1^3*a2*a3^3*a4^4*b1^3*b2^5*b3^3*b4^2 +
20*a1^3*a2^5*a3^3*a4^2*b1^3*b2*b3^3*b4^4 + 20*a1^3*a2*a3^3*a4^2*b1^3*b2^5*b3^3*b4^4
20*(a2^4 + b2^4)*(a4^2 + b4^2)*a1^3*a2*a3^3*a4^2*b1^3*b2*b3^3*b4^2
-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 + 2*q4)*q1^3*q2*q3^3*q4^2

c20142=sum(c20.find(a1^3*a4^3*a2^4*w0))+sum(c20.find(a1^3*a4^3*b2^4*w0))
c20142
c20142.factor()
s20142=-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*q1^3*q2^2*q3*q4^3
s20142
#This is the sum of the terms with a1^3, a4^3 and either a3^4 or b3^4, its \
factorization, and its substitution.
20*a1^3*a2^4*a3^5*a4^3*b1^3*b2^2*b3*b4^3 + 20*a1^3*a2^2*a3^5*a4^3*b1^3*b2^4*b3*b4^3 +
20*a1^3*a2^4*a3^5*a4^3*b1^3*b2^2*b3^5*b4^3 + 20*a1^3*a2^2*a3^5*a4^3*b1^3*b2^4*b3^5*b4^3
20*(a3^4 + b3^4)*(a2^2 + b2^2)*a1^3*a2^2*a3^5*a4^3*b1^3*b2^2*b3*b4^3
-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*q1^3*q2^2*q3*q4^3

c20143=sum(c20.find(a1^3*a4^3*a3^4*w0))+sum(c20.find(a1^3*a4^3*b3^4*w0))
c20143
c20143.factor()
s20143=-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*q1^3*q2*q3^2*q4^3
s20143
#This is the sum of the terms with a1^3, a4^3 and either a3^4 or b3^4, its \
factorization, and its substitution.
20*a1^3*a2^5*a3^4*a4^3*b1^3*b2*b3^2*b4^3 + 20*a1^3*a2*a3^4*a4^3*b1^3*b2^5*b3^2*b4^3 +
20*a1^3*a2^5*a3^2*a4^3*b1^3*b2*b3^4*b4^3 + 20*a1^3*a2*a3^2*a4^3*b1^3*b2^5*b3^4*b4^3
20*(a2^4 + b2^4)*(a3^2 + b3^2)*a1^3*a2*a3^2*a4^3*b1^3*b2*b3^2*b4^3
-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*q1^3*q2*q3^2*q4^3

c20231=sum(c20.find(a2^3*a3^3*a1^4*w0))+sum(c20.find(a2^3*a3^3*b1^4*w0))
c20231
c20231.factor()
s20231=-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^3*q4
s20231
#This is the sum of the terms with a2^3, a3^3 and either a1^4 or b1^4, its \
factorization, and its substitution.
20*a1^4*a2^3*a3^3*a4^5*b1^2*b2^3*b3^3*b4 + 20*a1^2*a2^3*a3^3*a4^5*b1^4*b2^3*b3^3*b4 +
20*a1^4*a2^3*a3^3*a4*b1^2*b2^3*b3^3*b4^5 + 20*a1^2*a2^3*a3^3*a4*b1^4*b2^3*b3^3*b4^5
20*(a4^4 + b4^4)*(a1^2 + b1^2)*a1^2*a2^3*a3^3*a4*b1^2*b2^3*b3^3*b4
-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3^3*q4

c20234=sum(c20.find(a2^3*a3^3*a4^4*w0))+sum(c20.find(a2^3*a3^3*b4^4*w0))
c20234
c20234.factor()
s20234=-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p4^2 + 2*q4)*q1*q2^3*q3^3*q4^2
s20234
#This is the sum of the terms with a2^3, a3^3 and either a4^4 or b4^4, its \
factorization, and its substitution.
20*a1^5*a2^3*a3^3*a4^4*b1*b2^3*b3^3*b4^2 + 20*a1*a2^3*a3^3*a4^4*b1^5*b2^3*b3^3*b4^2 +
20*a1^5*a2^3*a3^3*a4^2*b1*b2^3*b3^3*b4^4 + 20*a1*a2^3*a3^3*a4^2*b1^5*b2^3*b3^3*b4^4
20*(a1^4 + b1^4)*(a4^2 + b4^2)*a1*a2^3*a3^3*a4^2*b1*b2^3*b3^3*b4^2
-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p4^2 + 2*q4)*q1*q2^3*q3^3*q4^2

c20241=sum(c20.find(a2^3*a4^3*a1^4*w0))+sum(c20.find(a2^3*a4^3*b1^4*w0))
c20241

```

```

c20241.factor()
s20241=-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3*q4^3
s20241
#This is the sum of the terms with a2^3, a4^3 and either a1^4 or b1^4, its \
factorization , and its substitution.
20*a1^4*a2^3*a3^5*a4^3*b1^2*b2^3*b3*b4^3 + 20*a1^2*a2^3*a3^5*a4^3*b1^4*b2^3*b3*b4^3 +
20*a1^4*a2^3*a3^5*a4^3*b1^2*b2^3*b3^5*b4^3 + 20*a1^2*a2^3*a3^5*a4^3*b1^4*b2^3*b3^5*b4^3
20*(a3^4 + b3^4)*(a1^2 + b1^2)*a1^2*a2^3*a3^5*a4^3*b1^2*b2^3*b3*b4^3
-20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3*q4^3

c20243=sum(c20.find(a2^3*a4^3*a3^4*w0))+sum(c20.find(a2^3*a4^3*b3^4*w0))
c20243
c20243.factor()
s20243=-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p3^2 + 2*q3)*q1*q2^3*q3^2*q4^3
s20243
#This is the sum of the terms with a2^3, a4^3 and either a2^4 or b2^4, its \
factorization , and its substitution.
20*a1^5*a2^3*a3^4*a4^3*b1*b2^3*b3^2*b4^3 + 20*a1*a2^3*a3^4*a4^3*b1^5*b2^3*b3^2*b4^3 +
20*a1^5*a2^3*a3^2*a4^3*b1*b2^3*b3^4*b4^3 + 20*a1*a2^3*a3^2*a4^3*b1^5*b2^3*b3^4*b4^3
20*(a1^4 + b1^4)*(a3^2 + b3^2)*a1*a2^3*a3^2*a4^3*b1*b2^3*b3^2*b4^3
-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p3^2 + 2*q3)*q1*q2^3*q3^2*q4^3

c20341=sum(c20.find(a3^3*a4^3*a1^4*w0))+sum(c20.find(a3^3*a4^3*b1^4*w0))
c20341
c20341.factor()
s20341=-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*q1^2*q2^2*q3^3*q4^3
s20341
#This is the sum of the terms with a3^3, a4^3 and either a1^4 or b1^4, its \
factorization , and its substitution.
20*a1^4*a2^5*a3^3*a4^3*b1^2*b2^2*b3^3*b4^3 + 20*a1^2*a2^5*a3^3*a4^3*b1^4*b2^2*b3^3*b4^3 +
20*a1^4*a2^3*a3^3*a4^3*b1^2*b2^5*b3^3*b4^3 + 20*a1^2*a2^3*a3^3*a4^3*b1^4*b2^5*b3^3*b4^3
20*(a2^4 + b2^4)*(a1^2 + b1^2)*a1^2*a2^3*a3^3*a4^3*b1^2*b2^2*b3^3*b4^3
-20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 + 2*q1)*q1^2*q2^2*q3^3*q4^3

c20342=sum(c20.find(a3^3*a4^3*a2^4*w0))+sum(c20.find(a3^3*a4^3*b2^4*w0))
c20342
c20342.factor()
s20342=-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*q1*q2^2*q3^3*q4^3
s20342
#This is the sum of the terms with a3^3, a4^3 and either a2^4 or b2^4, its \
factorization , and its substitution.
20*a1^5*a2^4*a3^3*a4^3*b1*b2^2*b3^3*b4^3 + 20*a1*a2^4*a3^3*a4^3*b1^5*b2^2*b3^3*b4^3 +
20*a1^5*a2^2*a3^3*a4^3*b1*b2^4*b3^3*b4^3 + 20*a1*a2^2*a3^3*a4^3*b1^5*b2^4*b3^3*b4^3
20*(a1^4 + b1^4)*(a2^2 + b2^2)*a1*a2^2*a3^3*a4^3*b1*b2^2*b3^3*b4^3
-20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*q1*q2^2*q3^3*q4^3

bool(c20==c20123+c20124+c20132+c20134+c20142+c20143+c20231+c20234+c20241+c20243\
+c20341+c20342)
#This is a check to make sure as we seperated the groups we used every term in \
c20 once and only once.
True

120=s20123+s20124+s20132+s20134+s20142+s20143+s20231+s20234+s20241+s20243+\
s20341+s20342
120
1020=120(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
bool(1020==c20)
#This line brings all the group substitutions we created for c20 back into one \
expression and checks to make sure that when back substitute we end up with \
c20.

```

```

-20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4 - 20*((p4^2 + 2*q4)^2 -
2*q4^2)*(p2^2 + 2*q2)*q1^3*q2^2*q3^3*q4 - 20*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 +
2*q1)*q1^2*q2^3*q3^3*q4 - 20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p4^2 + 2*q4)*q1^3*q2^3*q3*q4^2 -
20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3*q4^2 - 20*((p1^2 + 2*q1)^2 -
2*q1^2)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3*q4^2 - 20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 +
2*q2)*q1^3*q2^2*q3^3*q4^3 - 20*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*q1^2*q2^3*q3*q4^3 -
20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4^3 - 20*((p1^2 + 2*q1)^2 -
2*q1^2)*(p3^2 + 2*q3)*q1^2*q2^3*q3^2*q4^3 - 20*((p2^2 + 2*q2)^2 - 2*q2^2)*(p1^2 +
2*q1)*q1^2*q2^2*q3^3*q4^3 - 20*((p1^2 + 2*q1)^2 - 2*q1^2)*(p2^2 + 2*q2)*q1^2*q2^2*q3^3*q4^3
True

```

```

c28=sum(f10.find(28*w0))
c28
c28.factor()
s28=28*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*\
q4^2
s28

```

```

#This finds all our terms in f10 that have a 20 in them and then adds them \
together, its factorization, and its substitution.

```

```

28*a1^4*a2^4*a3^4*a4^4*b1^2*b2^2*b3^2*b4^2 + 28*a1^2*a2^4*a3^4*a4^4*b1^4*b2^2*b3^2*b4^2 +
28*a1^4*a2^2*a3^4*a4^4*b1^2*b2^4*b3^2*b4^2 + 28*a1^2*a2^2*a3^4*a4^4*b1^4*b2^4*b3^2*b4^2 +
28*a1^4*a2^4*a3^2*a4^4*b1^2*b2^2*b3^4*b4^2 + 28*a1^2*a2^4*a3^2*a4^4*b1^4*b2^2*b3^4*b4^2 +
28*a1^4*a2^2*a3^2*a4^4*b1^2*b2^4*b3^4*b4^2 + 28*a1^2*a2^2*a3^2*a4^4*b1^4*b2^4*b3^4*b4^2 +
28*a1^4*a2^4*a3^4*a4^2*b1^2*b2^2*b3^2*b4^4 + 28*a1^2*a2^4*a3^4*a4^2*b1^4*b2^2*b3^2*b4^4 +
28*a1^4*a2^2*a3^4*a4^2*b1^2*b2^4*b3^2*b4^4 + 28*a1^2*a2^2*a3^4*a4^2*b1^4*b2^4*b3^2*b4^4 +
28*a1^4*a2^4*a3^2*a4^2*b1^2*b2^2*b3^4*b4^4 + 28*a1^2*a2^4*a3^2*a4^2*b1^4*b2^2*b3^4*b4^4 +
28*a1^4*a2^2*a3^2*a4^2*b1^2*b2^4*b3^4*b4^4 + 28*a1^2*a2^2*a3^2*a4^2*b1^4*b2^4*b3^4*b4^4
28*(a1^2 + b1^2)*(a2^2 + b2^2)*(a3^2 + b3^2)*(a4^2 +
b4^2)*a1^2*a2^2*a3^2*a4^2*b1^2*b2^2*b3^2*b4^2
28*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4^2

```

```

l28=s28
l28
l028=l28(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
bool(l028==c28)

```

```

#This line brings all the group substitutions we created for c28 back into one \
expression and checks to make sure that when back substitute we end up with \
c28.

```

```

28*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^2*q3^2*q4^2
True

```

```

c32=sum(f10.find(32*w0))
c32

```

```

#This finds all our terms in f10 that have a 32 in them and then adds them \
together. The groups we form will have one of the four a variables or its \
corresponding b variable to the fifth power.

```

```

32*a1^3*a2^3*a3^3*a4^5*b1^3*b2^3*b3^3*b4 + 32*a1^3*a2^3*a3^5*a4^3*b1^3*b2^3*b3^3*b4^3 +
32*a1^3*a2^5*a3^3*a4^3*b1^3*b2^3*b3^3*b4^3 + 32*a1^5*a2^3*a3^3*a4^3*b1^3*b2^3*b3^3*b4^3 +
32*a1^3*a2^3*a3^3*a4^3*b1^5*b2^3*b3^3*b4^3 + 32*a1^3*a2^3*a3^3*a4^3*b1^3*b2^5*b3^3*b4^3 +
32*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^5*b4^3 + 32*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^3*b4^5

```

```

c321=sum(c32.find(a1^5*w0))+sum(c32.find(b1^5*w0))
c321
c321.factor()
s321=32*((p1^2 + 2*q1)^2 - 2*q1^2)*q1*q2^3*q3^3*q4^3
s321

```

```

#This is the sum of the terms with either a1^4 or b1^4, its factorization, and \
its substitution.

```

```

32*a1^5*a2^3*a3^3*a4^3*b1*b2^3*b3^3*b4^3 + 32*a1*a2^3*a3^3*a4^3*b1^5*b2^3*b3^3*b4^3
32*(a1^4 + b1^4)*a1*a2^3*a3^3*a4^3*b1*b2^3*b3^3*b4^3
32*((p1^2 + 2*q1)^2 - 2*q1^2)*q1*q2^3*q3^3*q4^3

```

```

c322=sum(c32.find(a2^5*w0))+sum(c32.find(b2^5*w0))
c322
c322.factor()
s322=32*((p2^2 + 2*q2)^2 - 2*q2^2)*q1^3*q2*q3^3*q4^3
s322

```

```

#This is the sum of the terms with either a2^4 or b2^4, its factorization, and \
its substitution.

```

```

32*a1^3*a2^5*a3^3*a4^3*b1^3*b2*b3^3*b4^3 + 32*a1^3*a2*a3^3*a4^3*b1^3*b2^5*b3^3*b4^3
32*(a2^4 + b2^4)*a1^3*a2*a3^3*a4^3*b1^3*b2*b3^3*b4^3
32*((p2^2 + 2*q2)^2 - 2*q2^2)*q1^3*q2*q3^3*q4^3

```

```

c323=sum(c32.find(a3^5*w0))+sum(c32.find(b3^5*w0))
c323
c323.factor()
s323=32*((p3^2 + 2*q3)^2 - 2*q3^2)*q1^3*q2^3*q3*q4^3
s323

```

```

#This is the sum of the terms with either a3^4 or b3^4, its factorization, and \
its substitution.

```

```

32*a1^3*a2^3*a3^5*a4^3*b1^3*b2^3*b3*b4^3 + 32*a1^3*a2^3*a3*a4^3*b1^3*b2^3*b3^5*b4^3
32*(a3^4 + b3^4)*a1^3*a2^3*a3*a4^3*b1^3*b2^3*b3*b4^3
32*((p3^2 + 2*q3)^2 - 2*q3^2)*q1^3*q2^3*q3*q4^3

```

```

c324=sum(c32.find(a4^5*w0))+sum(c32.find(b4^5*w0))
c324
c324.factor()
s324=32*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4
s324

```

```

#This is the sum of the terms with either a4^4 or b4^4, its factorization, and \
its substitution.

```

```

32*a1^3*a2^3*a3^3*a4^5*b1^3*b2^3*b3^3*b4 + 32*a1^3*a2^3*a3^3*a4*b1^3*b2^3*b3^3*b4^5
32*(a4^4 + b4^4)*a1^3*a2^3*a3^3*a4*b1^3*b2^3*b3^3*b4
32*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4

```

```

bool(c32==c321+c322+c323+c324)

```

```

#This is a check to make sure as we seperated the groups we used every term in \
c32 once and only once.

```

```

True

```

```

l32=s321+s322+s323+s324
l32

```

```

l032=l32(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)

```

```

bool(l032==c32)

```

```

#This line brings all the group substitutions we created for c32 back into one \
expression and checks to make sure that when back substitute we end up with \
c32.

```

```

32*((p4^2 + 2*q4)^2 - 2*q4^2)*q1^3*q2^3*q3^3*q4 + 32*((p3^2 + 2*q3)^2 -
2*q3^2)*q1^3*q2^3*q3^3*q4^3 + 32*((p2^2 + 2*q2)^2 - 2*q2^2)*q1^3*q2^3*q3^3*q4^3 + 32*((p1^2 +
2*q1)^2 - 2*q1^2)*q1*q2^3*q3^3*q4^3

```

```

True

```

```

c46=sum(f10.find(46*w0))
c46

```

```

#This finds all our terms in f10 that have a 46 in them and then adds them \
together. The groups we form will have one of the four variables to the \
third power.

```

$46*a_1^3*a_2^4*a_3^4*a_4^4*b_1^3*b_2^2*b_3^2*b_4^2 + 46*a_1^4*a_2^3*a_3^4*a_4^4*b_1^2*b_2^3*b_3^2*b_4^2 + 46*a_1^2*a_2^3*a_3^4*a_4^4*b_1^4*b_2^3*b_3^2*b_4^2 + 46*a_1^4*a_2^4*a_3^3*a_4^4*b_1^2*b_2^2*b_3^3*b_4^2 + 46*a_1^2*a_2^4*a_3^3*a_4^4*b_1^4*b_2^2*b_3^3*b_4^2 + 46*a_1^4*a_2^2*a_3^3*a_4^4*b_1^2*b_2^4*b_3^3*b_4^2 + 46*a_1^2*a_2^2*a_3^3*a_4^4*b_1^4*b_2^4*b_3^3*b_4^2 + 46*a_1^3*a_2^4*a_3^2*a_4^4*b_1^3*b_2^2*b_3^4*b_4^2 + 46*a_1^4*a_2^3*a_3^2*a_4^4*b_1^2*b_2^3*b_3^4*b_4^2 + 46*a_1^2*a_2^3*a_3^2*a_4^4*b_1^4*b_2^3*b_3^4*b_4^2 + 46*a_1^4*a_2^2*a_3^2*a_4^4*b_1^2*b_2^4*b_3^4*b_4^2 + 46*a_1^2*a_2^2*a_3^2*a_4^4*b_1^4*b_2^4*b_3^4*b_4^2 + 46*a_1^4*a_2^4*a_3^4*a_4^3*b_1^2*b_2^2*b_3^2*b_4^3 + 46*a_1^2*a_2^4*a_3^4*a_4^3*b_1^4*b_2^2*b_3^2*b_4^3 + 46*a_1^4*a_2^2*a_3^4*a_4^3*b_1^2*b_2^4*b_3^2*b_4^3 + 46*a_1^2*a_2^2*a_3^4*a_4^3*b_1^4*b_2^4*b_3^2*b_4^3 + 46*a_1^4*a_2^4*a_3^2*a_4^3*b_1^2*b_2^2*b_3^4*b_4^3 + 46*a_1^2*a_2^4*a_3^2*a_4^3*b_1^4*b_2^2*b_3^4*b_4^3 + 46*a_1^4*a_2^2*a_3^2*a_4^3*b_1^2*b_2^4*b_3^4*b_4^3 + 46*a_1^2*a_2^2*a_3^2*a_4^3*b_1^4*b_2^4*b_3^4*b_4^3 + 46*a_1^3*a_2^4*a_3^4*a_4^2*b_1^3*b_2^2*b_3^2*b_4^4 + 46*a_1^4*a_2^3*a_3^4*a_4^2*b_1^2*b_2^3*b_3^2*b_4^4 + 46*a_1^2*a_2^3*a_3^4*a_4^2*b_1^4*b_2^3*b_3^2*b_4^4 + 46*a_1^4*a_2^4*a_3^3*a_4^2*b_1^2*b_2^2*b_3^3*b_4^4 + 46*a_1^2*a_2^4*a_3^3*a_4^2*b_1^4*b_2^2*b_3^3*b_4^4 + 46*a_1^4*a_2^2*a_3^3*a_4^2*b_1^2*b_2^4*b_3^3*b_4^4 + 46*a_1^2*a_2^2*a_3^3*a_4^2*b_1^4*b_2^4*b_3^3*b_4^4 + 46*a_1^3*a_2^4*a_3^2*a_4^2*b_1^3*b_2^2*b_3^4*b_4^4 + 46*a_1^4*a_2^3*a_3^2*a_4^2*b_1^2*b_2^3*b_3^4*b_4^4 + 46*a_1^2*a_2^3*a_3^2*a_4^2*b_1^4*b_2^3*b_3^4*b_4^4 + 46*a_1^4*a_2^4*a_3^2*a_4^2*b_1^2*b_2^4*b_3^4*b_4^4 + 46*a_1^2*a_2^4*a_3^2*a_4^2*b_1^4*b_2^4*b_3^4*b_4^4$

```

c461=sum(c46.find(a1^3*w0))
c461
c461.factor()
s461=-46*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4^2
s461
#This is the sum of the terms with a1^3, its factorization, and its \
substitution.

```

$46*a_1^3*a_2^4*a_3^4*a_4^4*b_1^3*b_2^2*b_3^2*b_4^2 + 46*a_1^3*a_2^2*a_3^4*a_4^4*b_1^3*b_2^4*b_3^2*b_4^2 + 46*a_1^3*a_2^4*a_3^2*a_4^4*b_1^3*b_2^2*b_3^4*b_4^2 + 46*a_1^3*a_2^2*a_3^2*a_4^4*b_1^3*b_2^4*b_3^4*b_4^2 + 46*a_1^3*a_2^4*a_3^2*a_4^2*b_1^3*b_2^2*b_3^2*b_4^4 + 46*a_1^3*a_2^2*a_3^2*a_4^2*b_1^3*b_2^4*b_3^2*b_4^4 + 46*a_1^3*a_2^2*a_3^2*a_4^2*b_1^3*b_2^2*b_3^4*b_4^4 + 46*a_1^3*a_2^2*a_3^2*a_4^2*b_1^3*b_2^4*b_3^4*b_4^4 + 46*(a2^2 + b2^2)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^3*a2^2*a3^2*a4^2*b1^3*b2^2*b3^2*b4^2 - 46*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4^2$

```

c462=sum(c46.find(a2^3*w0))
c462
c462.factor()
s462=-46*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2*q4^2
s462
#This is the sum of the terms with a2^3, its factorization, and its \
substitution.

```

$46*a_1^4*a_2^3*a_3^4*a_4^4*b_1^2*b_2^3*b_3^2*b_4^2 + 46*a_1^2*a_2^3*a_3^4*a_4^4*b_1^4*b_2^3*b_3^2*b_4^2 + 46*a_1^4*a_2^3*a_3^2*a_4^4*b_1^2*b_2^3*b_3^4*b_4^2 + 46*a_1^2*a_2^3*a_3^2*a_4^4*b_1^4*b_2^3*b_3^4*b_4^2 + 46*a_1^4*a_2^3*a_3^4*a_4^2*b_1^2*b_2^2*b_3^2*b_4^4 + 46*a_1^2*a_2^3*a_3^4*a_4^2*b_1^4*b_2^2*b_3^2*b_4^4 + 46*a_1^4*a_2^3*a_3^2*a_4^2*b_1^2*b_2^4*b_3^2*b_4^4 + 46*a_1^2*a_2^3*a_3^2*a_4^2*b_1^4*b_2^4*b_3^2*b_4^4 + 46*(a1^2 + b1^2)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^2*a2^3*a3^2*a4^2*b1^2*b2^3*b3^2*b4^2 - 46*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2*q4^2$

```

c463=sum(c46.find(a3^3*w0))
c463
c463.factor()
s463=-46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^3*q4^2
s463
#This is the sum of the terms with a3^3, its factorization, and its \
substitution.

```

$46*a_1^4*a_2^4*a_3^3*a_4^4*b_1^2*b_2^2*b_3^3*b_4^2 + 46*a_1^2*a_2^4*a_3^3*a_4^4*b_1^4*b_2^2*b_3^3*b_4^2 + 46*a_1^4*a_2^2*a_3^3*a_4^4*b_1^2*b_2^4*b_3^3*b_4^2 + 46*a_1^2*a_2^2*a_3^3*a_4^4*b_1^4*b_2^4*b_3^3*b_4^2 + 46*a_1^4*a_2^4*a_3^3*a_4^2*b_1^2*b_2^2*b_3^3*b_4^4 + 46*a_1^2*a_2^4*a_3^3*a_4^2*b_1^4*b_2^2*b_3^3*b_4^4 + 46*a_1^4*a_2^2*a_3^3*a_4^2*b_1^2*b_2^4*b_3^3*b_4^4 + 46*a_1^2*a_2^2*a_3^3*a_4^2*b_1^4*b_2^4*b_3^3*b_4^4 + 46*(a1^2 + b1^2)*(a2^2 + b2^2)*(a4^2 + b4^2)*a1^2*a2^2*a3^3*a4^2*b1^2*b2^2*b3^3*b4^2 - 46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^3*q4^2$

```

c464=sum(c46.find(a4^3*w0))
c464

```

```

c464.factor()
s464=-46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4^3
s464
#This is the sum of the terms with a4^3, its factorization, and its \
substitution.
46*a1^4*a2^4*a3^4*a4^3*b1^2*b2^2*b3^2*b4^3 + 46*a1^2*a2^4*a3^4*a4^3*b1^4*b2^2*b3^2*b4^3 +
46*a1^4*a2^2*a3^4*a4^3*b1^2*b2^4*b3^2*b4^3 + 46*a1^2*a2^2*a3^4*a4^3*b1^4*b2^4*b3^2*b4^3 +
46*a1^4*a2^4*a3^2*a4^3*b1^2*b2^2*b3^4*b4^3 + 46*a1^2*a2^4*a3^2*a4^3*b1^4*b2^2*b3^4*b4^3 +
46*a1^4*a2^2*a3^2*a4^3*b1^2*b2^4*b3^4*b4^3 + 46*a1^2*a2^2*a3^2*a4^3*b1^4*b2^4*b3^4*b4^3
46*(a1^2 + b1^2)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^2*a2^2*a3^2*a4^3*b1^2*b2^2*b3^2*b4^3
-46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q3^2*q4^3

bool(c46==c461+c462+c463+c464)
#This is a check to make sure as we seperated the groups we used every term in \
c46 once and only once.
True

146=s461+s462+s463+s464
146
1046=146 (p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
bool(1046==c46)
#This line brings all the group substitutions we created for c46 back into one \
expression and checks to make sure that when back substitute we end up with \
c46.
-46*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2*q4^2 - 46*(p1^2 + 2*q1)*(p3^2
+ 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2*q4^2 - 46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 +
2*q4)*q1^2*q2^2*q3^3*q4^2 - 46*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 +
2*q3)*q1^2*q2^2*q3^2*q4^3
True

c70=sum(f10.find(70*w0))
c70
#This finds all our terms in f10 that have a 70 in them and then adds them \
together. The groups we form will have two of the four a variables to the \
third power.
70*a1^3*a2^3*a3^4*a4^4*b1^3*b2^3*b3^2*b4^2 + 70*a1^3*a2^4*a3^3*a4^4*b1^3*b2^2*b3^3*b4^2 +
70*a1^4*a2^3*a3^3*a4^4*b1^2*b2^3*b3^3*b4^2 + 70*a1^2*a2^3*a3^3*a4^4*b1^4*b2^3*b3^3*b4^2 +
70*a1^3*a2^2*a3^3*a4^4*b1^3*b2^4*b3^3*b4^2 + 70*a1^3*a2^3*a3^2*a4^4*b1^3*b2^3*b3^4*b4^2 +
70*a1^3*a2^4*a3^4*a4^3*b1^3*b2^2*b3^2*b4^3 + 70*a1^4*a2^3*a3^4*a4^3*b1^2*b2^3*b3^2*b4^3 +
70*a1^2*a2^3*a3^4*a4^3*b1^4*b2^3*b3^2*b4^3 + 70*a1^3*a2^2*a3^4*a4^3*b1^3*b2^4*b3^2*b4^3 +
70*a1^4*a2^4*a3^3*a4^3*b1^2*b2^2*b3^3*b4^3 + 70*a1^2*a2^4*a3^3*a4^3*b1^4*b2^2*b3^3*b4^3 +
70*a1^4*a2^2*a3^3*a4^3*b1^2*b2^4*b3^3*b4^3 + 70*a1^2*a2^2*a3^3*a4^3*b1^4*b2^4*b3^3*b4^3 +
70*a1^3*a2^4*a3^2*a4^3*b1^3*b2^2*b3^4*b4^3 + 70*a1^4*a2^3*a3^2*a4^3*b1^2*b2^3*b3^4*b4^3 +
70*a1^2*a2^3*a3^2*a4^3*b1^4*b2^3*b3^4*b4^3 + 70*a1^4*a2^2*a3^2*a4^3*b1^2*b2^4*b3^4*b4^3 +
70*a1^3*a2^3*a3^4*a4^2*b1^3*b2^3*b3^2*b4^4 + 70*a1^3*a2^4*a3^3*a4^2*b1^3*b2^2*b3^3*b4^4 +
70*a1^4*a2^3*a3^3*a4^2*b1^2*b2^3*b3^3*b4^4 + 70*a1^2*a2^3*a3^3*a4^2*b1^4*b2^3*b3^3*b4^4 +
70*a1^3*a2^2*a3^3*a4^2*b1^3*b2^4*b3^3*b4^4 + 70*a1^3*a2^3*a3^2*a4^2*b1^3*b2^3*b3^4*b4^4

c7012=sum(c70.find(a1^3*a2^3*w0))
c7012
c7012.factor()
s7012=70*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2*q4^2
s7012
#This is the sum of the terms with a1^3 and a2^3, its factorization, and its \
substitution.
70*a1^3*a2^3*a3^4*a4^4*b1^3*b2^3*b3^2*b4^2 + 70*a1^3*a2^3*a3^2*a4^4*b1^3*b2^3*b3^4*b4^2 +
70*a1^3*a2^3*a3^4*a4^2*b1^3*b2^3*b3^2*b4^4 + 70*a1^3*a2^3*a3^2*a4^2*b1^3*b2^3*b3^4*b4^4
70*(a3^2 + b3^2)*(a4^2 + b4^2)*a1^3*a2^3*a3^2*a4^2*b1^3*b2^3*b3^2*b4^2
70*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2*q4^2

```



```

c7013=sum(c70.find(a1^3*a3^3*w0))
c7013
c7013.factor()
s7013=70*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3*q4^2
s7013
#This is the sum of the terms with a1^3 and a3^3, its factorization, and its \
substitution.
70*a1^3*a2^4*a3^3*a4^4*b1^3*b2^2*b3^3*b4^2 + 70*a1^3*a2^2*a3^3*a4^4*b1^3*b2^4*b3^3*b4^2 +
70*a1^3*a2^4*a3^3*a4^2*b1^3*b2^2*b3^3*b4^4 + 70*a1^3*a2^2*a3^3*a4^2*b1^3*b2^4*b3^3*b4^4
70*(a2^2 + b2^2)*(a4^2 + b4^2)*a1^3*a2^2*a3^3*a4^2*b1^3*b2^2*b3^3*b4^2
70*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^3*q2^2*q3^3*q4^2

c7014=sum(c70.find(a1^3*a4^3*w0))
c7014
c7014.factor()
s7014=70*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4^3
s7014
#This is the sum of the terms with a1^3 and a4^3, its factorization, and its \
substitution.
70*a1^3*a2^4*a3^4*a4^3*b1^3*b2^2*b3^2*b4^3 + 70*a1^3*a2^2*a3^4*a4^3*b1^3*b2^4*b3^2*b4^3 +
70*a1^3*a2^4*a3^2*a4^3*b1^3*b2^2*b3^4*b4^3 + 70*a1^3*a2^2*a3^2*a4^3*b1^3*b2^4*b3^4*b4^3
70*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^3*a2^2*a3^2*a4^3*b1^3*b2^2*b3^2*b4^3
70*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4^3

c7023=sum(c70.find(a2^3*a3^3*w0))
c7023
c7023.factor()
s7023=70*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3*q4^2
s7023
#This is the sum of the terms with a2^3 and a3^3, its factorization, and its \
substitution.
70*a1^4*a2^3*a3^3*a4^4*b1^2*b2^3*b3^3*b4^2 + 70*a1^2*a2^3*a3^3*a4^4*b1^4*b2^3*b3^3*b4^2 +
70*a1^4*a2^3*a3^3*a4^2*b1^2*b2^3*b3^3*b4^4 + 70*a1^2*a2^3*a3^3*a4^2*b1^4*b2^3*b3^3*b4^4
70*(a1^2 + b1^2)*(a4^2 + b4^2)*a1^2*a2^3*a3^3*a4^2*b1^2*b2^3*b3^3*b4^2
70*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3*q4^2

c7024=sum(c70.find(a2^3*a4^3*w0))
c7024
c7024.factor()
s7024=70*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^3*q3^2*q4^3
s7024
#This is the sum of the terms with a2^3 and a4^3, its factorization, and its \
substitution.
70*a1^4*a2^3*a3^4*a4^3*b1^2*b2^3*b3^2*b4^3 + 70*a1^2*a2^3*a3^4*a4^3*b1^4*b2^3*b3^2*b4^3 +
70*a1^4*a2^3*a3^2*a4^3*b1^2*b2^3*b3^4*b4^3 + 70*a1^2*a2^3*a3^2*a4^3*b1^4*b2^3*b3^4*b4^3
70*(a1^2 + b1^2)*(a3^2 + b3^2)*a1^2*a2^3*a3^2*a4^3*b1^2*b2^3*b3^2*b4^3
70*(p1^2 + 2*q1)*(p3^2 + 2*q3)*q1^2*q2^3*q3^2*q4^3

c7034=sum(c70.find(a3^3*a4^3*w0))
c7034
c7034.factor()
s7034=70*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^3*q4^3
s7034
#This is the sum of the terms with a3^3 and a4^3, its factorization, and its \
substitution.
70*a1^4*a2^4*a3^3*a4^3*b1^2*b2^2*b3^3*b4^3 + 70*a1^2*a2^4*a3^3*a4^3*b1^4*b2^2*b3^3*b4^3 +
70*a1^4*a2^2*a3^3*a4^3*b1^2*b2^4*b3^3*b4^3 + 70*a1^2*a2^2*a3^3*a4^3*b1^4*b2^4*b3^3*b4^3
70*(a1^2 + b1^2)*(a2^2 + b2^2)*a1^2*a2^2*a3^3*a4^3*b1^2*b2^2*b3^3*b4^3
70*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^3*q4^3

bool(c70==c7012+c7013+c7014+c7023+c7024+c7034)

```



```

#This is a check to make sure as we seperated the groups we used every term in \
c70 once and only once.
True

170=s7012+s7013+s7014+s7023+s7024+s7034
170
1070=170 (p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
bool(1070==c70)
#This line brings all the group substitutions we created for c70 back into one \
expression and checks to make sure that when back substitute we end up with \
c70.
70*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^3*q3^2*q4^2 + 70*(p2^2 + 2*q2)*(p4^2 +
2*q4)*q1^3*q2^2*q3^3*q4^2 + 70*(p1^2 + 2*q1)*(p4^2 + 2*q4)*q1^2*q2^3*q3^3*q4^2 + 70*(p2^2
+ 2*q2)*(p3^2 + 2*q3)*q1^3*q2^2*q3^2*q4^3 + 70*(p1^2 + 2*q1)*(p3^2 +
2*q3)*q1^2*q2^3*q3^2*q4^3 + 70*(p1^2 + 2*q1)*(p2^2 + 2*q2)*q1^2*q2^2*q3^3*q4^3
True

c104=sum(f10.find(104*w0))
c104
#This finds all our terms in f10 that have a 104 in them and then adds them \
together. The groups we form will have one of the four a variables or its \
corresponding b variable to the forth power.
104*a1^3*a2^3*a3^3*a4^4*b1^3*b2^3*b3^3*b4^2 + 104*a1^3*a2^3*a3^4*a4^3*b1^3*b2^3*b3^2*b4^3
+ 104*a1^3*a2^4*a3^3*a4^3*b1^3*b2^2*b3^3*b4^3 +
104*a1^4*a2^3*a3^3*a4^3*b1^2*b2^3*b3^3*b4^3 + 104*a1^2*a2^3*a3^3*a4^3*b1^4*b2^3*b3^3*b4^3
+ 104*a1^3*a2^2*a3^3*a4^3*b1^3*b2^4*b3^3*b4^3 +
104*a1^3*a2^3*a3^2*a4^3*b1^3*b2^3*b3^4*b4^3 + 104*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^3*b4^4

c1041=sum(c104.find(a1^4*w0))+sum(c104.find(b1^4*w0))
c1041
c1041.factor()
s1041=-104*(p1^2 + 2*q1)*q1^2*q2^3*q3^3*q4^3
s1041
#This is the sum of the terms with either a1^4 or b1^4, its factorization, and \
its substitution.
104*a1^4*a2^3*a3^3*a4^3*b1^2*b2^3*b3^3*b4^3 + 104*a1^2*a2^3*a3^3*a4^3*b1^4*b2^3*b3^3*b4^3
104*(a1^2 + b1^2)*a1^3*a2^3*a3^3*a4^3*b1^2*b2^3*b3^3*b4^3
-104*(p1^2 + 2*q1)*q1^2*q2^3*q3^3*q4^3

c1042=sum(c104.find(a2^4*w0))+sum(c104.find(b2^4*w0))
c1042
c1042.factor()
s1042=-104*(p2^2 + 2*q2)*q1^3*q2^2*q3^3*q4^3
s1042
#This is the sum of the terms with either a2^4 or b2^4, its factorization, and \
its substitution.
104*a1^3*a2^4*a3^3*a4^3*b1^3*b2^2*b3^3*b4^3 + 104*a1^3*a2^2*a3^3*a4^3*b1^3*b2^4*b3^3*b4^3
104*(a2^2 + b2^2)*a1^3*a2^2*a3^3*a4^3*b1^3*b2^2*b3^3*b4^3
-104*(p2^2 + 2*q2)*q1^3*q2^2*q3^3*q4^3

c1043=sum(c104.find(a3^4*w0))+sum(c104.find(b3^4*w0))
c1043
c1043.factor()
s1043=-104*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4^3
s1043
#This is the sum of the terms with either a3^4 or b3^4, its factorization, and \
its substitution.
104*a1^3*a2^3*a3^4*a4^3*b1^3*b2^3*b3^2*b4^3 + 104*a1^3*a2^3*a3^2*a4^3*b1^3*b2^3*b3^4*b4^3
104*(a3^2 + b3^2)*a1^3*a2^3*a3^2*a4^3*b1^3*b2^3*b3^2*b4^3

```

$-104*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4^3$

```
c1044=sum(c104.find(a4^4*w0))+sum(c104.find(b4^4*w0))
c1044
c1044.factor()
s1044=-104*(p4^2 + 2*q4)*q1^3*q2^3*q3^3*q4^2
s1044
```

```
#This is the sum of the terms with either a4^4 or b4^4, its factorization, and \
its substitution.
```

```
104*a1^3*a2^3*a3^3*a4^4*b1^3*b2^3*b3^3*b4^2 + 104*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^3*b4^4
104*(a4^2 + b4^2)*a1^3*a2^3*a3^3*a4^2*b1^3*b2^3*b3^3*b4^2
-104*(p4^2 + 2*q4)*q1^3*q2^3*q3^3*q4^2
```

```
bool(c104==c1041+c1042+c1043+c1044)
```

```
#This is a check to make sure as we seperated the groups we used every term in \
c104 once and only once.
```

```
True
```

```
l104=s1041+s1042+s1043+s1044
```

```
l104
```

```
l0104=l104(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
```

```
bool(l0104==c104)
```

```
#This line brings all the group substitutions we created for c104 back into one\
expression and checks to make sure that when back substitute we end up with\
c104.
```

```
-104*(p4^2 + 2*q4)*q1^3*q2^3*q3^3*q4^2 - 104*(p3^2 + 2*q3)*q1^3*q2^3*q3^2*q4^3 - 104*(p2^2
+ 2*q2)*q1^3*q2^2*q3^3*q4^3 - 104*(p1^2 + 2*q1)*q1^2*q2^3*q3^3*q4^3
```

```
True
```

```
c152=sum(f10.find(152*w0))
```

```
c152
```

```
s152=152*q1^3*q2^3*q3^3*q4^3
```

```
s152
```

```
#This finds all our terms in f10 that have a 152 in them and then adds them \
together, its factorization, and its substitution.
```

```
152*a1^3*a2^3*a3^3*a4^3*b1^3*b2^3*b3^3*b4^3
152*q1^3*q2^3*q3^3*q4^3
```

```
l152=s152
```

```
l152
```

```
l0152=l152(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3\
=-1*a3*b3, q4=-1*a4*b4)
```

```
bool(l0152==c152)
```

```
#This line brings all the group substitutions we created for c152 back into one\
expression and checks to make sure that when back substitute we end up with\
c152.
```

```
152*q1^3*q2^3*q3^3*q4^3
```

```
True
```

```
c1=f10 - c2 - c4 - c7 - c12 - c20 - c28 - c32 - c46 - c70 - c104 - c152
```

```
c1
```

```
#This finds all our terms in f11 that have a 1 in them and then adds them \
together. The first groups we form will have one of the four a variables to \
the third power and either a second a variable or its corresponding b \
variable to the sixth power.
```

```
a1^3*a2^4*a3^4*a4^6*b1^3*b2^2*b3^2 + a1^4*a2^3*a3^4*a4^6*b1^2*b2^3*b3^2 +
a1^2*a2^3*a3^4*a4^6*b1^4*b2^3*b3^2 + a1^3*a2^2*a3^4*a4^6*b1^3*b2^4*b3^2 +
a1^4*a2^4*a3^3*a4^6*b1^2*b2^2*b3^3 + a1^2*a2^4*a3^3*a4^6*b1^4*b2^2*b3^3 +
a1^4*a2^2*a3^3*a4^6*b1^2*b2^4*b3^3 + a1^2*a2^2*a3^3*a4^6*b1^4*b2^4*b3^3 +
```



```

s114=-((p4^2 + 2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1\
^3*q2^2*q3^2
s114
#This is the sum of the terms with a1^3 and either a4^6 or b4^6, its \
factorization, and its substitution.
a1^3*a2^4*a3^4*a4^6*b1^3*b2^2*b3^2 + a1^3*a2^2*a3^4*a4^6*b1^3*b2^4*b3^2 +
a1^3*a2^4*a3^2*a4^6*b1^3*b2^2*b3^4 + a1^3*a2^2*a3^2*a4^6*b1^3*b2^4*b3^4 +
a1^3*a2^4*a3^4*b1^3*b2^2*b3^2*b4^6 + a1^3*a2^2*a3^4*b1^3*b2^4*b3^2*b4^6 +
a1^3*a2^4*a3^2*b1^3*b2^2*b3^4*b4^6 + a1^3*a2^2*a3^2*b1^3*b2^4*b3^4*b4^6
(a4^4 - a4^2*b4^2 + b4^4)*(a2^2 + b2^2)*(a3^2 + b3^2)*(a4^2 +
b4^2)*a1^3*a2^2*a3^2*b1^3*b2^2*b3^2
-((p4^2 + 2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2

c121=sum(c1.find(a2^3*a1^6*w0))+sum(c1.find(a2^3*b1^6*w0))
c121
c121.factor()
s121=-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q2\
^3*q3^2*q4^2
s121
#This is the sum of the terms with a2^3 and either a1^6 or b1^6, its \
factorization, and its substitution.
a1^6*a2^3*a3^4*a4^4*b2^3*b3^2*b4^2 + a2^3*a3^4*a4^4*b1^6*b2^3*b3^2*b4^2 +
a1^6*a2^3*a3^2*a4^4*b2^3*b3^4*b4^2 + a2^3*a3^2*a4^4*b1^6*b2^3*b3^4*b4^2 +
a1^6*a2^3*a3^4*a4^2*b2^3*b3^2*b4^4 + a2^3*a3^4*a4^2*b1^6*b2^3*b3^2*b4^4 +
a1^6*a2^3*a3^2*a4^2*b2^3*b3^4*b4^4 + a2^3*a3^2*a4^2*b1^6*b2^3*b3^4*b4^4
(a1^4 - a1^2*b1^2 + b1^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*(a4^2 +
b4^2)*a2^3*a3^2*a4^2*b2^3*b3^2*b4^2
-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q2^3*q3^2*q4^2

c123=sum(c1.find(a2^3*a3^6*w0))+sum(c1.find(a2^3*b3^6*w0))
c123
c123.factor()
s123=-((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1\
^2*q2^3*q4^2
s123
#This is the sum of the terms with a2^3 and either a3^6 or b3^6, its \
factorization, and its substitution.
a1^4*a2^3*a3^6*a4^4*b1^2*b2^3*b4^2 + a1^2*a2^3*a3^6*a4^4*b1^4*b2^3*b4^2 +
a1^4*a2^3*a4^4*b1^2*b2^3*b3^6*b4^2 + a1^2*a2^3*a4^4*b1^4*b2^3*b3^6*b4^2 +
a1^4*a2^3*a3^6*a4^2*b1^2*b2^3*b4^4 + a1^2*a2^3*a3^6*a4^2*b1^4*b2^3*b4^4 +
a1^4*a2^3*a4^2*b1^2*b2^3*b3^6*b4^4 + a1^2*a2^3*a4^2*b1^4*b2^3*b3^6*b4^4
(a3^4 - a3^2*b3^2 + b3^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*(a4^2 +
b4^2)*a1^2*a2^3*a4^2*b1^2*b2^3*b4^2
-((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q4^2

c124=sum(c1.find(a2^3*a4^6*w0))+sum(c1.find(a2^3*b4^6*w0))
c124
c124.factor()
s124=-((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1\
^2*q2^3*q3^2
s124
#This is the sum of the terms with a2^3 and either a4^6 or b4^6, its \
factorization, and its substitution.
a1^4*a2^3*a3^4*a4^6*b1^2*b2^3*b3^2 + a1^2*a2^3*a3^4*a4^6*b1^4*b2^3*b3^2 +
a1^4*a2^3*a3^2*a4^6*b1^2*b2^3*b3^4 + a1^2*a2^3*a3^2*a4^6*b1^4*b2^3*b3^4 +
a1^4*a2^3*a3^4*b1^2*b2^3*b3^2*b4^6 + a1^2*a2^3*a3^4*b1^4*b2^3*b3^2*b4^6 +
a1^4*a2^3*a3^2*b1^2*b2^3*b3^4*b4^6 + a1^2*a2^3*a3^2*b1^4*b2^3*b3^4*b4^6
(a4^4 - a4^2*b4^2 + b4^4)*(a1^2 + b1^2)*(a3^2 + b3^2)*(a4^2 +
b4^2)*a1^2*a2^3*a3^2*b1^2*b2^3*b3^2
-((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2

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c131=sum(c1.find(a3^3*a1^6*w0))+sum(c1.find(a3^3*b1^6*w0))
c131
c131.factor()
s131=-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q2\
^2*q3^3*q4^2
s131
#This is the sum of the terms with a3^3 and either a1^6 or b1^6, its \
factorization, and its substitution.
a1^6*a2^4*a3^3*a4^4*b2^2*b3^3*b4^2 + a2^4*a3^3*a4^4*b1^6*b2^2*b3^3*b4^2 +
a1^6*a2^2*a3^3*a4^4*b2^4*b3^3*b4^2 + a2^2*a3^3*a4^4*b1^6*b2^4*b3^3*b4^2 +
a1^6*a2^4*a3^3*a4^2*b2^2*b3^3*b4^4 + a2^4*a3^3*a4^2*b1^6*b2^2*b3^3*b4^4 +
a1^6*a2^2*a3^3*a4^2*b2^4*b3^3*b4^4 + a2^2*a3^3*a4^2*b1^6*b2^4*b3^3*b4^4
(a1^4 - a1^2*b1^2 + b1^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a4^2 +
b4^2)*a2^2*a3^3*a4^2*b2^2*b3^3*b4^2
-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q2^2*q3^3*q4^2

c132=sum(c1.find(a3^3*a2^6*w0))+sum(c1.find(a3^3*b2^6*w0))
c132
c132.factor()
s132=-((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1\
^2*q3^3*q4^2
s132
#This is the sum of the terms with a3^3 and either a2^6 or b2^6, its \
factorization, and its substitution.
a1^4*a2^6*a3^3*a4^4*b1^2*b3^3*b4^2 + a1^2*a2^6*a3^3*a4^4*b1^4*b3^3*b4^2 +
a1^4*a3^3*a4^4*b1^2*b2^6*b3^3*b4^2 + a1^2*a3^3*a4^4*b1^4*b2^6*b3^3*b4^2 +
a1^4*a2^6*a3^3*a4^2*b1^2*b3^3*b4^4 + a1^2*a2^6*a3^3*a4^2*b1^4*b3^3*b4^4 +
a1^4*a3^3*a4^2*b1^2*b2^6*b3^3*b4^4 + a1^2*a3^3*a4^2*b1^4*b2^6*b3^3*b4^4
(a2^4 - a2^2*b2^2 + b2^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a4^2 +
b4^2)*a1^2*a3^3*a4^2*b1^2*b3^3*b4^2
-((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q3^3*q4^2

c134=sum(c1.find(a3^3*a4^6*w0))+sum(c1.find(a3^3*b4^6*w0))
c134
c134.factor()
s134=-((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1\
^2*q2^2*q3^3
s134
#This is the sum of the terms with a3^3 and either a4^6 or b4^6, its \
factorization, and its substitution.
a1^4*a2^4*a3^3*a4^6*b1^2*b2^2*b3^3 + a1^2*a2^4*a3^3*a4^6*b1^4*b2^2*b3^3 +
a1^4*a2^2*a3^3*a4^6*b1^2*b2^4*b3^3 + a1^2*a2^2*a3^3*a4^6*b1^4*b2^4*b3^3 +
a1^4*a2^4*a3^3*b1^2*b2^2*b3^3*b4^6 + a1^2*a2^4*a3^3*b1^4*b2^2*b3^3*b4^6 +
a1^4*a2^2*a3^3*b1^2*b2^4*b3^3*b4^6 + a1^2*a2^2*a3^3*b1^4*b2^4*b3^3*b4^6
(a4^4 - a4^2*b4^2 + b4^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a4^2 +
b4^2)*a1^2*a2^2*a3^3*b1^2*b2^2*b3^3
-((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^3

c141=sum(c1.find(a4^3*a1^6*w0))+sum(c1.find(a4^3*b1^6*w0))
c141
c141.factor()
s141=-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q2\
^2*q3^2*q4^3
s141
#This is the sum of the terms with a4^3 and either a1^6 or b1^6, its \
factorization, and its substitution.
a1^6*a2^4*a3^4*a4^3*b2^2*b3^2*b4^3 + a2^4*a3^4*a4^3*b1^6*b2^2*b3^2*b4^3 +
a1^6*a2^2*a3^4*a4^3*b2^4*b3^2*b4^3 + a2^2*a3^4*a4^3*b1^6*b2^4*b3^2*b4^3 +
a1^6*a2^4*a3^2*a4^3*b2^2*b3^4*b4^3 + a2^4*a3^2*a4^3*b1^6*b2^2*b3^4*b4^3 +
a1^6*a2^2*a3^2*a4^3*b2^4*b3^4*b4^3 + a2^2*a3^2*a4^3*b1^6*b2^4*b3^4*b4^3
(a1^4 - a1^2*b1^2 + b1^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a3^2 +

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$$b3^2)*a2^2*a3^2*a4^3*b2^2*b3^2*b4^3$$

$$-((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q2^2*q3^2*q4^3$$

c142=sum(c1.find(a4^3*a2^6*w0))+sum(c1.find(a4^3*b2^6*w0))

c142

c142.factor()

$$s142=-((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q3^2*q4^3$$

s142

#This is the sum of the terms with a4^3 and either a2^6 or b2^6, its factorization, and its substitution.

$$a1^4*a2^6*a3^4*a4^3*b1^2*b3^2*b4^3 + a1^2*a2^6*a3^4*a4^3*b1^4*b3^2*b4^3 +$$

$$a1^4*a3^4*a4^3*b1^2*b2^6*b3^2*b4^3 + a1^2*a3^4*a4^3*b1^4*b2^6*b3^2*b4^3 +$$

$$a1^4*a2^6*a3^2*a4^3*b1^2*b3^4*b4^3 + a1^2*a2^6*a3^2*a4^3*b1^4*b3^4*b4^3 +$$

$$a1^4*a3^2*a4^3*b1^2*b2^6*b3^4*b4^3 + a1^2*a3^2*a4^3*b1^4*b2^6*b3^4*b4^3$$

$$(a2^4 - a2^2*b2^2 + b2^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^2*a3^2*a4^3*b1^2*b3^2*b4^3$$

$$-((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q3^2*q4^3$$

c143=sum(c1.find(a4^3*a3^6*w0))+sum(c1.find(a4^3*b3^6*w0))

c143

c143.factor()

$$s143=-((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q4^3$$

s143

#This is the sum of the terms with a4^3 and either a3^6 or b3^6, its factorization, and its substitution.

$$a1^4*a2^4*a3^6*a4^3*b1^2*b2^2*b4^3 + a1^2*a2^4*a3^6*a4^3*b1^4*b2^2*b4^3 +$$

$$a1^4*a2^2*a3^6*a4^3*b1^2*b2^4*b4^3 + a1^2*a2^2*a3^6*a4^3*b1^4*b2^4*b4^3 +$$

$$a1^4*a2^4*a4^3*b1^2*b2^2*b3^6*b4^3 + a1^2*a2^4*a4^3*b1^4*b2^2*b3^6*b4^3 +$$

$$a1^4*a2^2*a4^3*b1^2*b2^4*b3^6*b4^3 + a1^2*a2^2*a4^3*b1^4*b2^4*b3^6*b4^3$$

$$(a3^4 - a3^2*b3^2 + b3^4)*(a1^2 + b1^2)*(a2^2 + b2^2)*(a3^2 + b3^2)*a1^2*a2^2*a4^3*b1^2*b2^2*b4^3$$

$$-((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q4^3$$

c01=c1-c112-c113-c114-c121-c123-c124-c131-c132-c134-c141-c142-c143

c01

#This gets all the terms from c1 that have not been assigned to a group. The groups we form will have one of the four a variables or its corresponding b variable to the forth power and either a second a variable or its corresponding b variable to the forth power.

$$a1^4*a2^4*a3^5*a4^5*b1^2*b2^2*b3*b4 + a1^2*a2^4*a3^5*a4^5*b1^4*b2^2*b3*b4 +$$

$$a1^4*a2^2*a3^5*a4^5*b1^2*b2^4*b3*b4 + a1^2*a2^2*a3^5*a4^5*b1^4*b2^4*b3*b4 +$$

$$a1^4*a2^5*a3^4*a4^5*b1^2*b2*b3^2*b4 + a1^2*a2^5*a3^4*a4^5*b1^4*b2*b3^2*b4 +$$

$$a1^5*a2^4*a3^4*a4^5*b1*b2^2*b3^2*b4 + a1*a2^4*a3^4*a4^5*b1^5*b2^2*b3^2*b4 +$$

$$a1^5*a2^2*a3^4*a4^5*b1*b2^4*b3^2*b4 + a1*a2^2*a3^4*a4^5*b1^5*b2^4*b3^2*b4 +$$

$$a1^4*a2^5*a3^2*a4^5*b1^2*b2^5*b3^2*b4 + a1^2*a2^5*a3^2*a4^5*b1^4*b2^5*b3^2*b4 +$$

$$a1^5*a2^4*a3^2*a4^5*b1*b2^2*b3^4*b4 + a1*a2^4*a3^2*a4^5*b1^5*b2^2*b3^4*b4 +$$

$$a1^5*a2^2*a3^2*a4^5*b1*b2^4*b3^4*b4 + a1*a2^2*a3^2*a4^5*b1^5*b2^4*b3^4*b4 +$$

$$a1^4*a2^5*a3^2*a4^5*b1^2*b2^5*b3^4*b4 + a1^2*a2^5*a3^2*a4^5*b1^4*b2^5*b3^4*b4 +$$

$$a1^5*a2^4*a3^2*a4^5*b1*b2^2*b3^5*b4 + a1*a2^4*a3^2*a4^5*b1^5*b2^2*b3^5*b4 +$$

$$a1^4*a2^2*a3^2*a4^5*b1*b2^4*b3^5*b4 + a1*a2^2*a3^2*a4^5*b1^5*b2^4*b3^5*b4 +$$

$$a1^4*a2^5*a3^5*a4^4*b1^2*b2^2*b3*b4^2 + a1^2*a2^5*a3^5*a4^4*b1^4*b2^2*b3*b4^2 +$$

$$a1^4*a2^2*a3^5*a4^4*b1^2*b2^4*b3*b4^2 + a1^2*a2^2*a3^5*a4^4*b1^4*b2^4*b3*b4^2 +$$

$$a1^4*a2^4*a3^5*a4^4*b1*b2^2*b3^5*b4^2 + a1*a2^4*a3^5*a4^4*b1^5*b2^2*b3^5*b4^2 +$$

$$a1^4*a2^2*a3^5*a4^4*b1*b2^4*b3^5*b4^2 + a1*a2^2*a3^5*a4^4*b1^5*b2^4*b3^5*b4^2 +$$

$$a1^4*a2^5*a3^4*a4^4*b1^2*b2^5*b3^2*b4^2 + a1^2*a2^5*a3^4*a4^4*b1^4*b2^5*b3^2*b4^2 +$$

$$a1^4*a2^2*a3^4*a4^4*b1^2*b2^5*b3^4*b4^2 + a1^2*a2^2*a3^4*a4^4*b1^4*b2^5*b3^4*b4^2 +$$

$$a1^4*a2^4*a3^4*a4^4*b1*b2^2*b3^5*b4^2 + a1*a2^4*a3^4*a4^4*b1^5*b2^2*b3^5*b4^2 +$$

$$a1^4*a2^2*a3^4*a4^4*b1*b2^4*b3^5*b4^2 + a1*a2^2*a3^4*a4^4*b1^5*b2^4*b3^5*b4^2 +$$

$$a1^4*a2^5*a3^2*a4^4*b1^2*b2^5*b3^4*b4^2 + a1^2*a2^5*a3^2*a4^4*b1^4*b2^5*b3^4*b4^2 +$$

$$a1^4*a2^2*a3^2*a4^4*b1^2*b2^5*b3^6*b4^2 + a1^2*a2^2*a3^2*a4^4*b1^4*b2^5*b3^6*b4^2 +$$

$a1^4 a2^5 a3^2 a4^5 b1^2 b2 b3^4 b4 + a1^2 a2^5 a3^2 a4^5 b1^4 b2 b3^4 b4 +$
 $a1^4 a2^2 a3^2 a4^5 b1^2 b2^5 b3^4 b4 + a1^2 a2^2 a3^2 a4^5 b1^4 b2^5 b3^4 b4 +$
 $a1^4 a2^5 a3^4 a4^5 b1^2 b2 b3^2 b4^5 + a1^2 a2^5 a3^4 a4^5 b1^4 b2 b3^2 b4^5 +$
 $a1^4 a2^2 a3^4 a4^5 b1^2 b2^5 b3^2 b4^5 + a1^2 a2^2 a3^4 a4^5 b1^4 b2^5 b3^2 b4^5 +$
 $a1^4 a2^5 a3^2 a4^5 b1^2 b2 b3^4 b4^5 + a1^2 a2^5 a3^2 a4^5 b1^4 b2 b3^4 b4^5 +$
 $a1^4 a2^2 a3^2 a4^5 b1^2 b2^5 b3^4 b4^5 + a1^2 a2^2 a3^2 a4^5 b1^4 b2^5 b3^4 b4^5$
 $(a2^4 + b2^4)(a4^4 + b4^4)(a1^2 + b1^2)(a3^2 + b3^2)a1^2 a2^2 a3^2 a4^5 b1^2 b2 b3^2 b4$
 $((p2^2 + 2*q2)^2 - 2*q2^2)((p4^2 + 2*q4)^2 - 2*q4^2)(p1^2 + 2*q1)(p3^2 +$
 $2*q3)*q1^2*q2^2*q3^2*q4$

```

c0114=sum(c01.find(a1^4*a4^4*w0))+sum(c01.find(a1^4*b4^4*w0))+sum(c01.find(b1\
^4*b4^4*w0))+sum(c01.find(b1^4*a4^4*w0))
c0114
c0114.factor()
s0114=((p2^2 + 2*q2)^2 - 2*q2^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p1^2 + 2*q1)*(p4^2\
+ 2*q4)*q1^2*q2^2*q3^2*q4^2
s0114

```

#This is the sum of the terms with either a1^4 or b1^4 and either a4^4 or b4^4,\ its factorization, and its substitution.

$a1^4 a2^5 a3^5 a4^4 b1^2 b2 b3^4 b4^2 + a1^2 a2^5 a3^5 a4^4 b1^4 b2 b3^4 b4^2 +$
 $a1^4 a2^2 a3^5 a4^4 b1^2 b2^5 b3^4 b4^2 + a1^2 a2^2 a3^5 a4^4 b1^4 b2^5 b3^4 b4^2 +$
 $a1^4 a2^5 a3^4 a4^4 b1^2 b2 b3^5 b4^2 + a1^2 a2^5 a3^4 a4^4 b1^4 b2 b3^5 b4^2 +$
 $a1^4 a2^2 a3^4 a4^4 b1^2 b2^5 b3^5 b4^2 + a1^2 a2^2 a3^4 a4^4 b1^4 b2^5 b3^5 b4^2 +$
 $a1^4 a2^5 a3^5 a4^2 b1^2 b2 b3^4 b4^4 + a1^2 a2^5 a3^5 a4^2 b1^4 b2 b3^4 b4^4 +$
 $a1^4 a2^2 a3^5 a4^2 b1^2 b2^5 b3^4 b4^4 + a1^2 a2^2 a3^5 a4^2 b1^4 b2^5 b3^4 b4^4 +$
 $a1^4 a2^5 a3^4 a4^2 b1^2 b2 b3^5 b4^4 + a1^2 a2^5 a3^4 a4^2 b1^4 b2 b3^5 b4^4 +$
 $a1^4 a2^2 a3^4 a4^2 b1^2 b2^5 b3^5 b4^4 + a1^2 a2^2 a3^4 a4^2 b1^4 b2^5 b3^5 b4^4$
 $(a2^4 + b2^4)(a3^4 + b3^4)(a1^2 + b1^2)(a4^2 + b4^2)a1^2 a2^2 a3^4 a4^2 b1^2 b2 b3^4 b4^2$
 $((p2^2 + 2*q2)^2 - 2*q2^2)((p3^2 + 2*q3)^2 - 2*q3^2)(p1^2 + 2*q1)(p4^2 +$
 $2*q4)*q1^2*q2^2*q3^2*q4^2$

```

c0123=sum(c01.find(a2^4*a3^4*w0))+sum(c01.find(a2^4*b3^4*w0))+sum(c01.find(b2\
^4*b3^4*w0))+sum(c01.find(b2^4*a3^4*w0))
c0123
c0123.factor()
s0123=((p1^2 + 2*q1)^2 - 2*q1^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p2^2 + 2*q2)*(p3^2\
+ 2*q3)*q1*q2^2*q3^2*q4
s0123

```

#This is the sum of the terms with either a2^4 or b2^4 and either a3^4 or b3^4,\ its factorization, and its substitution.

$a1^5 a2^4 a3^4 a4^5 b1^2 b2^2 b3^2 b4 + a1^2 a2^4 a3^4 a4^5 b1^5 b2^2 b3^2 b4 +$
 $a1^5 a2^2 a3^4 a4^5 b1^2 b2^4 b3^2 b4 + a1^2 a2^2 a3^4 a4^5 b1^5 b2^4 b3^2 b4 +$
 $a1^5 a2^4 a3^2 a4^5 b1^2 b2^2 b3^4 b4 + a1^2 a2^4 a3^2 a4^5 b1^5 b2^2 b3^4 b4 +$
 $a1^5 a2^2 a3^2 a4^5 b1^2 b2^4 b3^4 b4 + a1^2 a2^2 a3^2 a4^5 b1^5 b2^4 b3^4 b4 +$
 $a1^5 a2^4 a3^4 a4^5 b1^2 b2^2 b3^2 b4^5 + a1^2 a2^4 a3^4 a4^5 b1^5 b2^2 b3^2 b4^5 +$
 $a1^5 a2^2 a3^4 a4^5 b1^2 b2^4 b3^2 b4^5 + a1^2 a2^2 a3^4 a4^5 b1^5 b2^4 b3^2 b4^5 +$
 $a1^5 a2^4 a3^2 a4^5 b1^2 b2^2 b3^4 b4^5 + a1^2 a2^4 a3^2 a4^5 b1^5 b2^2 b3^4 b4^5 +$
 $a1^5 a2^2 a3^2 a4^5 b1^2 b2^4 b3^4 b4^5 + a1^2 a2^2 a3^2 a4^5 b1^5 b2^4 b3^4 b4^5$
 $(a1^4 + b1^4)(a4^4 + b4^4)(a2^2 + b2^2)(a3^2 + b3^2)a1^2 a2^2 a3^2 a4^5 b1^2 b2^2 b3^2 b4$
 $((p1^2 + 2*q1)^2 - 2*q1^2)((p4^2 + 2*q4)^2 - 2*q4^2)(p2^2 + 2*q2)(p3^2 +$
 $2*q3)*q1*q2^2*q3^2*q4$

```

c0124=sum(c01.find(a2^4*a4^4*w0))+sum(c01.find(a2^4*b4^4*w0))+sum(c01.find(b2\
^4*b4^4*w0))+sum(c01.find(b2^4*a4^4*w0))
c0124
c0124.factor()
s0124=((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*(p4^2\
+ 2*q4)*q1*q2^2*q3^2*q4^2
s0124

```

#This is the sum of the terms with either a2^4 or b2^4 and either a4^4 or b4^4,\ its factorization, and its substitution.

```

a1^5*a2^4*a3^5*a4^4*b1*b2^2*b3*b4^2 + a1*a2^4*a3^5*a4^4*b1^5*b2^2*b3*b4^2 +
a1^5*a2^2*a3^5*a4^4*b1*b2^4*b3*b4^2 + a1*a2^2*a3^5*a4^4*b1^5*b2^4*b3*b4^2 +
a1^5*a2^4*a3^4*a4^4*b1*b2^2*b3^5*b4^2 + a1*a2^4*a3^4*a4^4*b1^5*b2^2*b3^5*b4^2 +
a1^5*a2^2*a3^4*a4^4*b1*b2^4*b3^5*b4^2 + a1*a2^2*a3^4*a4^4*b1^5*b2^4*b3^5*b4^2 +
a1^5*a2^4*a3^5*a4^2*b1*b2^2*b3*b4^4 + a1*a2^4*a3^5*a4^2*b1^5*b2^2*b3*b4^4 +
a1^5*a2^2*a3^5*a4^2*b1*b2^4*b3*b4^4 + a1*a2^2*a3^5*a4^2*b1^5*b2^4*b3*b4^4 +
a1^5*a2^4*a3^4*a2*b1*b2^2*b3^5*b4^4 + a1*a2^4*a3^4*a2*b1^5*b2^2*b3^5*b4^4 +
a1^5*a2^2*a3^4*a2*b1*b2^4*b3^5*b4^4 + a1*a2^2*a3^4*a2*b1^5*b2^4*b3^5*b4^4
(a1^4 + b1^4)*(a3^4 + b3^4)*(a2^2 + b2^2)*(a4^2 + b4^2)*a1*a2^2*a3^4*a4^2*b1*b2^2*b3*b4^2
((p1^2 + 2*q1)^2 - 2*q1^2)*((p3^2 + 2*q3)^2 - 2*q3^2)*(p2^2 + 2*q2)*(p4^2 +
2*q4)*q1*q2^2*q3*q4^2

```

```

c0134=sum(c01.find(a3^4*a4^4*w0))+sum(c01.find(a3^4*b4^4*w0))+sum(c01.find(b3\
^4*b4^4*w0))+sum(c01.find(b3^4*a4^4*w0))
c0134
c0134.factor()
s0134=((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*(p4^2\
+ 2*q4)*q1*q2^2*q3^2*q4^2
s0134
#This is the sum of the terms with either a3^4 or b3^4 and either a4^4 or b4^4,\
its factorization, and its substitution.

```

```

a1^5*a2^5*a3^4*a4^4*b1*b2*b3^2*b4^2 + a1*a2^5*a3^4*a4^4*b1^5*b2*b3^2*b4^2 +
a1^5*a2^3*a3^4*a4^4*b1*b2^5*b3^2*b4^2 + a1*a2^3*a3^4*a4^4*b1^5*b2^5*b3^2*b4^2 +
a1^5*a2^5*a3^2*a4^4*b1*b2*b3^4*b4^2 + a1*a2^5*a3^2*a4^4*b1^5*b2*b3^4*b4^2 +
a1^5*a2^3*a3^2*a4^4*b1*b2^5*b3^4*b4^2 + a1*a2^3*a3^2*a4^4*b1^5*b2^5*b3^4*b4^2 +
a1^5*a2^5*a3^4*a4^2*b1*b2*b3^2*b4^4 + a1*a2^5*a3^4*a4^2*b1^5*b2*b3^2*b4^4 +
a1^5*a2^3*a3^4*a4^2*b1*b2^5*b3^2*b4^4 + a1*a2^3*a3^4*a4^2*b1^5*b2^5*b3^2*b4^4 +
a1^5*a2^5*a3^2*a4^2*b1*b2*b3^4*b4^4 + a1*a2^5*a3^2*a4^2*b1^5*b2*b3^4*b4^4 +
a1^5*a2^3*a3^2*a4^2*b1*b2^5*b3^4*b4^4 + a1*a2^3*a3^2*a4^2*b1^5*b2^5*b3^4*b4^4
(a1^4 + b1^4)*(a2^4 + b2^4)*(a3^2 + b3^2)*(a4^2 + b4^2)*a1*a2^2*a3^2*a4^2*b1*b2^2*b3^2*b4^2
((p1^2 + 2*q1)^2 - 2*q1^2)*((p2^2 + 2*q2)^2 - 2*q2^2)*(p3^2 + 2*q3)*(p4^2 +
2*q4)*q1*q2^2*q3^2*q4^2

```

```

bool(c1==c0112+c0113+c0114+c0123+c0124+c0134 - (-c112 - c113 - c114 - c121 - c123 - c124 - \
c131 - c132 - c134 - c141 - c142 - c143))
#This is a check to make sure as we seperated the groups we used every term in \
c1 once and only once.

```

True

```

l1=s0112+s0113+s0114+s0123+s0124+s0134+s112+s113+s114+s121+s123+s124+s131+s132+\
s134+s141+s142+s143

```

l1

```

l01=l1(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3=-1*\
a3*b3, q4=-1*a4*b4)

```

```
bool(l01==c1)
```

```

#This line brings all the group substitutions we created for c1 back into one \
expression and checks to make sure that when back substitute we end up with \
c1.

```

```

-((p4^2 + 2*q4)^2 - 3*q4^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q3^2 -
((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q3^2 -
((p4^2 + 2*q4)^2 - 3*q4^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q2^2*q3^3 -
((p3^2 + 2*q3)^2 - 3*q3^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q2^2*q4^2 -
((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^2*q2^3*q4^2 -
((p2^2 + 2*q2)^2 - 3*q2^2)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q1^3*q3^2*q4^2 -
((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p3^2 + 2*q3)*(p4^2 + 2*q4)*q2^3*q3^2*q4^2 -
((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q1^2*q3^3*q4^2 -
((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p4^2 + 2*q4)*q2^2*q3^3*q4^2 -
((p3^2 + 2*q3)^2 - 3*q3^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q2^2*q4^3 -
((p2^2 + 2*q2)^2 - 3*q2^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q1^2*q3^2*q4^3 -
((p1^2 + 2*q1)^2 - 3*q1^2)*(p1^2 + 2*q1)*(p2^2 + 2*q2)*(p3^2 + 2*q3)*q2^2*q3^2*q4^3 +
((p3^2 + 2*q3)^2 - 2*q3^2)*((p4^2 + 2*q4)^2 - 2*q4^2)*(p1^2 + 2*q1)*(p2^2 +

```


$$\begin{aligned}
& 2^*q3^2 * q1^3 * q2^3 * q3 * q4^3 - ((p2^2 + 2^*q2)^2 - 3^*q2^2) * (p1^2 + 2^*q1) * (p2^2 + 2^*q2) * (p3^2 + \\
& 2^*q3) * q1^2 * q3^2 * q4^3 + 2^*((p2^2 + 2^*q2)^2 - 3^*q2^2) * (p2^2 + 2^*q2) * (p3^2 + \\
& 2^*q3) * q1^3 * q3^2 * q4^3 + 12^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * (p1^2 + 2^*q1) * (p3^2 + \\
& 2^*q3) * q1^2 * q2 * q3^2 * q4^3 - 20^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * (p3^2 + 2^*q3) * q1^3 * q2 * q3^2 * q4^3 - \\
& ((p1^2 + 2^*q1)^2 - 3^*q1^2) * (p1^2 + 2^*q1) * (p2^2 + 2^*q2) * (p3^2 + 2^*q3) * q2^2 * q3^2 * q4^3 + \\
& 12^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * (p2^2 + 2^*q2) * (p3^2 + 2^*q3) * q1 * q2^2 * q3^2 * q4^3 + 2^*((p1^2 + \\
& 2^*q1)^2 - 3^*q1^2) * (p1^2 + 2^*q1) * (p3^2 + 2^*q3) * q2^3 * q3^2 * q4^3 - 20^*((p1^2 + 2^*q1)^2 - \\
& 2^*q1^2) * (p3^2 + 2^*q3) * q1 * q2^3 * q3^2 * q4^3 + 2^*((p2^2 + 2^*q2)^2 - 3^*q2^2) * (p1^2 + 2^*q1) * (p2^2 \\
& + 2^*q2) * q1^2 * q3^3 * q4^3 - 4^*((p2^2 + 2^*q2)^2 - 3^*q2^2) * (p2^2 + 2^*q2) * q1^3 * q3^3 * q4^3 - \\
& 20^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * (p1^2 + 2^*q1) * q1^2 * q2 * q3^3 * q4^3 + 32^*((p2^2 + 2^*q2)^2 - \\
& 2^*q2^2) * q1^3 * q2 * q3^3 * q4^3 + 2^*((p1^2 + 2^*q1)^2 - 3^*q1^2) * (p1^2 + 2^*q1) * (p2^2 + \\
& 2^*q2) * q2^2 * q3^3 * q4^3 - 20^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * (p2^2 + 2^*q2) * q1 * q2^2 * q3^3 * q4^3 - \\
& 4^*((p1^2 + 2^*q1)^2 - 3^*q1^2) * (p1^2 + 2^*q1) * q2^3 * q3^3 * q4^3 + 32^*((p1^2 + 2^*q1)^2 - \\
& 2^*q1^2) * q1 * q2^3 * q3^3 * q4^3 + ((p3^2 + 2^*q3)^2 - 2^*q3^2) * ((p4^2 + 2^*q4)^2 - 2^*q4^2) * (p1^2 + \\
& 2^*q1) * (p2^2 + 2^*q2) * q1^2 * q2^2 * q3 * q4 - 2^*((p3^2 + 2^*q3)^2 - 2^*q3^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * (p2^2 + 2^*q2) * q1^3 * q2^2 * q3 * q4 - 2^*((p3^2 + 2^*q3)^2 - 2^*q3^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * (p1^2 + 2^*q1) * q1^2 * q2^3 * q3 * q4 + 4^*((p3^2 + 2^*q3)^2 - 2^*q3^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * q1^3 * q2^3 * q3 * q4 + ((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p4^2 + 2^*q4)^2 - 2^*q4^2) * (p1^2 + \\
& 2^*q1) * (p3^2 + 2^*q3) * q1^2 * q2 * q3^2 * q4 - 2^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * (p3^2 + 2^*q3) * q1^3 * q2 * q3^2 * q4 + ((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * (p2^2 + 2^*q2) * (p3^2 + 2^*q3) * q1 * q2^2 * q3^2 * q4 - 2^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p4^2 + \\
& 2^*q4)^2 - 2^*q4^2) * (p3^2 + 2^*q3) * q1 * q2^3 * q3^2 * q4 - 2^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p4^2 + \\
& 2^*q4)^2 - 2^*q4^2) * (p1^2 + 2^*q1) * q1^2 * q2 * q3^3 * q4 + 4^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p4^2 + \\
& 2^*q4)^2 - 2^*q4^2) * q1^3 * q2 * q3^3 * q4 - 2^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * (p2^2 + 2^*q2) * q1 * q2^2 * q3^3 * q4 + 4^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p4^2 + 2^*q4)^2 - \\
& 2^*q4^2) * q1 * q2^3 * q3^3 * q4 + ((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p3^2 + 2^*q3)^2 - 2^*q3^2) * (p1^2 + \\
& 2^*q1) * (p4^2 + 2^*q4) * q1^2 * q2 * q3 * q4^2 - 2^*((p2^2 + 2^*q2)^2 - 2^*q2^2) * ((p3^2 + 2^*q3)^2 - \\
& 2^*q3^2) * (p4^2 + 2^*q4) * q1^3 * q2 * q3 * q4^2 + ((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p3^2 + 2^*q3)^2 - \\
& 2^*q3^2) * (p2^2 + 2^*q2) * (p4^2 + 2^*q4) * q1 * q2^2 * q3 * q4^2 - 2^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p3^2 + \\
& 2^*q3)^2 - 2^*q3^2) * (p4^2 + 2^*q4) * q1 * q2^3 * q3 * q4^2 + ((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p2^2 + \\
& 2^*q2)^2 - 2^*q2^2) * (p3^2 + 2^*q3) * (p4^2 + 2^*q4) * q1 * q2 * q3^2 * q4^2 - 2^*((p1^2 + 2^*q1)^2 - \\
& 2^*q1^2) * ((p2^2 + 2^*q2)^2 - 2^*q2^2) * (p4^2 + 2^*q4) * q1 * q2 * q3^3 * q4^2 - 2^*((p2^2 + 2^*q2)^2 - \\
& 2^*q2^2) * ((p3^2 + 2^*q3)^2 - 2^*q3^2) * (p1^2 + 2^*q1) * q1^2 * q2 * q3 * q4^3 + 4^*((p2^2 + 2^*q2)^2 - \\
& 2^*q2^2) * ((p3^2 + 2^*q3)^2 - 2^*q3^2) * q1^3 * q2 * q3 * q4^3 - 2^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p3^2 + \\
& 2^*q3)^2 - 2^*q3^2) * (p2^2 + 2^*q2) * q1 * q2^2 * q3 * q4^3 + 4^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p3^2 + \\
& 2^*q3)^2 - 2^*q3^2) * q1 * q2^3 * q3 * q4^3 - 2^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p2^2 + 2^*q2)^2 - \\
& 2^*q2^2) * (p3^2 + 2^*q3) * q1 * q2 * q3^2 * q4^3 + 4^*((p1^2 + 2^*q1)^2 - 2^*q1^2) * ((p2^2 + 2^*q2)^2 - \\
& 2^*q2^2) * q1 * q2 * q3^3 * q4^3
\end{aligned}$$

```

ll=1(p1=a1+b1, p2=a2+b2, p3=a3+b3, p4=a4+b4, q1=-1*a1*b1, q2=-1*a2*b2, q3=-1*a3\
*b3, q4=-1*a4*b4)

```

```
bool(ll==f10)
```

```

#This is a check to make sure that when back substitute we end up with the \
original coefficient.

```

```
True
```

```
m=l.expand()
```

```
m
```

```
latex(m)
```

```

#This is the expansion of our coefficient and a latex output for easy copying \
to the thesis.

```

$$\begin{aligned}
& p1^2 * p2^2 * p3^4 * p4^4 * q1^2 * q2^2 * q3 * q4 + p1^2 * p2^4 * p3^2 * p4^4 * q1^2 * q2 * q3^2 * q4 + \\
& p1^4 * p2^2 * p3^2 * p4^4 * q1 * q2^2 * q3^2 * q4 + p1^2 * p2^2 * p4^4 * p3^4 * p4^2 * q1^2 * q2 * q3 * q4^2 + \\
& p1^4 * p2^2 * p3^4 * p4^2 * q1 * q2^2 * q3 * q4^2 + p1^4 * p2^4 * p3^2 * p4^2 * q1 * q2 * q3^2 * q4^2 - \\
& p2^2 * p3^2 * p4^6 * q1^3 * q2^2 * q3^2 - p1^2 * p3^2 * p4^6 * q1^2 * q2^3 * q3^2 - \\
& p1^2 * p2^2 * p4^6 * q1^2 * q2^2 * q3^3 + 5 * p1^2 * p2^2 * p3^2 * p4^4 * q1^2 * q2^2 * q3^2 * q4 - \\
& p2^2 * p3^6 * p4^2 * q1^3 * q2^2 * q4^2 - p1^2 * p3^6 * p4^2 * q1^2 * q2^3 * q4^2 + \\
& 5 * p1^2 * p2^2 * p3^4 * p4^2 * q1^2 * q2^2 * q3 * q4^2 - p2^6 * p3^2 * p4^2 * q1^3 * q3^2 * q4^2 + \\
& 5 * p1^2 * p2^4 * p3^2 * p4^2 * q1^2 * q2 * q3^2 * q4^2 + 5 * p1^4 * p2^2 * p3^2 * p4^2 * q1 * q2^2 * q3^2 * q4^2 -
\end{aligned}$$

$$\begin{aligned}
& p_1^6 p_3^2 p_4^2 q_2^3 q_3^2 q_4^2 - p_1^2 p_2^6 p_4^2 q_1^2 q_3^3 q_4^2 - \\
& p_1^6 p_2^2 p_4^2 q_2^2 q_3^3 q_4^2 - p_1^2 p_2^2 p_3^6 q_1^2 q_2^2 q_4^3 - \\
& p_1^2 p_2^6 p_3^2 q_1^2 q_3^2 q_4^3 - p_1^6 p_2^2 p_3^2 q_2^2 q_3^2 q_4^3 - 2^* p_3^2 p_4^6 q_1^3 q_2^3 q_3^2 \\
& - 2^* p_2^2 p_4^6 q_1^3 q_2^2 q_3^3 - 2^* p_1^2 p_4^6 q_1^2 q_2^3 q_3^3 - \\
& 6^* p_2^2 p_3^2 p_4^4 q_1^3 q_2^2 q_3^2 q_4 - 6^* p_1^2 p_3^2 p_4^4 q_1^2 q_2^3 q_3^2 q_4 - \\
& 6^* p_1^2 p_2^2 p_4^4 q_1^2 q_2^2 q_3^3 q_4 - 2^* p_3^6 p_4^2 q_1^3 q_2^3 q_4^2 - \\
& 6^* p_2^2 p_3^4 p_4^2 q_1^3 q_2^2 q_3^3 q_4^2 - 6^* p_1^2 p_3^4 p_4^2 q_1^2 q_2^3 q_3^3 q_4^2 - \\
& 6^* p_2^4 p_3^2 p_4^2 q_1^3 q_2^2 q_3^2 q_4^2 + 12^* p_1^2 p_2^2 p_3^2 p_4^2 q_1^2 q_2^2 q_3^2 q_4^2 - \\
& 6^* p_1^4 p_3^2 p_4^2 q_1^2 q_2^3 q_3^2 q_4^2 - 2^* p_2^6 p_4^2 q_1^3 q_3^3 q_4^2 - \\
& 6^* p_1^2 p_2^4 p_4^2 q_1^2 q_2^2 q_3^3 q_4^2 - 6^* p_1^4 p_2^2 p_4^2 q_1^2 q_2^2 q_3^3 q_4^2 - \\
& 2^* p_1^6 p_4^2 q_2^3 q_3^3 q_4^2 - 2^* p_2^2 p_3^6 q_1^3 q_2^2 q_4^3 - 2^* p_1^2 p_3^6 q_1^2 q_2^3 q_4^3 - \\
& 6^* p_1^2 p_2^2 p_3^4 q_1^2 q_2^2 q_3^3 q_4^3 - 2^* p_2^6 p_3^2 q_1^3 q_3^2 q_4^3 - \\
& 6^* p_1^2 p_2^4 p_3^2 q_1^2 q_2^2 q_3^2 q_4^3 - 6^* p_1^4 p_2^2 p_3^2 q_1^2 q_2^2 q_3^2 q_4^3 - \\
& 2^* p_1^6 p_3^2 q_2^3 q_3^2 q_4^3 - 2^* p_1^2 p_2^6 q_1^2 q_3^3 q_4^3 - 2^* p_1^6 p_2^2 q_2^2 q_3^3 q_4^3 - \\
& 4^* p_4^6 q_1^3 q_2^3 q_3^3 - 12^* p_3^2 p_4^4 q_1^3 q_2^3 q_3^2 q_4 - 12^* p_2^2 p_4^4 q_1^3 q_2^2 q_3^3 q_4 - \\
& 12^* p_1^2 p_4^4 q_1^2 q_2^3 q_3^3 q_4 - 12^* p_3^4 p_4^2 q_1^3 q_2^3 q_3^3 q_4^2 - \\
& 31^* p_2^2 p_3^2 p_4^2 q_1^3 q_2^2 q_3^2 q_4^2 - 31^* p_1^2 p_3^2 p_4^2 q_1^2 q_2^3 q_3^2 q_4^2 - \\
& 12^* p_2^4 p_4^2 q_1^3 q_2^2 q_3^3 q_4^2 - 31^* p_1^2 p_2^2 p_4^2 q_1^2 q_2^2 q_3^3 q_4^2 - \\
& 12^* p_1^4 p_4^2 q_1^2 q_2^3 q_3^3 q_4^2 - 4^* p_3^6 q_1^3 q_2^3 q_4^3 - 12^* p_2^2 p_3^4 q_1^3 q_2^2 q_3^3 q_4^3 - \\
& 12^* p_1^2 p_3^4 q_1^2 q_2^3 q_3^3 q_4^3 - 12^* p_2^4 p_3^2 q_1^3 q_2^2 q_3^2 q_4^3 - \\
& 31^* p_1^2 p_2^2 p_3^2 q_1^2 q_2^2 q_3^2 q_4^3 - 12^* p_1^4 p_3^2 q_1^2 q_2^3 q_3^2 q_4^3 - \\
& 4^* p_2^6 q_1^3 q_3^3 q_4^3 - 12^* p_1^2 p_2^4 q_1^2 q_2^2 q_3^3 q_4^3 - 12^* p_1^4 p_2^2 q_1^2 q_2^2 q_3^3 q_4^3 - \\
& 4^* p_1^6 q_2^3 q_3^3 q_4^3 - 24^* p_4^4 q_1^3 q_2^3 q_3^3 q_4 - 46^* p_3^2 p_4^2 q_1^3 q_2^3 q_3^2 q_4^2 - \\
& 46^* p_2^2 p_4^2 q_1^3 q_2^2 q_3^3 q_4^2 - 46^* p_1^2 p_4^2 q_1^2 q_2^3 q_3^3 q_4^2 - \\
& 24^* p_3^4 q_1^3 q_2^3 q_3^3 q_4^3 - 46^* p_2^2 p_3^2 q_1^3 q_2^2 q_3^2 q_4^3 - \\
& 46^* p_1^2 p_3^2 q_1^2 q_2^3 q_3^2 q_4^3 - 24^* p_2^4 q_1^3 q_2^2 q_3^3 q_4^3 - \\
& 46^* p_1^2 p_2^2 q_1^2 q_2^2 q_3^3 q_4^3 - 24^* p_1^4 q_1^2 q_2^3 q_3^3 q_4^3 - 60^* p_4^2 q_1^3 q_2^3 q_3^3 q_4^2 \\
& - 60^* p_3^2 q_1^3 q_2^3 q_3^2 q_4^3 - 60^* p_2^2 q_1^3 q_2^2 q_3^3 q_4^3 - 60^* p_1^2 q_1^2 q_2^3 q_3^3 q_4^3 \\
& - 56^* q_1^3 q_2^3 q_3^3 q_4^3
\end{aligned}$$

$$\begin{aligned}
& p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^4} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}} q_{\{4\}} + p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}} q_{\{3\}^2} q_{\{4\}} + p_{\{1\}^4} p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} + p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} + p_{\{1\}^4} p_{\{2\}^2} p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} + p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} + p_{\{1\}^4} p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} - p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^6} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^2} - p_{\{1\}^2} p_{\{3\}^2} p_{\{4\}^6} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^2} - p_{\{1\}^2} p_{\{2\}^2} p_{\{4\}^6} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^3} + 5 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} - p_{\{2\}^2} p_{\{3\}^6} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^2} q_{\{4\}^2} - p_{\{1\}^2} p_{\{3\}^6} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{4\}^2} + 5 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}} q_{\{4\}^2} - p_{\{2\}^6} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^3} q_{\{3\}^2} q_{\{4\}^2} + 5 \setminus, p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} + 5 \setminus, p_{\{1\}^4} p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} - p_{\{1\}^6} p_{\{3\}^2} p_{\{4\}^2} q_{\{2\}^3} q_{\{3\}^2} q_{\{4\}^2} - p_{\{1\}^2} p_{\{2\}^6} p_{\{4\}^2} q_{\{1\}^2} q_{\{3\}^3} q_{\{4\}^2} - p_{\{1\}^6} p_{\{2\}^2} p_{\{4\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} - p_{\{4\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} - p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^6} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}^2} - p_{\{2\}^6} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}^2} - 2 \setminus, p_{\{3\}^2} p_{\{4\}^6} q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^2} - 2 \setminus, p_{\{2\}^2} p_{\{4\}^6} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^3} - 2 \setminus, p_{\{1\}^2} p_{\{4\}^6} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^3} - 6 \setminus, p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}} - 6 \setminus, p_{\{1\}^2} p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^2} q_{\{4\}} - 6 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}} - 2 \setminus, p_{\{3\}^6} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^3} q_{\{4\}^2} - 6 \setminus, p_{\{2\}^2} p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}} q_{\{4\}^2} - 6 \setminus, p_{\{1\}^2} p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}} q_{\{4\}^2} - 6 \setminus, p_{\{2\}^2} p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} + 12 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}^2} - 6 \setminus, p_{\{1\}^4} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^2} q_{\{4\}} - 6 \setminus, p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} - 2 \setminus, p_{\{1\}^6} p_{\{4\}^2} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^2} - 2 \setminus, p_{\{2\}^2} p_{\{3\}^6} q_{\{1\}^3} q_{\{2\}^2} q_{\{4\}^3} - 2 \setminus, p_{\{1\}^2} p_{\{3\}^6} q_{\{1\}^2} q_{\{2\}^3} q_{\{4\}^3} - 6 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^4}
\end{aligned}$$

$$\begin{aligned}
& q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}} q_{\{4\}^3} - 2 \setminus, p_{\{2\}^6} p_{\{3\}^2} q_{\{1\}^3} q_{\{3\}^2} \\
& q_{\{4\}^3} - 6 \setminus, p_{\{1\}^2} p_{\{2\}^4} p_{\{3\}^2} q_{\{1\}^2} q_{\{2\}} q_{\{3\}^2} q_{\{4\}^3} - 6 \setminus, \\
& p_{\{1\}^4} p_{\{2\}^2} p_{\{3\}^2} q_{\{1\}} q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}^3} - 2 \setminus, p_{\{1\}^6} \\
& p_{\{3\}^2} q_{\{2\}^3} q_{\{3\}^2} q_{\{4\}^3} - 2 \setminus, p_{\{1\}^2} p_{\{2\}^6} q_{\{1\}^2} q_{\{3\}^3} \\
& q_{\{4\}^3} - 2 \setminus, p_{\{1\}^6} p_{\{2\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^3} - 4 \setminus, p_{\{4\}^6} \\
& q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^3} - 12 \setminus, p_{\{3\}^2} p_{\{4\}^4} q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^2} \\
& q_{\{4\}} - 12 \setminus, p_{\{2\}^2} p_{\{4\}^4} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}} - 12 \setminus, p_{\{1\}^2} \\
& p_{\{4\}^4} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}} - 12 \setminus, p_{\{3\}^4} p_{\{4\}^2} q_{\{1\}^3} \\
& q_{\{2\}^3} q_{\{3\}} q_{\{4\}^2} - 31 \setminus, p_{\{2\}^2} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^2} \\
& q_{\{3\}^2} q_{\{4\}^2} - 31 \setminus, p_{\{1\}^2} p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^2} \\
& q_{\{4\}^2} - 12 \setminus, p_{\{2\}^4} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}} q_{\{3\}^3} q_{\{4\}^2} - 31 \setminus, \\
& p_{\{1\}^2} p_{\{2\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} - 12 \setminus, p_{\{1\}^4} \\
& p_{\{4\}^2} q_{\{1\}} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^2} - 4 \setminus, p_{\{3\}^6} q_{\{1\}^3} q_{\{2\}^3} \\
& q_{\{4\}^3} - 12 \setminus, p_{\{2\}^2} p_{\{3\}^4} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}} q_{\{4\}^3} - 12 \setminus, \\
& p_{\{1\}^2} p_{\{3\}^4} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}} q_{\{4\}^3} - 12 \setminus, p_{\{2\}^4} p_{\{3\}^2} \\
& q_{\{1\}^3} q_{\{2\}} q_{\{3\}^2} q_{\{4\}^3} - 31 \setminus, p_{\{1\}^2} p_{\{2\}^2} p_{\{3\}^2} q_{\{1\}^2} \\
& q_{\{2\}^2} q_{\{3\}^2} q_{\{4\}^3} - 12 \setminus, p_{\{1\}^4} p_{\{3\}^2} q_{\{1\}} q_{\{2\}^3} q_{\{3\}^2} \\
& q_{\{4\}^3} - 4 \setminus, p_{\{2\}^6} q_{\{1\}^3} q_{\{3\}^3} q_{\{4\}^3} - 12 \setminus, p_{\{1\}^2} p_{\{2\}^4} \\
& q_{\{1\}^2} q_{\{2\}} q_{\{3\}^3} q_{\{4\}^3} - 12 \setminus, p_{\{1\}^4} p_{\{2\}^2} q_{\{1\}} q_{\{2\}^2} q_{\{3\}^3} \\
& q_{\{4\}^3} - 4 \setminus, p_{\{1\}^6} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^3} - 24 \setminus, p_{\{4\}^4} q_{\{1\}^3} \\
& q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}} - 46 \setminus, p_{\{3\}^2} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^2} \\
& q_{\{4\}^2} - 46 \setminus, p_{\{2\}^2} p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^2} - 46 \setminus, \\
& p_{\{1\}^2} p_{\{4\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^2} - 24 \setminus, p_{\{3\}^4} q_{\{1\}^3} \\
& q_{\{2\}^3} q_{\{3\}} q_{\{4\}^3} - 46 \setminus, p_{\{2\}^2} p_{\{3\}^2} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^2} \\
& q_{\{4\}^3} - 46 \setminus, p_{\{1\}^2} p_{\{3\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^2} q_{\{4\}^3} - 24 \setminus, \\
& p_{\{2\}^4} q_{\{1\}^3} q_{\{2\}} q_{\{3\}^3} q_{\{4\}^3} - 46 \setminus, p_{\{1\}^2} p_{\{2\}^2} q_{\{1\}^2} \\
& q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^3} - 24 \setminus, p_{\{1\}^4} q_{\{1\}} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^3} - 60 \\
& \setminus, p_{\{4\}^2} q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^2} - 60 \setminus, p_{\{3\}^2} q_{\{1\}^3} q_{\{2\}^3} \\
& q_{\{3\}^2} q_{\{4\}^3} - 60 \setminus, p_{\{2\}^2} q_{\{1\}^3} q_{\{2\}^2} q_{\{3\}^3} q_{\{4\}^3} - 60 \setminus, \\
& p_{\{1\}^2} q_{\{1\}^2} q_{\{2\}^3} q_{\{3\}^3} q_{\{4\}^3} - 56 \setminus, q_{\{1\}^3} q_{\{2\}^3} q_{\{3\}^3} \\
& q_{\{4\}^3}
\end{aligned}$$