University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2018

Assignment # 4

You are expected to work on this assignment prior to your tutorial during the week of Feb. 5th. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 4, Sections: 4.5, 4.6 (ONLY Thm 4.31), 4.7 (OMIT Thm 4.35), 5.1 (OMIT Thm 5.4 - we never mix variables;*if we perform a u-subst. to a definite integral then our u integrand* ***must*** *have corresponding u integration limits if we keep our integral in definite form*).

HOMEWORK:

At the beginning of your TUTORIAL during the week of Feb. 12th you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

1. Textbook Section 4.5 - # 30, 32, 38, 54, 58.
2. Textbook Section 5.1 - # 38, 40, 46.
3. Let $x > 0$. Prove that the value of the following expression does not depend on x: $\displaystyle\int_0^x \frac{1}{1+t^4}dt + \frac{1}{3}\int_0^{\frac{1}{x^3}} \frac{1}{1+t^{\frac{4}{3}}}dt$. Fully justify your argument.
4. Let $a \in \mathbb{R}$. Suppose that f is continuous on $[-a, a]$. Prove the following statements. Use **only** the subst. rule and integration properties. Do not use FTOC I.
   (a) If f is an even function on $[-a, a]$ then $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.
   (b) If f is an odd function on $[-a, a]$ then $\displaystyle\int_{-a}^{a} f(x)dx = 0$.
   (c) Use the above properties to evaluate $\displaystyle\int_{-1}^{1} \frac{\tan(x)}{1+x^2+x^4}dx$.

EXERCISES: You do not need to submit solutions to the following problems but you should make sure that you are able to answer them.

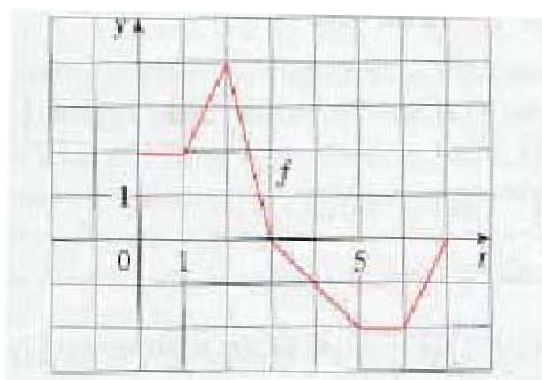
1. Textbook Section 4.5 - # 20-23, 25, 27, 28, 33-37, 40-45, 47, 49, 51, 53, 59, 61, 64, 75, 76. — You get better at integrating by practicing!

2. Textbook Section 4.7 - # 1(a)-(h), 17, 29, 37, 39, 40, 46, 41, 46, 43, 47, 50, 69.

3. Textbook Section 5.1 - # 1(c)(d)(f)-(h), 21-37, 39, 41, 43, 45 — You get better at integrating by practicing!

4. Let $a, b \in \mathbb{R}$, $a < b$. Let f be a function such that f' is continuous on $[a, b]$. Prove that $\displaystyle\int_a^b f(t)f'(t)dt = \frac{1}{2}\left(f^2(b) - f^2(a)\right)$.

5. Let $g(x) = \int_0^x f(t)dt$ where f is the function whose graph is shown below.



   (a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$ and $g(6)$.

   (b) On what interval is g increasing?

   (c) Where does g have a maximum value?

   (d) Sketch a rough graph of g.

6. Find $h'(2)$ for $h(x) = \left(\displaystyle\int_1^x \frac{1}{2+\sin^2(t)}dt\right)^3$. Make sure to justify your work. *(Hint : Do not evaluate these integrals.)*

7. Suppose that g is continuous on $\mathbb{R}$. Find all functions g such that $\displaystyle\int_0^x tg(t)dt = x + x^2$ for $x > 0$.

8. Prove that the value of the $\displaystyle\int_{-\cos(x)}^{\sin(x)} \frac{1}{\sqrt{1-t^2}}dt$, $x \in (0, \frac{\pi}{2})$ does <u>not</u> depend on x.

9. Suppose that f is a continuous function and that for $x > 0$,

$$\int_0^x tf(t)d = x\sin(x) + \cos(x) - 1.$$

   (a) Find $f(\pi)$.

   (b) Calculate $f'(x)$.

10. On what interval is the curve $y = \displaystyle\int_0^x \frac{t^2}{t^2+t+2}dt$ concave down?

11. The natural logarithm may be defined as an area accumulation function. Namely, for $x > 0$ the natural logarithm function is defined by $\ln(x) = \displaystyle\int_1^x \frac{1}{t}dt$. Prove each of the following from Section 4.7 of your textbook using this new definition of $\ln(x)$. # 71-74.

12. Evaluate the following :

   (a) $\displaystyle\int_0^1 \frac{e^{\tan^{-1}(x)}}{x^2+1}dx.$

   (b) $\displaystyle\int_0^1 \frac{1+\ln(x)}{x}dx.$

   (c) $\displaystyle\int (ax+b)^{\frac{3}{4}}dx$, where $a, b \in \mathbb{R}^+$.

   (d) $\displaystyle\int \frac{x+e^{2x}}{x^2+e^{2x}}dx.$

   (e) $\displaystyle\int x\sin^3(x^2)\cos(x^2)dx.$

   (f) $\displaystyle\int \frac{g(x)g'(x)}{\sqrt{1+g^2(x)}}dx$, where $g'(x)$ is continuous.

   (g) $\int_0^{\frac{\pi}{2}} \cos(x)\sin^5(x)dx$. Use algebra to rewrite the integrand and use the u-substitution $u = \cos(x)$