A category-mistake in the classical labor theory of value: identification and resolution

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Abstract

The classical labor theory of value generates two well-known antinomies: Ricardo's problem of an invariable measure of value and Marx's transformation problem. I show that both antinomies are generated by the same category-mistake of expecting a technical measure of labor cost to function as a total measure of labor cost. This category-mistake is the deep conceptual generator of the two hundred year history of the 'value controversy'. Once identified we can avoid the category-mistake, which yields a labor theory of value with an invariable measure of value and without a transformation problem.

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This paper diagnoses a conceptual mistake. To expose the mistake we need the help of some formality. So we begin by translating the classical concept of 'labor value' into linear production theory.

1. The definition of 'labor value'

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{l} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & 0 \\ 0 & 0 & a_{3,3} \\ l_1 & l_2 & l_3 \end{bmatrix}$$

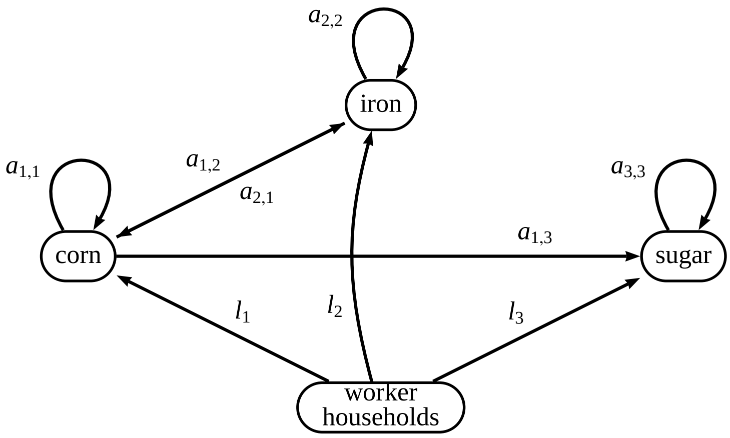

Figure 1: An input-output matrix for an example 3-sector economy depicted as a directed graph.

Figure 1 depicts an input-output matrix that specifies the relative quantities of labor and commodity inputs that must be combined in order to produce commodity outputs. The input-output matrix specifies the 'technology' or 'technique' that prevails in an economy in a given period of time.

The technique immediately tells us that l_i units (say, hours) of *direct labor* are required to *produce* commodity i. But we can also calculate the *total* direct and indirect labor required to *reproduce* commodity i, which is the labor, operating not just in sector i but also in parallel in the other sectors of the economy that is simultaneously supplied to replace *all* the direct and indirect commodity inputs used-up during the production of 1 unit of commodity i.

Marx, following the Ricardian socialist, Thomas Hodgskin (Hodgskin, 1825; Perelman, 1987), illustrated this concept of 'total labor' (both direct and indirect labor) in terms of a contrast between 'coexisting labor' and 'antecedent labor':

> '[Raw] cotton, yarn, fabric, are not only produced one after the other and from one another, but they are produced and reproduced *simultaneously*, alongside one another. What appears as the effect of antecedent labor, if one considers the production process of the individual commodity, presents itself at the same time as the effect of coexisting labor, if one considers the *reproduction process* of the commodity, that is, if one considers this production process in its continuous motion and in the entirety of its conditions, and not merely an isolated action or a limited part of it. There exists not only a cycle comprising various phases, but all the phases of the commodity are simultaneously produced in the various spheres and branches of production.' (Marx, 2000)

Commodities require different quantities of coexisting labor for their reproduction and hence vary in their 'difficulty of production' (Ricardo, [1817] 1996). The classical labor theory of value is founded on this objective cost property of commodities: the *labor-value* of commodity-type A is the total coexisting labor required to reproduce one unit of A.

We can formally define a labor-value as follows: imagine 1 unit of commodity i has been produced. How much coexisting labor did this production require? We answer the question as follows: consider the technology as a directed graph (see figure 1) and, starting at sector i, recursively trace all input paths backwards in the directed graph, summing direct labor inputs along the way. This procedure is known as 'vertical integration' (Pasinetti, 1980) since we sum 'backwards' in the 'vertical' chain of production.

For example, production of unit i requires direct labor l_i plus a bundle of input commodities $\mathbf{A}^{(i)}$ (i.e., column i of matrix $\mathbf{A}$, which represents all the input paths to sector i). During production of unit i the input bundle is simultaneously replaced by an expenditure of indirect labor $\mathbf{lA}^{(i)}$ operating in parallel in other sectors. But this production itself requires as input another bundle of commodities $\mathbf{AA}^{(i)}$, which are also simultaneously replaced by the expenditure of an additional amount of indirect labor $\mathbf{lAA}^{(i)}$ operating in parallel. To count all the coexisting labor, λ_i, we must continue the sum; that is,

$$\begin{aligned}\lambda_i &= l_i + \mathbf{lA}^{(i)} + \mathbf{lAA}^{(i)} + \mathbf{lA}^2\mathbf{A}^{(i)} + \ldots \\ &= l_i + \mathbf{l}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots)\mathbf{A}^{(i)} \\ &= l_i + \mathbf{l}(\sum_{n=0}^{\infty} \mathbf{A}^n)\mathbf{A}^{(i)}. \end{aligned} \tag{1}$$

This is an infinite sum. The infinite series converges to a finite value if the technique is economically productive (see Lancaster (1968)). Let the vector $\boldsymbol{\lambda}$ denote the coexisting labor required to reproduce unit bundle $\mathbf{u} = [1]$; then, from equation (1),

$$\boldsymbol{\lambda} = \mathbf{l} + \mathbf{l}(\sum_{n=0}^{\infty} \mathbf{A}^n)\mathbf{A} = \mathbf{l} \sum_{n=0}^{\infty} \mathbf{A}^n.$$

The Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$ is an alternative representation of the infinite series; hence, $\boldsymbol{\lambda} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ and the vector of coexisting labor required to reproduce unit commodities is – as we'd expect – identical to the standard, and well-known, modern formula for labor-values, $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$; that is:

Definition 1. *Standard labor-values*, $\mathbf{v}$, are given by

$$\mathbf{v} = \mathbf{vA} + \mathbf{l}. \tag{2}$$

A labor-value is often interpreted in terms of antecedent labor as the sum of past labor 'embodied' in means of production ($\mathbf{vA}$) plus the addition of present 'living' labor ($\mathbf{l}$). In this paper, however, we always interpret labor-values in terms of coexisting labor. Hence the total coexisting labor, v_i, breaks down into indirect labor, $\mathbf{vA}^{(i)}$, and direct labor, l_i.

This equation was probably first written down by Dmitriev (1868 – 1913) who translated the classical concept of 'labor embodied' into a mathematical formula (Nuti, 1974; Dmitriev, 1974). Dmitriev's formula is now standard (e.g., Sraffa (1960); Samuelson (1971); Pasinetti (1977); Steedman (1981)).

Now that we've defined labor-values let's turn to two famous antinomies of the classical labor theory of value.

2. Ricardo's problem of an invariable measure of value

Consider a tree A that is twice the height of tree B. At a later date tree A is three times the height of tree B. Assume we only know the *relative* change in heights. Does this change indicate that tree A has increased in size, tree B has decreased in size, or some combination of these causes? To answer we need an *absolute* measure of height that is *invariable* over time.

The 'meter' is such an invariable standard. We measure the absolute height of tree A and B in meters, both before and after the change. Then we can unambiguously determine the cause of the variation in relative heights.

The definition and adoption of the meter – an invariable standard measure of length – in 1793 by post-revolutionary France was accompanied by much theoretical debate and reflection (Roncaglia, 2005, pg. 192).

Ricardo, a contemporary of these events, recognizes that an objective theory of economic value requires an analogous standard of measurement. But Ricardo cannot identify such a standard.

Market prices – whether stated in terms of exchange ratios between commodities (e.g., a piece of cloth exchanges for a certain quantity of leather) or in terms of a money-commodity (e.g., a piece of cloth exchanges for 2 ounces of gold) – cannot function as a standard because prices merely indicate relative values.

> 'If for example a piece of cloth is now the value of 2 ounces of gold and was formerly the value of four I cannot positively say that the cloth is only half as valuable as before, because it is possible that the gold may be twice as valuable as before.' (Ricardo, 2005)

The cause of an altered exchange ratio between the chosen standard (or *numéraire*) (e.g., units of leather, or ounces of gold) and the commodity whose value we wish to measure (e.g., a piece of cloth) might be due to an alteration in the absolute value of the standard itself. Attempting to use market price to measure absolute value is analogous to picking the height of a specific tree to function as an invariable standard of length. Between measurements the chosen tree might grow.

Perhaps we shouldn't try to find a standard? This is not an option because, lacking an invariable standard, the theory of value collapses into subjectivity, leaving 'every one to chuse his own measure of value' (Ricardo, 2005, pg. 370). In consequence, public statements about objective value, such as 'commodity A is now less valuable than one year ago', would, strictly speaking, be nonsense. Ricardo therefore looks beyond exchange ratios in the marketplace to seek a 'standard in nature' (Ricardo, 2005, pg. 381).

In Ricardo's thought the problem of an invariable standard and the aim of elucidating the underlying laws that regulate prices are closely identified (Sraffa (2005), pg. xli). An important bedrock of Ricardo's theory is that a reproducible commodity's natural price is regulated by its 'difficulty of production' measured in labour time (e.g., Ricardo ([1817] 1996, Ch. 4)). Natural prices are stable exchange ratios that are independent of 'accidental and temporary deviations' between supply and demand (Ricardo, [1817] 1996, Ch. 5). And reproducible commodities are those 'that may be multiplied ... almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them' (Ricardo, [1817] 1996, pg. 18). Ricardo maintains that the 'natural price of commodities ... always ultimately governs their market prices' (Ricardo, [1817] 1996, Ch. 16). For example, in conditions of constant 'difficulty of production' market prices gravitate toward their natural prices due to profit-seeking behavior (Wright, 2008, 2011).

Natural prices, or 'prices of production' (Marx, [1894] 1971), are equilibrium prices, which we can state in terms of linear production theory as

$$\mathbf{p} = (\mathbf{pA} + \mathbf{l}w)(1 + r), \tag{3}$$

where $\mathbf{p}$ is a vector of prices (measured, say, in dollars), w a wage rate (dollars per hour), and r a uniform 'rate of profit' or percentage interest-rate on the money invested to fund the period of production. Equation (3) simply states that production price p_i of commodity-type i has three components: (i) the cost of the input bundle, $\mathbf{pA}^{(i)}$, paid to other sectors of production, (ii) the wage costs, $l_i w$, paid to workers in sector i, and (iii) the profits, $(\mathbf{pA}^{(i)} + l_i w)r$, received by capitalists, as owners of firms in this sector, on the money-capital they advance to pay input and direct labor costs (collectively, the cost-price).

Ricardo believes that if we had 'possession of the knowledge of the law which regulates the exchangeable-value of commodities [that is, production prices], we should be only one step from the discovery of the measure of absolute value'. Now if 'difficulty of production', measured in units of labor, in fact regulates production prices then, in theory, we can measure (absolute) labor-values to unambiguously determine the cause of variations in (relative) prices. We would then have found a 'standard in nature' and Ricardo could 'speak of the variation of other things, without embarrassing myself on every occasion with the consideration of the possible alteration in the value of the medium in which price and value are estimated' (Ricardo, [1817] 1996, Ch. 1).

In fact, in some special cases labor-values do vary one-to-one with production prices. For instance, Smith ([1776] 1994) restricts the applicability of a labor theory of value to an 'early and rude state of society' that precedes the 'accumulation of stock' where profits are absent and 'the whole produce of labor belongs to the laborer'. In these circumstances production price is simply the wage bill of the total coexisting labor required to reproduce the commodity; that is,

Proposition 1. $r = 0$ implies $\mathbf{p} = w\mathbf{v}$ (see appendix for proof).

So prices are proportional to labor-values with constant of proportionality w. Hence (relative) production prices vary in lock-step with (absolute) labor-values.

Ricardo notes that if the ratio of 'fixed capital' (i.e., the input bundle) to 'circulating capital' (i.e., the real wage bundle for 'the support of labor') is identical in all sectors then production prices are proportional to labor-values (Ricardo, [1817] 1996, pg. 31). Define $\bar{\mathbf{w}} = (1/\mathbf{lq}^{\mathrm{T}})\mathbf{w}$ as the real wage bundle consumed per unit of labor supplied, where $\mathbf{q}$ is the scale of production or gross product. Then Ricardo's ratio, in terms of labor-values, is

$$k = \frac{\mathbf{vA}^{(i)}}{\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i},$$

where $\mathbf{vA}^{(i)}$ is the labor-value of the input bundle and $\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$ is the labor-value of the real wage consumed by workers in sector i. Marx would later call this ratio the technical or organic 'composition of capital' (Marx, [1894] 1971, Ch. 8). A uniform organic composition of capital implies price-value proportionality; that is,

Proposition 2. $\mathbf{vA} = k\, \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}$ implies $\mathbf{p} = \alpha\mathbf{v}$, where $\alpha = w(1 + r)/(1 - k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} r)$ (see appendix for proof).

Proposition 2 confirms Ricardo's proposition. Again, in these special circumstances, production prices vary in lock-step with labor-values. Ricardo therefore claims that 'the quantity of labour bestowed on a commodity ... is under many circumstances an invariable standard' (Ricardo, [1817] 1996, pg. 19).

But apart from 'many' special cases there exists an infinite number of cases where production prices fail to vary one-to-one with labor-values. The reason is very simple: production prices, $\mathbf{p}$, are a function of the profit-rate, r, but labor-values, $\mathbf{v}$, are not. Hence a variation in the profit-rate alters prices but leaves labor-values entirely unchanged. As Ricardo (2005) clearly identifies: price depends on the distribution of income (i.e., how the net product is distributed in the form of wage and profit income) but 'difficulty of production', a purely technical measure of direct and indirect labor costs, does not; therefore, production prices have an additional degree-of-freedom unrelated to labor-values. In general, *the relative value of a commodity varies independently of its absolute value*.

This is very perplexing, since it's analogous to discovering that the relative size of two trees can change even though their absolute sizes, measured in meters, remain unaltered. Such a discovery would imply the meter is not an invariable standard of size, or that one's theory of size is flawed. Ricardo's problem of an invariable standard of value arises, therefore, because his labor theory of value cannot fully account for production prices. The profit component of price appears to be unrelated to any objective labor cost.

Ricardo understands the necessity for an invariable standard in his theoretical framework yet simultaneously understands the conditions that prevent this necessity from being met. Faced with a contradiction he is forced to draw the negative conclusion that there cannot be an invariable standard of value. Although 'the great cause of the variation of commodities is the greater or less quantity of labour that may be necessary to produce them' there is another 'less powerful cause of their variation' (Ricardo, 2005, pg. 404). The 'less powerful cause', that is income distribution, is an additional factor that interferes with the theoretical and practical requirement of measuring how changes in labour productivity affect production prices (Colliot-Thélène, 1979). Ricardo therefore retreats to an objective theory of value that is necessarily approximate. He proposes to rank all possible 'imperfect' standards of value according to the extent they minimize the effect of changes in the distribution of income (Ricardo (2005, pg. 405) and Sraffa (2005)). But despite Ricardo's efforts he bequeathed an unstable theoretical system that eventually led to the rejection of his theory of value (Rubin (1979), Ch. 33).

3. Marx's transformation problem

Marx ([1887] 1954) explicitly assumes prices are proportional to labor-values in Volume I of *Capital*. On this basis profit is the money representation of the unpaid or 'surplus labour' of the working class. Hence profit, and its rate, directly relate to objective labor costs. But Marx must establish the generality of this proposition in the case of (non-proportional) production prices. He tackles the issue in unfinished notes published as Volume III of *Capital* (Marx, [1894] 1971). He proposes that *aggregates* of labor-values and production prices are proportional, even though individual prices and labor-values diverge, and therefore total profit remains the money representation of total surplus labor.

Let's reproduce Marx's reasoning in terms of our formal model. For Marx the labor-value of a 'commodity is equal to the value of the constant capital contained in it, plus the value of the variable capital reproduced in it, plus the increment – the surplus-value produced – of this variable capital' (Marx, [1894] 1971, Ch. 8). So we can write labor-value v_i as

$$v_i = \mathbf{v}\mathbf{A}^{(i)} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i + (1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}) l_i,$$

Note that v_i consists of three components: (i) constant capital, which is the labor-value of the input bundle, $\mathbf{v}\mathbf{A}^{(i)}$, (ii) variable capital, $\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$, which is labor-value of the real wage, and (iii) surplus labor, $(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}) l_i$, which is the fraction of labor supplied that capitalists receive in the form of commodities purchased with profit income. (This breakdown is equivalent to standard formula (2) for labor-value.) Marx defines the 'rate of surplus-value' or 'degree of exploitation' as the ratio of surplus-labor to variable capital,

$$\theta = \frac{1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}}{\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}},$$

which he assumes, for simplicity, to be the same for all sectors. The degree of exploitation, θ, is a distributional variable – a high (resp. low) θ implies capitalists receive a larger (resp. smaller) share of the fruits of labor.

Now, according to Marx, only 'living labor' creates surplus-value. So the quantity of surplus-labor, and therefore profit, produced in each sector depends on the variable, not the constant, capital. Marx considers an initial situation of prices proportional to labor-values. In these circumstances a sector's profit-rate can be expressed as the ratio of surplus-labor to the labor-value of the constant and variable capital,

$$r_i = \frac{(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}) l_i}{\mathbf{v}\mathbf{A}^{(i)} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i} = \theta \frac{1}{\frac{\mathbf{v}\mathbf{A}^{(i)}}{\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i} + 1}.$$

Hence in this initial situation profit-rates are equal only if the organic composition of capitals, that is the ratios $\mathbf{v}\mathbf{A}^{(i)}/\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$ are equal, for all i and j. But they are not equal; hence, 'in the different spheres of production with the same degree of exploitation, we find considerably different rates of profit corresponding to the different organic composition of these capitals' (Marx, [1894] 1971, pg. 155).

'The rates of profit prevailing in the various branches of production are originally very different' (Marx, [1894] 1971, pg. 158) but the different rates 'are equalized by competition to a single general [uniform] rate of profit' (Marx, [1894] 1971, pg. 158) during the formation of production prices. Marx proposes that the formation of a uniform profit-rate *conservatively redistributes* the surplus-labor (in the form of commodities purchased with profit income) amongst capitalist owners, at which point, 'although in selling their commodities the capitalists of various spheres of production recover the value of the capital consumed in their production, they do not secure the surplus-value [i.e., surplus-labor], and consequently the profit, created in their own sphere by the production of these commodities.' (Marx, [1894] 1971, pg. 158). Marx proposes that capitalists share the available pool of surplus-labor in proportion to the size of the money-capitals they advance rather than the size of the (value-creating) workforces they employ.

Marx provides numerical examples and formulae to demonstrate how surplus-labor is redistributed. He computes a uniform (labor-value) profit-rate, r_v, by dividing the aggregate surplus-labor by the aggregate labor-value of constant and variable capital,

$$r_v = \frac{(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}) \mathbf{l}\mathbf{q}^{\mathrm{T}}}{\mathbf{v}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} \mathbf{l}\mathbf{q}^{\mathrm{T}}}, \tag{4}$$

where $\mathbf{lq}^{\mathrm{T}}$ is the total labor supplied to the economy and $\mathbf{vAq}^{\mathrm{T}}$ is the labor-value of the total constant capital. Marx states that the (labor-value) profit-rate, r_v, is identical to the uniform (money) profit-rate, r, that obtains once production prices have formed. He defines 'price of production' as the *initial* cost-price of a commodity, which is proportional to labor-value, marked-up by the uniform profit-rate, r_v. Let α be the constant of proportionality, measured in money units per labor unit. Then we can write Marx's production prices as

$$\mathbf{p}^{\star} = \alpha\left(\mathbf{vA} + \mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\right)(1 + r_v). \tag{5}$$

'Hence, the price of production of a commodity is equal to its cost-price plus the profit, allotted to it in per cent, in accordance with the general rate of profit, or, in other words, to its cost-price plus the average profit [i.e., r_v]' (Marx, [1894] 1971, pg. 157).

Marx's production prices $\mathbf{p}^{\star}$ are not proportional to labor-values. So 'one portion of the commodities is sold above its [labor-]value in the same proportion in which the other is sold below it. And it is only the sale of the commodities at such prices that enables the rate of profit for capitals [to be uniform], regardless of their different organic composition' (Marx, [1894] 1971, pg. 157).

In Marx's view production prices scramble and obscure the source of profit in surplus-labor. But the labour theory of value continues to hold in the aggregate because the 'transformation' from unequal profit-rates to production prices is conservative: nominal price changes neither create or destroy surplus-labor but merely redistribute it. So Marx claimed that three aggregate equalities are invariant over the transformation: (i) the (money) profit-rate, r, is equal to the (labor-value) profit-rate, r_v; (ii) 'the sum of the profits in all spheres of production must equal the sum of the surplus-values', (Marx, [1894] 1971, pg. 173); and (iii) 'the sum of the prices of production of the total social product equal the sum of its [labor-]value' (Marx, [1894] 1971, pg. 173) (here Marx assumes, for simplicity, that $\alpha = 1$).

Marx's 'prices of production' are computed from the assumption that money and labor-value profit-rates are equal and therefore equality (i) is true by definition. Also, Marx's prices $\mathbf{p}^{\star}$ satisfy equalities (ii) and (iii) (see Proposition 4 in the appendix). Hence the contradiction between labor-values and (non-proportional) production prices appears to be resolved: aggregate prices are proportional to aggregate labor-values and profit is, after all, a money representation of surplus-labor.

But Marx immediately critiques his own derivation. He observes that:

> 'we had originally assumed that the cost-price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as a cost-price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing the price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost-price, and to bear in mind that there is always the possibility of an error if the cost-price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point' (Marx, [1894] 1971, pg. 165).

The transformation procedure, like the whole of Volume III of *Capital*, is unfinished. Marx pinpoints a potential source of error but doesn't pursue it. But of course his critics did, beginning with von Bortkiewicz (1975) in 1898.

The problem that Marx highlights is that his 'prices of production', defined by equation (5), are calculated on the basis of *untransformed* cost-prices, $\alpha(\mathbf{vA}+\mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}))$, which are proportional to labour-value. Marx realized that 'the magnitudes on the basis of which surplus-value has been redistributed – that is, capital advanced, measured in [labor-]value – are not identical to the prices at which elements of capital are bought on the market. He therefore admits that the prices previously calculated must be adjusted' (Lippi, 1979).

Production prices are defined by equation (3), and not Marx's equation (5), when we make the adjustment. The transformation problem is then the logical impossibility of Marx's aggregate equalities. In fact, we can deduce:

Proposition 3. All Marx's equalities are true if the economy satisfies the special condition

$$\mathbf{v}\left(\mathbf{I} - (\mathbf{A} + \bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1 + r)\right)\mathbf{q}^{\mathrm{T}} = 0;$$

otherwise all Marx's equalities are not true (see appendix for proof).

Proposition 3 specifies a macroeconomic constraint between labor-values, income distribution and the scale of production. Some cases that satisfy the constraint include zero profit, a uniform organic composition of capital, or a scale of production in certain special proportions (for further details see Abraham-Frois and Berrebi (1997, Ch. 6)). But there is no economic reason why the constraint should hold, especially as income distribution and the scale of production may vary independently of labor-values. In general, a conservative transformation that maintains a quantitative correspondence between labor costs and money costs does not exist and therefore 'there is no rigorous quantitative connection between the labour time accounts arising from embodied labour coefficients and the phenomenal world of money price accounts' (Foley, 2000).

This transformation problem is the primary reason for the modern rejection of the logical possibility of a labor theory of value. The debate has generated a large literature spanning over one hundred years. Steedman (1981) provides the definitive statement of the negative consequences for Marx's value theory. First, the theory is *internally inconsistent* because Marx 'assumes that [r_v] is the rate of profit but then derives the result that prices diverge from [labor-]values, which means precisely, in general, that [r_v] is not the rate of profit' (Steedman, 1981, pg. 31). Second, the theory is *redundant* because 'profits and prices *cannot* be derived from the ordinary value schema, that [r_v] is *not* the rate of profit and that total profit is *not* equal to surplus value' (Steedman, 1981, pg. 48). Steedman notes, following Samuelson (1971), that given a technique and a real wage (the 'physical schema') one can determine (a) profits and prices and (b) labor-values. But due to the non-satisfaction of the condition in Proposition 3 there is, in general, 'no way' of relating (a) and (b). Despite Marx's efforts a theory of value based exclusively on labor-cost cannot account for price phenomena.

4. Total labor costs

Now that we've stated the problems we can turn to understanding why they exist. Clearly, prices and labor-values are incommensurable because a price depends on a profit-rate but a labor-value does not. But we need to dig deeper to discover the fundamental reason why money costs and labor costs diverge. First, we'll examine two related properties of labor-values, in the context of an economy where capitalist profits are absent, which are subtle and normally overlooked.

4.1. The independence of labor-values from the real wage

Figure 2 describes a 'worker only' economy in terms of a social accounting matrix, which consists of a technology matrix augmented with information that specifies the distribution of the real wage to worker households per unit of labor supplied, $\bar{\mathbf{w}}$.

In section 1 we interpreted the computation of a labor-value as a procedure of vertical integration that recursively traces input paths 'backwards' in a directed graph. If we perform this procedure in the context of a social accounting matrix we immediately notice that some input paths are ignored. Specifically, the real wage inputs to worker households, depicted as dashed arcs in figure 2, are not traced backwards. So the labor supplied to produce the real wage, which maintains and reproduces the working class, is excluded as a component of the labor cost of commodity i. Why is this coexisting labor not counted?

A labor-value is the answer to the question, 'What is the total coexisting labor required to reproduce 1 unit of a commodity?' But it is not the answer to the question, 'What is the total coexisting labor required to *both*

$$\begin{bmatrix} \mathbf{A} & \bar{\mathbf{w}}^{\mathrm{T}} \\ \mathbf{l} & 0 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \bar{w}_1 \\ a_{2,1} & a_{2,2} & 0 & \bar{w}_2 \\ 0 & 0 & a_{3,3} & \bar{w}_3 \\ l_1 & l_2 & l_3 & 0 \end{bmatrix}$$

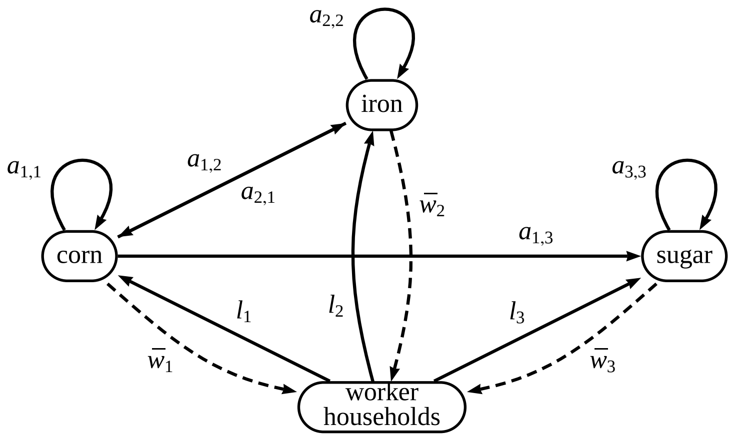

Figure 2: A social accounting matrix for an example 3-sector worker-only economy depicted as a directed graph.

reproduce 1 unit of a commodity *and* reproduce the labor that reproduced that unit?' It would make no sense to measure the cost of reproducing the very resource that serves as the measure of cost. This would be like measuring the height of a tree with a meter rod and including the length of the rod as part of the tree's height.

We can look at this another way. Any system of measurement defines a standard unit (e.g., the 'meter'). We do not ask, 'How many meters are in one meter?' since the measure of the standard unit is by definition a unit of the standard. In a labor theory of value the question, 'What is the labor-value of one unit of direct labor?' is similarly ill-formed: the real cost of 1 hour of labor, *measured by labor time*, is 1 hour. No further reduction is possible or required. The self-identity of the standard of measure is a conceptual necessity in any system of measurement. So whether workers consume one bushel or a thousand bushels of corn to supply a unit of direct labor makes no difference to the labor-value of that unit of direct labor: an hour of labor-time is an hour of labor-time, period. The procedure of vertical integration, when applied to a social accounting matrix, therefore always terminates at labor inputs and does not further reduce labor inputs to the real wage.

For example, Marx notes that the expression 'labor-value of labor-power', where labor-power is the capacity to supply labor, denotes the 'difficulty of production' of the real wage, which is the conventional level of consumption that reproduces the working class. In contrast, the expression 'labor-value of labor' is an oxymoron: 'the value of labor is only an irrational expression for the value of labor-power'. The expression, taken literally, is analogous to querying the color of a logarithm (Marx, [1894] 1971) or the time on the sun (Pollock, 2004). 'Labor is the substance, and the immanent measure of value, but *has itself no value*.' (Marx, [1887] 1954, pg. 503).

4.2. Labor-values as total labor costs

Labor-values, then, exclude as a conceptual necessity the reproduction costs of labor (i.e., the coexisting labor required to reproduce the real wage). In the context of a worker-only economy the procedure of vertical integration therefore reduces *all* real costs (such as corn, iron and sugar) to quantities of direct labor *except* the reproduction cost of labor. Hence labor-values, $\mathbf{v}$, are 'total labor costs':

Definition 2. A commodity's *total labor cost* is (i) a measure of the coexisting labor required to reproduce it that (ii) *only* excludes the reproduction cost of labor.

4.3. 'That early and rude state'

The classical proposition that equilibrium prices of reproducible goods are proportional to labor-values in an 'early and rude state' (see Proposition 1) that precedes the 'accumulation of stock' (Smith, [1776] 1994), and therefore capitalist profit, is not controversial. Indeed, in the context of static, equilibrium models, even critics of a labor theory of value accept this (e.g., Samuelson (1971); Steedman (1981); Roemer (1982)). In this situation money costs are proportional to labor costs, that is $\mathbf{p} = w\mathbf{v}$, because both accounting systems apply the same accounting convention: all commodities are reduced to a scalar measure of total cost – either total money or total labor cost. The dual accounting systems are mutually consistent and can therefore be related by the price of labor, w. So in a worker-only economy the production price of a commodity is the wage bill of the total coexisting labor required to reproduce it. Commodities that require more of society's labor-time to reproduce sell at higher prices in equilibrium.

Now let's introduce capitalist profit income and determine exactly why this simple relationship breaks down.

5. Labor-values and profit income

Money functions as a means of payment in the hands of purchasers, either consumers spending for personal consumption or firms buying factors of production. In contrast, money, in the hands of capitalist owners of firms, functions as 'money-capital' since its advance to production commands a return. Capitalists advance money-capital to cover input costs, $m_i = \mathbf{pA}^{(i)} + l_i w$, and receive profit income, $m_i r$, proportional to their advance. This profit mark-up, or price of money-capital, r, forms a component of the money cost of production. Equilibrium prices are then production prices given by equation (3).

$$\begin{bmatrix} \mathbf{A} & \overline{\mathbf{w}}^{\mathrm{T}} & \overline{\mathbf{c}}^{\mathrm{T}} \\ \mathbf{l} & 0 & 0 \\ \mathbf{m} & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \overline{w}_1 & \overline{c}_1 \\ a_{2,1} & a_{2,2} & 0 & \overline{w}_2 & \overline{c}_2 \\ 0 & 0 & a_{3,3} & \overline{w}_3 & \overline{c}_3 \\ l_1 & l_2 & l_3 & 0 & 0 \\ m_1 & m_2 & m_3 & 0 & 0 \end{bmatrix}$$

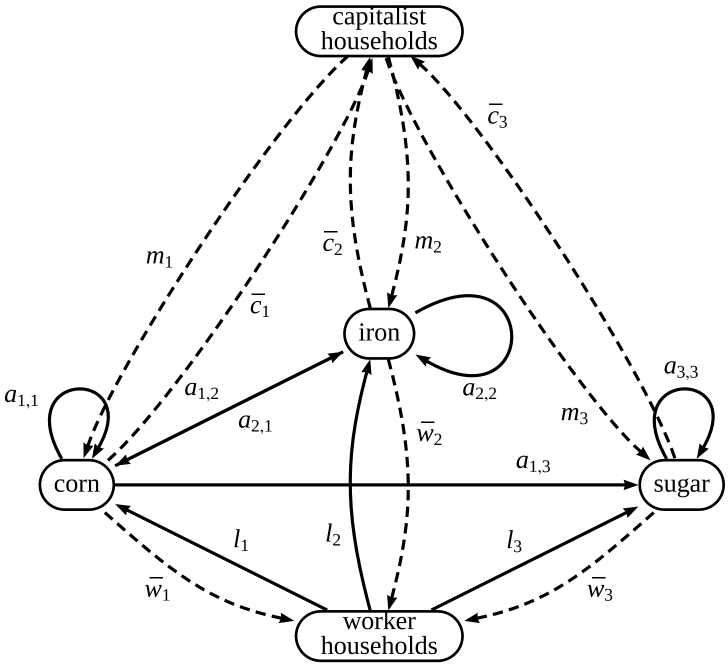

Figure 3: A social accounting matrix for an example 3-sector capitalist economy depicted as a directed graph.

Figure 3 describes an idealized capitalist economy in terms of a social accounting matrix. It's identical to the worker-only economy but with addition of new material relations between capitalist households and the system of production. As before, the production of a unit of corn directly requires $a_{1,1}$ units of corn, $a_{2,1}$ units of iron, and l_1 units of labor. But in capitalist conditions production also requires money-capital, m_i. The social accounting matrix therefore also specifies (i) money-capital requirement coefficients, $\mathbf{m}$, a vector of cost-prices that denotes the quantity of money-capital required to finance unit output of commodity i, and (ii) the distribution of consumption goods, $\overline{\mathbf{c}} = (1/\mathbf{mq}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}$, to capitalist households per unit of money-capital advanced, where $\mathbf{c}$ is the consumption bundle that capitalists consume and $\mathbf{mq}^{\mathrm{T}}$ is the total money-capital advanced during the period of production (not to be confused with the total stock of money in circulation since 'the same mass of actual money can ... represent very different masses of money-capital' (Marx, [1894] 1971, pg. 510)).

5.1. The divergence of technical and total labor costs

Let's reconsider the procedure of vertical integration when capitalist profit is present.

Production of a unit of commodity i requires direct labor l_i and a bundle of input commodities $\mathbf{A}^{(i)}$. The input bundle $\mathbf{A}^{(i)}$ is simultaneously replaced by the application of indirect labor $\mathbf{l}\mathbf{A}^{(i)}$. But production now additionally requires money-capital m_i (see the dashed input edges from capitalist households to the system of production in Figure 2).

Nobody 'makes' money-capital, even in circumstances where money is a commodity. Money-capital is not produced, but lent. Hence we assume the supply of money-capital does not incur direct labor costs. Including the labor of capital management would not alter the essential fact that 'in the price of commodities ... the profits of stock constitute a component part altogether different from the wages of labor, and regulated by quite different principles' (Smith, [1776] 1994).

But although there are no direct labor costs there are indirect labor costs associated with production financed by money-capital. Capitalists do not advance money-capital for free, either nominally or in real terms. In parallel with the production of unit i, and the supply of money-capital m_i, capitalists consume commodity bundle $m_i\bar{\mathbf{c}}$. So a quantity of coexisting labor, $m_i\mathbf{l}\bar{\mathbf{c}}^{\mathrm{T}}$, is indeed used-up during the advance of money-capital, specifically the coexisting labor employed to produce capitalist consumption goods.

The standard formula (2) for labor-value does not vertically integrate over the input paths corresponding to money-capital. Money-capital inputs are not part of the technique, and are therefore ignored, which is equivalent to treating money-capital inputs as an irreducible terminus on the same footing as the supply of labor (e.g., all the dashed input edges from capitalist households in Figure 3 are not traversed). In consequence, standard labor-values do not count the coexisting labor employed to produce capitalist consumption goods as a real cost of production.

Should this labor be counted as a cost?

Quite simply, the answer depends on what we want to measure. Standard labor-values are a purely technical measure of labor costs. For example, the reciprocal of a standard labor-value serves as a productivity index that measures the amount of the commodity produced by a unit of coexisting labor, independent of the wider institutional context in which this activity occurs. Standard labor-values therefore allow productivity comparisons across time independent of the distribution of income (e.g., see especially Flaschel (2010, Pt. 1)).

But if we want to measure *total labor costs* then, in the context of capitalist production, we cannot use standard labor-values. By definition total labor costs reduce *all* real costs to labor, except the cost of reproducing labor itself. But standard labor-values exclude the additional labor cost of reproducing the capitalist class; hence, they do not measure total labor costs. This is not a matter of interpretation but definition.

Note that the labor required to produce capitalist consumption goods is not a cost of reproducing labor and therefore necessarily excluded, as a conceptual necessity, from any definition of labor-value.

In a monetary production economy, like capitalism, money-capital is not a 'technical' requirement of production but nonetheless is a necessary material prerequisite to production. In capitalist conditions, a commodity cannot be produced without workers simultaneously performing 'tributary' or 'surplus' labor for a capitalist class. Standard labor-values, as a purely technical measure of labor cost, exclude this tributary labor as a real cost of production. A measure of total labor costs must include it.

Technical and total labor costs are accidentally identical in the case of a worker-only economy. But the presence of 'profits on stock' causes technical and total labor costs to diverge. If we aim to calculate the *total* coexisting labor required to reproduce a commodity then we must treat money-capital as a *bona fide* commodity and include its (indirect) labor cost as a real cost of production.

6. Total labor costs: nonstandard labor values

'Nonstandard' labor-values (Wright, 2007, 2009) are the labor-values that result when we include the real cost of capitalist consumption in the process of vertical integration. For example, returning to our example in figure 3, section 5.1, in addition to $\mathbf{l}\mathbf{A}^{(i)}$ labor used-up to replace input commodities $\mathbf{A}^{(i)}$ we now also count the $m_i \mathbf{l}\bar{\mathbf{c}}^{\mathrm{T}}$ labor simultaneously used-up to produce consumption goods $m_i \bar{\mathbf{c}}$.

The $n \times n$ matrix of capitalist consumption coefficients is

$$\mathbf{C} = \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} = [c_{i,j}],$$

where each $c_{i,j}$ is the quantity of commodity i capitalists consume per unit output of commodity j. Matrix $\mathbf{C}$ encapsulates the current real costs of supplying money-capital to fund production in the different sectors of the economy. The nonstandard approach reduces these capitalist consumption goods to their labor costs, as follows. Define the technique augmented by capitalist consumption as

$$\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C} = [\tilde{a}_{i,j}],$$

where each $\tilde{a}_{i,j} = a_{i,j} + c_{i,j}$ is the quantity of commodity i, including that consumed by capitalists, directly used-up per unit output of j. Now the production of commodity bundle $\mathbf{A}^{(i)} + m_i \bar{\mathbf{c}}^{\mathrm{T}} = \mathbf{A}^{(i)} + \mathbf{C}^{(i)}$ itself uses-up the bundle of input commodities $\tilde{\mathbf{A}}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)})$, which are simultaneously replaced with the expenditure of direct labor $\mathbf{l}\tilde{\mathbf{A}}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)})$ operating in parallel. To count all the coexisting labor we continue the sum; that is,

$$\begin{aligned}
\tilde{\lambda}_i &= l_i + \mathbf{l}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \mathbf{l}\tilde{\mathbf{A}}(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \mathbf{l}\tilde{\mathbf{A}}^2(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) + \ldots \\
&= l_i + \mathbf{l}(\mathbf{I} + \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^2 + \ldots)(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}) \\
&= l_i + \mathbf{l}(\sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n)(\mathbf{A}^{(i)} + \mathbf{C}^{(i)}).
\end{aligned}$$

So the vector $\tilde{\lambda}$ of coexisting labor required to reproduce a unit bundle $\mathbf{u} = [1]$ of commodities is

$$\tilde{\lambda} = \mathbf{l} + \mathbf{l}(\sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n)(\mathbf{A} + \mathbf{C}) = \mathbf{l} \sum_{n=0}^{\infty} \tilde{\mathbf{A}}^n.$$

We can write the infinite series as $\tilde{\lambda} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1} = \tilde{\mathbf{v}}$; and therefore:

Definition 3. *Nonstandard labor-values*, $\tilde{\mathbf{v}}$, are given by

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}. \tag{6}$$

(Compare the structural similarity of (6) to standard labor-value equation (2)).

This equation has a finite solution if the augmented matrix $\tilde{\mathbf{A}}$ has a dominant eigenvalue less than 1, which is the nonstandard analog of the requirement of a productive technique (Wright, 2007). A nonviable augmented matrix $\tilde{\mathbf{A}}$ indicates that more than 1 unit of commodity i is required to reproduce 1 unit of commodity i given the current rate of capitalist consumption. In such circumstances a self-reproducing equilibrium cannot be obtained by any possible combination of activity levels.

The standard formula for labor-values, $\mathbf{v} = \mathbf{v}\mathbf{A} + \mathbf{l}$, is a property of the technique. Labor costs are the sum of indirect labor, $\mathbf{v}\mathbf{A}$, plus direct labor, $\mathbf{l}$. In contrast, the nonstandard formula for labor-values, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}$, is a property of the social accounting matrix, including the real distribution of income. Labor costs are the sum of indirect labor, including the 'tributary' labor devoted to the production of capitalist consumption goods, $\tilde{\mathbf{v}}\tilde{\mathbf{A}}$, plus direct labor, $\mathbf{l}$. Standard labor-values view all household consumption (whether workers or capitalists) as net output and not a cost of production; nonstandard labor-values, in contrast, view worker consumption as net output and capitalist consumption as a cost of production. In general, $\tilde{\mathbf{v}} > \mathbf{v}$. In the special case of a worker-only economy standard and nonstandard labor-values happen to be identical.

The standard approach measures technical labor costs and the nonstandard approach measures total labor costs. Both are functions of real or 'physical' data alone and constitute entirely self-consistent labor-cost accounting schemes that complement rather than contradict each other. Both are required to answer the full range of questions that a labor theory of value poses.

Pasinetti (1988) first proposed more general measures of labor-value that include in their definition what is necessary for the reproduction of an economic system. In a model of a growing economy Pasinetti defines 'hyper-integrated labor coefficients' by extending vertical integration to additionally include the labor required to produce the commodities 'strictly necessary to expand such a circular process at a rate of growth'. Nonstandard labor-values are a kind of Pasinettian labor coefficient (see also the appendix of Wright (2009)).

Now that we've distinguished between technical and total labor costs we can understand the fundamental reason why money and labor costs diverge.

7. The category-mistake: conflating technical and total labor costs

Money-capital has a price, the profit-rate, which is a 'mark up' component of the money cost of a commodity. Money-capital also has a real cost, which, given our assumptions, is capitalist consumption. Production prices, as total money costs, include the profit-rate as a money cost of production, and therefore prices depend on the distribution of income. But standard labor-values, as technical labor costs, exclude the labor cost of money-capital as a real cost of production, and therefore labor-values are independent of the real distribution of income. The dual systems apply different cost accounting conventions. In consequence, there cannot be a one-to-one relationship between prices and labor-values: the profit-rate component of money costs refers to labor costs that are not counted. The asymmetrical treatment of the commodity money-capital – present as a money cost in the price system but absent as a real cost in the labor-value system – is the fundamental cause of the divergence of money and labor costs. A quantitative mismatch necessarily arises if *total* money costs are compared to *partial* labor costs.

The philosopher Gilbert Ryle ([1949] 1984) coined the term 'category-mistake' to denote the conceptual mistake of expecting some concept or thing to possess properties it cannot have. For example, John Doe may be a relative, friend, enemy or stranger to Richard Roe; but he cannot be any of these things to the 'Average Taxpayer'. So if 'John Doe continues to think of the Average Taxpayer as a fellow-citizen, he will tend to think of him as an elusive an insubstantial man, a ghost who is everywhere yet nowhere' (Ryle, [1949] 1984, pg. 18).

Both Ricardo's search for an invariable measure and Marx's transformation are theoretical attempts to find an 'elusive and insubstantial man', specifically an impossible search for a one-to-one relationship between total money costs and a technical (i.e., partial) measure of labor cost. The classical antinomies derive from the category-mistake of implicitly expecting technical labor costs to function as total labor costs. But they cannot. The deep conceptual mistake at the heart of the classical labor theory of value is the failure to distinguish between a technical and total concept of labor cost.

In many respects Ricardo views capitalism as a 'natural' order with economic laws both immutable and ultimately reducible to physical laws, such as 'the biological law of population and the physico-chemical law of the declining fertility of the soil' (Rubin, 1979, pg. 243). His search for a 'standard in nature' that is simultaneously independent of income distribution yet explains the structure of natural prices is consistent with this outlook. Relative values appear to vary independently of absolute values simply because Ricardo compares total money costs with partial labor costs. Marx inherited the contradiction from Smith and Ricardo. His transformation procedure is a valiant and inspired attempt to resolve it. But since Marx also does not distinguish between technical and total labor costs his transformation also takes aim at a ghost.

Category-mistakes generate theoretical difficulties that appear insoluble because they are ill-posed at their hidden conceptual foundations. Only conceptual analysis, that is the identification and removal of the underlying category-mistake, can resolve, or more accurately dissolve, the problems. The classical category-mistake has been, and continues to be, the major obstacle toward a deeper understanding of the relationship between social labor and monetary phenomena. For example, it has directed theoretical attention toward the antinomies and away from the existence of a simple one-to-one quantitative relation between production prices and labor costs.

Theorem 1. Production prices are proportional to total labor costs,

$$\mathbf{p} = \tilde{\mathbf{v}} w,$$

where $\tilde{\mathbf{v}}$ are nonstandard labor-values (see appendix for proof).

A commodity's production price is the wage bill of the total coexisting labor required to reproduce it. Commodities that require more labor time to produce sell at proportionally higher prices in equilibrium. The objective cost principle that regulates production prices, even in conditions of capitalist profit, is total labor cost.

The classical authors believed that natural prices diverge from labor costs due to 'profits on stock'. This premise has been universally accepted. But it's false. In fact, technical labor costs diverge from total labor costs due to 'profits on stock'. But natural prices – whether in an early and rude state or in our late and civilized times – are always proportional to total labor costs.

Now that we've identified the category-mistake we can view the classical antinomies in an entirely new light.

8. Dissolution of the problem of an invariable measure of value

There are two concepts of 'difficulty of production' that Ricardo conflates but we can now distinguish.

Standard labor-values, $\mathbf{v}$, measure 'difficulty of production' independent of an economy's institutional structure and distributive rules. A standard labor-value, v_i, is therefore a *counterfactual* measure of the total coexisting labor that would be required to reproduce commodity-type i if the net product were entirely consumed by workers and no 'tributary' labor was performed during the production of commodities.

Nonstandard labor-values, $\tilde{\mathbf{v}}$, measure 'difficulty of production' dependent on an economy's institutional structure and distributive rules. A nonstandard labor-value, $\tilde{v}_i$, is therefore an *actual* measure of the total coexisting labor required to reproduce commodity-type i given that the net product isn't entirely consumed by workers and additional 'tributary' labor is performed during the production of commodities.

Standard labor-values are an invariable measure of absolute value independent of the distribution of income. We can use standard labor-values to say, without embarrassment or equivocation, that 'commodity A is now less valuable than one year ago' in the strictly technical sense that commodity A requires less labor resources to reproduce than it once did. But it's a category-mistake to hope or expect, as Ricardo did, that this standard also explains the structure of natural prices.

In contrast, nonstandard labor-values are not a measure of absolute value independent of the real distribution of income. But they do explain the structure of natural prices in terms of 'difficulty of production', i.e. objective quantities of coexisting labor required to reproduce a commodity. Hence they provide that all-important one-to-one relation, required by a labor theory of value, between absolute values, measured in terms of labor time, and relative prices. Once the appropriate concept of 'difficulty of production' is applied then relative values do not vary independently of absolute values. There is no other 'less powerful cause' of the variation of relative values other than labor costs. Ricardo's problem is therefore dissolved.

9. Dissolution of the transformation problem

Let $\mathbf{n} = \mathbf{w} + \mathbf{c}$ be the net product of the economy, where $\mathbf{c}$ is the consumption bundle of capitalists. Marx splits the total working day, $\mathbf{l}\mathbf{q}^{\mathrm{T}} = \mathbf{v}\mathbf{n}^{\mathrm{T}}$ (see Proposition 5 in the appendix), into necessary labor, $\mathbf{v}\mathbf{w}^{\mathrm{T}}$, which is the part ‘technically necessary’ to reproduce workers, and surplus labor, $\mathbf{vn}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}$ $(= \mathbf{vc}^{\mathrm{T}})$, which is an additional part appropriated by capitalists. Marx’s normative point, among other things, is that production can occur without the performance of this surplus labor.

Nonstandard labor-values, by definition, include surplus labor as a cost of production. In consequence, they cannot split the working day into necessary and surplus parts. In terms of total labor costs the whole working day, $\mathbf{lq}^{\mathrm{T}} = \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$ (see Proposition 6 in the appendix), is ‘socially necessary’ to reproduce workers given that the real wage cannot be produced without the simultaneous performance of surplus labor for capitalists.

We can restate Marx’s concept of ‘surplus labor’ in terms of nonstandard and standard labor-values. Surplus labor is the difference between (i) the labor time socially necessary and (ii) the labor time technically necessary to reproduce workers, i.e. $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}$ (since $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = \mathbf{lq}^{\mathrm{T}} = \mathbf{vn}^{\mathrm{T}}$).

Splitting the working day this way is both logical and illuminating, regardless of any relationship it may have to the price system. But it’s a category-mistake to hope or expect, as Marx did, that a technical, and therefore partial, measure of surplus labor has a one-to-one relation with a total measure of money profit. Money profit, in fact, has a one-to-one relation with ‘total surplus labor’, $\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}} - \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$, not Marx’s surplus labor, $\mathbf{vn}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}$ (see Proposition 7 in the appendix).

In general, the Marxist tradition has accepted divergence of production prices from labor-values but defended conservation of labor-value in price, whereas critics have also accepted divergence but denied conservation of labor-value in price. But both sides of the argument are simultaneously correct and mistaken: once we measure in terms of total labor costs there is no divergence and there is aggregate conservation. Production prices represent total labor costs, i.e. nonstandard labor-values, and therefore capitalist profit is a money representation of labor time.

Corollary 1. All Marx’s equalities are true if labor-values measure total labor costs, specifically (i) the profit-rate equals the labor-value profit-rate, (ii) total profit is proportional to surplus labor, and (iii) total production price is proportional to total labor-value (see appendix for proof).

In consequence, the standard criticisms of the labor theory of value do not apply: nonstandard labor-values are not internally inconsistent, since the money profit-rate equals the labor-value profit-rate, nor redundant, since production prices can be derived from labor-values by scaling by the money wage w. Hence a theory of value based exclusively on labor cost can account for price phenomena: (nonstandard) labor-values and prices are ‘two sides of the same coin’. The transformation problem is therefore dissolved.

This conclusion, it should be emphasized, destroys the basis of any claim that a labor theory of value is logically incoherent because prices and labor-values are quantitatively incommensurable (e.g., Samuelson (1971); Lippi (1979); Steedman (1981)).

10. Conclusion

Ricardo’s problem of an invariable measure of value and Marx’s transformation problem are theoretical manifestations of an underlying category-mistake that conflates technical and total labor costs. The category-mistake has misdirected theoretical attention toward the antinomies and away from the fact that a commodity’s production price is the wage bill of the total coexisting labor required to reproduce it (Theorem 1). But once we avoid the category-mistake we reveal a new theoretical object: a more general labor theory of value with an invariable measure of value and without a transformation problem.

11. Appendix

Proposition 1. $r = 0$ implies $\mathbf{p} = w\mathbf{v}$.

Proof. Set $r = 0$ into price equation (3) to get $\mathbf{p} = \mathbf{pA} + \mathbf{l}w$ or $\mathbf{p} = w\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. Since $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ the conclusion follows. □

Proposition 2. $\mathbf{vA} = k\,\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}$ implies $\mathbf{p} = \alpha\mathbf{v}$, where $\alpha = w(1+r)/(1 - k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}r)$.

Proof. Write price equation (3) in series form:

$$\begin{aligned}\mathbf{p} &= (\mathbf{pA} + \mathbf{l}w)(1+r)\\ &= w(1+r)\mathbf{l}(\mathbf{I} - \mathbf{A}(1+r))^{-1}\\ &= w(1+r)\mathbf{l}\sum_{n=0}^{\infty}\mathbf{A}^n(1+r)^n.\end{aligned}$$

Let $k' = k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}$. Given uniformity, $\mathbf{vA} = k'\mathbf{l}$, and therefore,

$$\mathbf{p} = w(1+r)\frac{1}{k'}\sum_{n=0}^{\infty}\mathbf{vA}^{n+1}(1+r)^n. \tag{7}$$

Given uniformity and the definition of labor-value, $\mathbf{v} = \mathbf{vA} + \mathbf{l}$, then $\mathbf{vA} = (k'/k' + 1)\mathbf{v}$. Hence $\mathbf{vA}^2 = (k'/k' + 1)\mathbf{vA} = (k'/k' + 1)^2\mathbf{v}$ and by induction,

$$\mathbf{vA}^n = \left(\frac{k'}{k'+1}\right)^n\mathbf{v}.$$

Substitute into price equation (7) to get

$$\mathbf{p} = \left(w\sum_{n=0}^{\infty}\frac{(k')^n(1+r)^{n+1}}{(1+k')^{n+1}}\right)\mathbf{v}.$$

Given that $k'r < 1$ then the infinite sum converges to $(1+r)/(1-k'r)$. □

Proposition 3. All Marx's equalities are true if the economy satisfies the special condition

$$\mathbf{v}\Big(\mathbf{I} - (\mathbf{A} + \bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1+r)\Big)\mathbf{q}^{\mathrm{T}} = 0;$$

otherwise all Marx's equalities are not true.

Proof. If total profit is proportional to total surplus-labor then

$$(\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}}r = k(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{lq}^{\mathrm{T}}, \tag{8}$$

where k is a constant of proportionality. If the profit-rate equals the labor-value profit-rate then

$$r = \frac{(1 - \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{lq}^{\mathrm{T}}}{\mathbf{vAq}^{\mathrm{T}} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{lq}^{\mathrm{T}}}. \tag{9}$$

Substitute r from (9) into (8) to get

$$(\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}} = k(\mathbf{vAq}^{\mathrm{T}} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{lq}^{\mathrm{T}}). \tag{10}$$

If the total price of the gross product equals its labor-value then

$$\mathbf{pq}^{\mathrm{T}} = k\mathbf{vq}^{\mathrm{T}},$$

which given price equation (3) implies that

$$(\mathbf{pA} + \mathbf{l}w)\mathbf{q}^{\mathrm{T}}(1+r) = k\mathbf{vq}^{\mathrm{T}}. \tag{11}$$

Substitute (11) into (10) to get

$$\mathbf{vq}^{\mathrm{T}} = (\mathbf{vAq}^{\mathrm{T}} + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{lq}^{\mathrm{T}})(1+r),$$

which can be rearranged into the form

$$\mathbf{v}\Big(\mathbf{I} - (\mathbf{A} + \bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1+r)\Big)\mathbf{q}^{\mathrm{T}} = 0. \tag{12}$$

Hence Marx's three equalities imply (12). □

Lemma 1. The profit-rate is the price of capitalist consumption per unit of money-capital advanced, $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$.

Proof. Activity levels, in self-replacing equilibrium, are

$$\mathbf{q} = \mathbf{qA}^{\mathrm{T}} + \mathbf{w} + \mathbf{c},$$

where $\mathbf{w}$ is the real wage and $\mathbf{c}$ is capitalist consumption. Multiplying both sides by production prices gives

$$\mathbf{pq}^{\mathrm{T}} = \mathbf{pAq}^{\mathrm{T}} + \mathbf{pw}^{\mathrm{T}} + \mathbf{pc}^{\mathrm{T}}.$$

Workers spend their income on the real wage, $\mathbf{pw}^{\mathrm{T}} = \mathbf{lq}^{\mathrm{T}}w$, and therefore

$$\mathbf{pc}^{\mathrm{T}} = \mathbf{pq}^{\mathrm{T}} - (\mathbf{pAq}^{\mathrm{T}} + \mathbf{lq}^{\mathrm{T}}w).$$

Define $\mathbf{m} = \mathbf{pA} + \mathbf{l}w$ as the vector of cost prices. Then

$$\mathbf{pc}^{\mathrm{T}} = \mathbf{pq}^{\mathrm{T}} - \mathbf{mq}^{\mathrm{T}}. \tag{13}$$

Production price equation (3), in terms of cost prices, is

$$\mathbf{p} = \mathbf{m}(1+r).$$

Substitute into equation (13) to get

$$\mathbf{mc}^{\mathrm{T}}(1+r) = \mathbf{mq}^{\mathrm{T}}(1+r) - \mathbf{mq}^{\mathrm{T}}. \tag{14}$$

We define capitalist consumption per unit of money-capital advanced as $\bar{\mathbf{c}} = (1/\mathbf{mq}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}$, where $\mathbf{mq}^{\mathrm{T}}$ is the total money-capital advanced. Rearrange equation (14) and substitute for $\bar{\mathbf{c}}$ to get

$$\mathbf{m}\bar{\mathbf{c}}^{\mathrm{T}}(1+r) = r. \tag{15}$$

Since $\mathbf{m} = (1/(1+r))\mathbf{p}$ the conclusion follows. □

Theorem 1. Production prices are proportional to total labor costs,

$$\mathbf{p} = \tilde{\mathbf{v}}w,$$

where $\tilde{\mathbf{v}}$ are nonstandard labor-values.

Proof. Substitute $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$ from Lemma 1 into production price equation (3) to get

$$\mathbf{p} = (\mathbf{pA} + \mathbf{l}w) + (\mathbf{pA} + \mathbf{l}w)\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}. \tag{16}$$

Substitute $\mathbf{m} = \mathbf{pA} + \mathbf{l}w$ into (16) and rearrange,

$$\begin{aligned}\mathbf{p} &= (\mathbf{pA} + \mathbf{l}w) + \mathbf{m}\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}} \\ &= \mathbf{pA} + \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} + \mathbf{l}w \\ &= \mathbf{p}(\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}) + \mathbf{l}w \\ &= \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w.\end{aligned}$$

Hence $\mathbf{p} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}w = \tilde{\mathbf{v}}w$, by equation (6). □

Corollary 1. All Marx's equalities are true if labor-values measure total labor costs, specifically (i) the profit-rate equals the labor-value profit-rate, (ii) total profit is proportional to surplus labor, and (iii) total production price is proportional to total labor-value.

Proof. This is a trivial consequence of Theorem 1, i.e. the proportionality of production prices and total labor costs. □

Proposition 4. Marx's 'production prices', $\mathbf{p}^{\star}$, satisfy (i) the sum of profits is proportional to surplus labor,

$$\alpha(\mathbf{vA} + \mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}))r \propto \mathbf{lq}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}, \tag{17}$$

and (ii) i.e. the price of the gross product is proportional to its labor-value,

$$\mathbf{p}^{\star}\mathbf{q}^{\mathrm{T}} \propto \mathbf{vq}^{\mathrm{T}}. \tag{18}$$

Proof. Marx defines $r = r_v$ so we can write the LHS of (17) as

$$\alpha\frac{\mathbf{vA} + \mathbf{l}(\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})}{\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}}}(\mathbf{lq}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}) = \beta(\mathbf{lq}^{\mathrm{T}} - \mathbf{vw}^{\mathrm{T}}),$$

which establishes (i). And

$$\begin{aligned}\mathbf{p}^{\star}\mathbf{q}^{\mathrm{T}} &= \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}}) + \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{vw}^{\mathrm{T}})r_v \\ &= \alpha(\mathbf{vAq}^{\mathrm{T}} + \mathbf{lq}^{\mathrm{T}}) \\ &= \alpha\mathbf{vq}^{\mathrm{T}},\end{aligned}$$

which establishes (ii). □

Proposition 5. The total direct labor supplied equals the standard labor-value of the net product, $\mathbf{lq}^{\mathrm{T}} = \mathbf{vn}^{\mathrm{T}}$.

Proof. Since $\mathbf{q} = \mathbf{qA}^{\mathrm{T}} + \mathbf{n}^{\mathrm{T}}$ it follows that

$$\mathbf{v}(\mathbf{I} - \mathbf{A})\mathbf{q}^{\mathrm{T}} = \mathbf{vn}^{\mathrm{T}}. \tag{19}$$

But $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. Replace $\mathbf{v}$ on the LHS of (19) to get $\mathbf{lq}^{\mathrm{T}} = \mathbf{vn}^{\mathrm{T}}$. □

Proposition 6. The total direct labor supplied equals the nonstandard labor-value of the real wage, $\mathbf{lq}^{\mathrm{T}} = \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$.

Proof. Workers spend their income on the real wage, $\mathbf{lq}^{\mathrm{T}}w = \mathbf{pw}^{\mathrm{T}}$. Use Theorem 1 to substitute for $\mathbf{p}$ and the conclusion follows. □

Proposition 7. Money profit, $\mathbf{mq}^{\mathrm{T}}r$, is proportional to 'total surplus labor', $\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}} - \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$.

Proof. Since $\mathbf{n} = \mathbf{w} + \mathbf{c}$ and $\bar{\mathbf{c}} = (1/\mathbf{mq}^{\mathrm{T}})\mathbf{c}^{\mathrm{T}}$ then

$$\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} = \tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{mq}^{\mathrm{T}}$$

By Theorem 1,

$$\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} = \frac{1}{w}\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{mq}^{\mathrm{T}}.$$

By Lemma 1,

$$\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} = \frac{1}{w}\mathbf{mq}^{\mathrm{T}}r$$

and therefore $\mathbf{mq}^{\mathrm{T}}r = w(\tilde{\mathbf{v}}\mathbf{n}^{\mathrm{T}} - \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}})$. □

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