Step-by-step solution

Given a real number *a*, define $S \equiv \{x \mid x \text{ in } \mathbb{Q}, x < a\}$. By the definition of *S*, *a* is an upper bound of *S*. We will show that *a* is the least upper bound of *S* by assuming that this is false and deriving a contradiction.

Comment

Let *c* be an upper bound of *S* such that $c < a$. Since the set of rational numbers is dense in $\mathbb{R}$, there is a rational number *x* in (c, a). Therefore, *x* is a rational number and $x < a$, so *x* is in *S*. Moreover, since $x > c$, *c* is not an upper bound of *S* as originally assumed.

Comment

This contradiction implies that no number less than *a* is an upper bound of *S*. Thus, $a = \sup S$.

Comment

NEXT