CO-1
- Static Inertia force analysis
- Fly wheels choice

CO-2
- Balancing of rotating components
- Balancing of reciprocating components.

CO-3
- single degree of freedom free Vibrations (undamped)
- Single Degree of freedom free Vibrations (Damped).

CO-4
- Gyroscopes
- Governors.

CO - 1

COI - 1

KOM

It deals with motion of the mechanisms without considering force applied on it.

DOM

It completely by considering the forces applied on mechanism.

Theory of Machines
- Kinematics
- Dynamics
  - Static
  - Kinetic

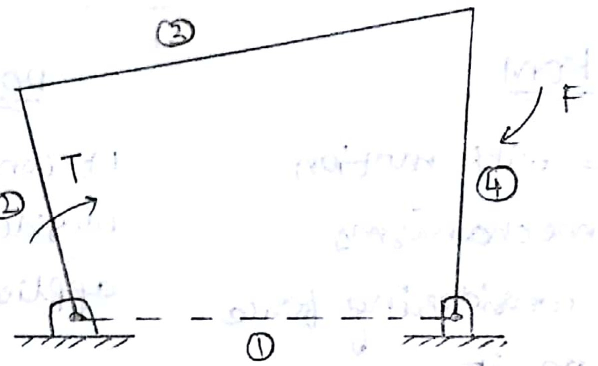
Static force Analysis:

When a machine is accelerated the components in the machine exert inertia forces. If there inertia forces are dominated by the externally applied forces then the inertial forces are neglected.

When the magnitude of inertia forces are considerably high then we will be going for inertia force analysis (or) Dynamic force analysis.

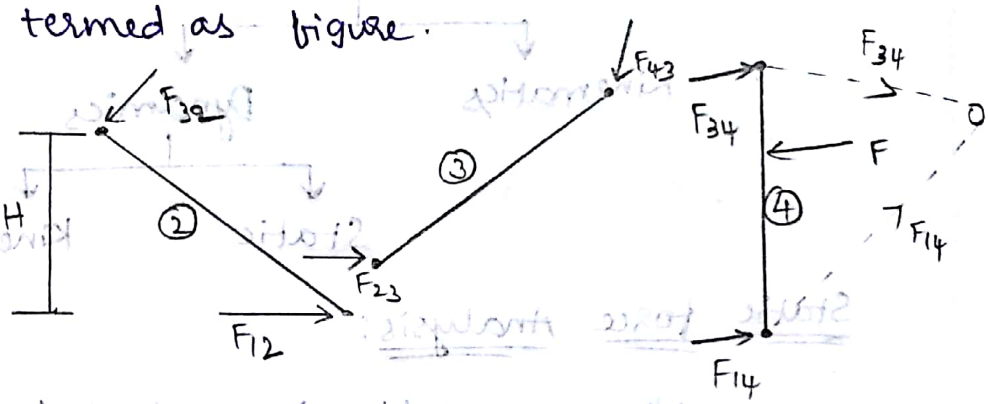
The Dynamic problem in D.F.A can be converted into static force analysis by applying DeAlemberts Principle.

Free body diagram of a four bar mechanism:



convention:

The force acting on the jth Link due to ith link is termed as figure.



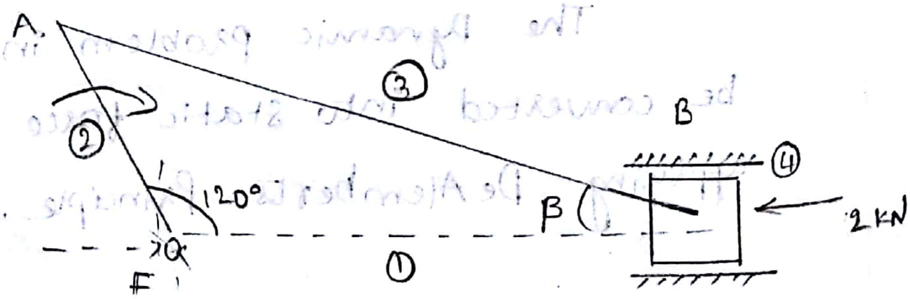
$$\Sigma F = 0$$

$$F_{12} + F_{32} + F_{23} + F_{43} + F_{34} + F_{14} = 0$$

$$\boxed{T = F_{32} \times H = F_{12} \times H}$$

Problems:

① A Slider crank mechanism with following dimensions is acted upon by force $F = 2\,KN$ $OA = 100\,mm$, $AB = 450\,mm$.



find the Torque T i.e., developed due to 2KN Force.

F_{12}

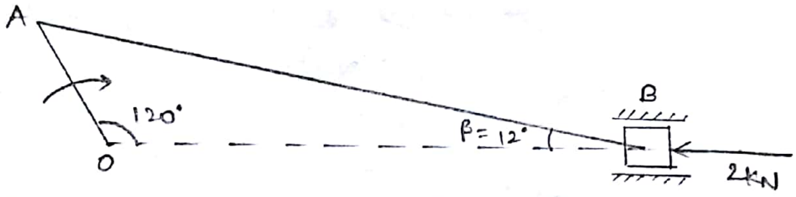
Scale :

100 mm = 2 cm

450 mm = 9 cm.

Step 1:

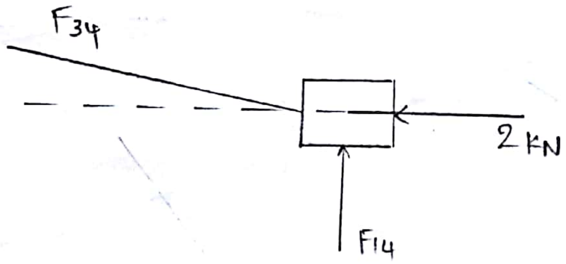
Draw the diagram in question according to its geometry.



$$\sin \beta = \frac{100}{450} \Rightarrow \beta = \sin^{-1}\left(\frac{100}{450}\right) = 12.8395°$$

Step 2:

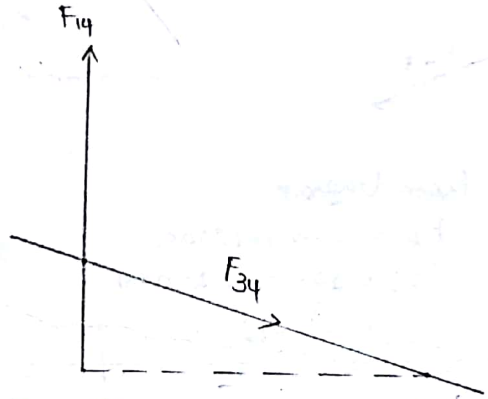
consider the link 4 due to link 4 has a force of magnitude and direction.



Scale

$x \times 400 = 2000$

1 cm = 400 N :



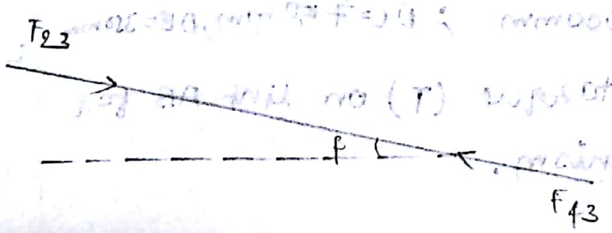
$F_{34} = 5.3$ cm

$F_{34} = 5.3 \times 400$ N

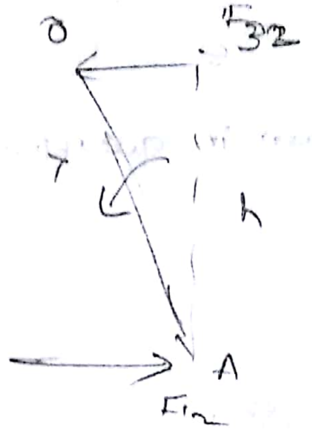
$F_{34} = 2120$ N

$\boxed{F_{34} = 21.2 \text{ kN}}$

Step 3: Considering the connecting rod.



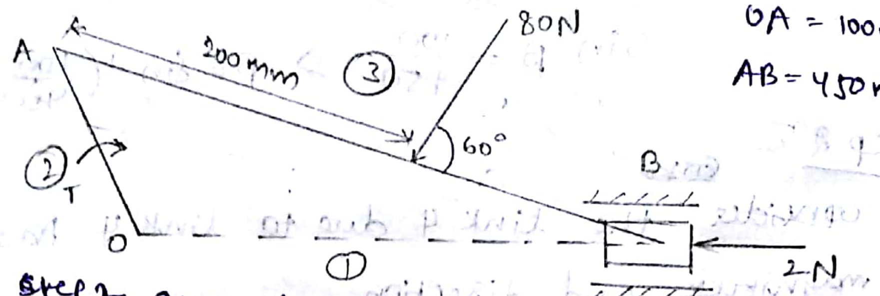
Step : 4



$F_{32} = 21.2\ kN$

$T = F_{32} \times h$.

②

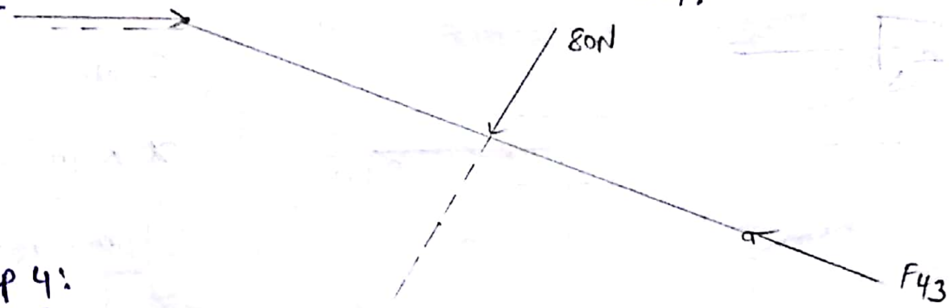


OA = 100mm

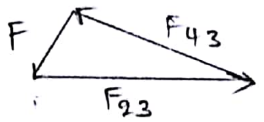
AB = 450 mm.

Step 1 and step 2 same as before Problem.

Step-3:



Step 4:



Scale:

1cm = 80N

$F_{43} = 21.2\ kN$

$F_{43} = 2120\ N$

$F_{43} = 2.65\ cm$

from diagram,

$F_{23} = 3\ cm$ (approx)

$F_{23} = 30 \times 80 = 2400N$

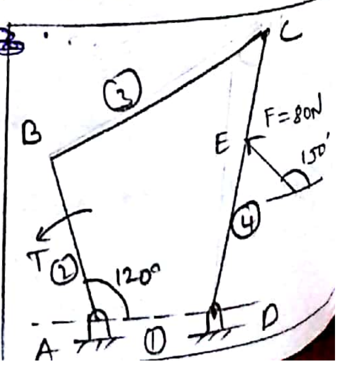
$\boxed{F_{23} = 24\ kN.}$

Problem ③: A four bar mechanism with following dimensio is acted upon by a force 80N at 150° on link DC.

~~AD=500mm~~ AD = 500mm, AB = 400mm; ~~BC~~

BC = 1000mm ; DC = 750 mm, DE = 350mm

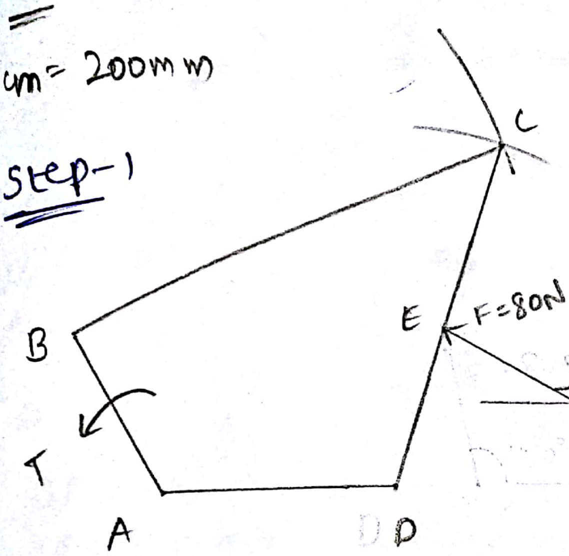
Determine torque (T) on link AB for this mechanism.



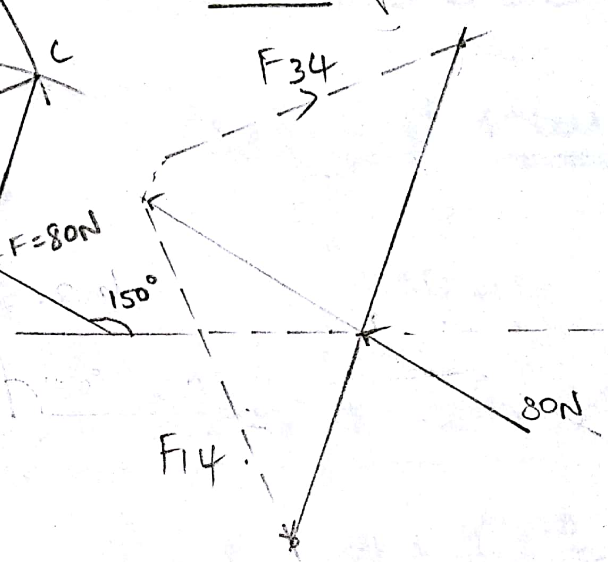
Scale:

1cm = 200mm

Step-1



Step-2:



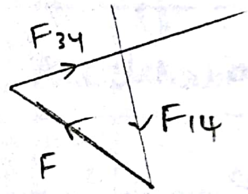
Scale: 1cm = 40N

80N = 2cm.

⇒ From diagram

$F_{34} = 1.35$ cm

$\boxed{F_{34} = 54N}$

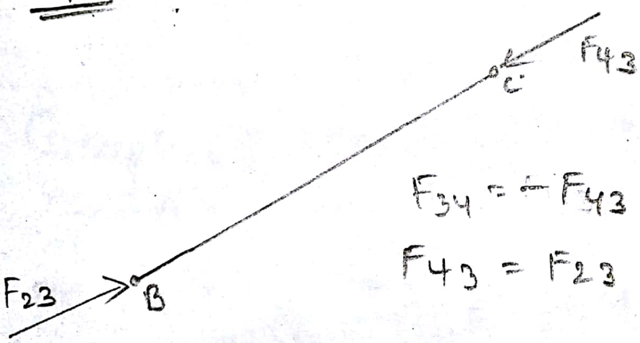


should be

$F_{34} = 46N$

$F_{14} = 64N$

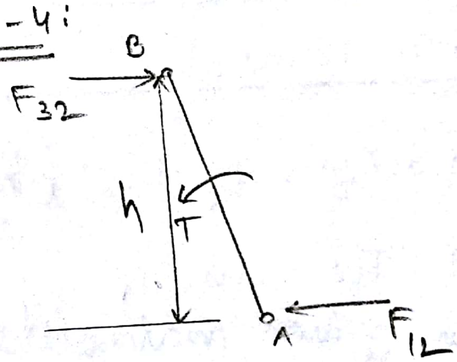
Step-3:



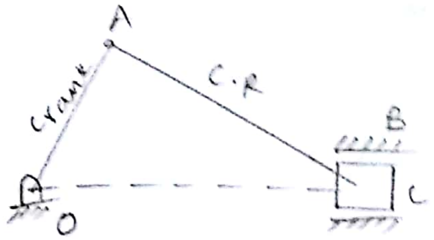
$F_{34} = -F_{43}$

$F_{43} = F_{23}$

Step-4:



Dynamics / Inertia Force Analysis



OA - crank
AB - C.R (connecting Rod)
BC - Slider / piston.

Analytical solution

	Velocity	Acceleration
Piston	$v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$	$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$
C.R	$\omega_c = \frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}$	$\alpha = \frac{\omega^2\sin\theta(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}$

Graphical solution (KLEIN's method):

Problem-1: In a slider crank mechanism having a stroke length of 30 cm and an obliquity ratio of 4. The crank is rotating uniformly in anti clockwise direction. The angular velocity of crank is 52 rad/sec. The crank has turned through an angle of 48.45° from inner dead centre (IDC) position. Determine using KLEIN's construction.

a) Acceleration of slider.

b) velocity of slider

Sol:- Stroke length = 30 cm

Obliquity ratio $\Rightarrow n = 4 = \frac{\text{length of the connecting Rod}}{\text{Radius of the crank}}$

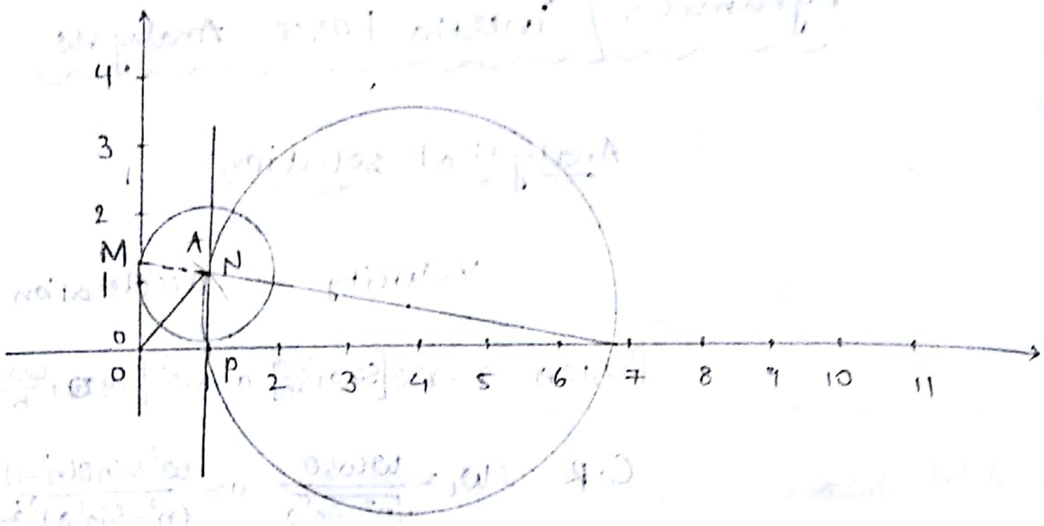
crank angle = 48.45°

velocity = 52 rad/sec.

$\boxed{l = 2 \times r} \Rightarrow 30\text{ cm} = 2 \times r \Rightarrow \boxed{r = 15\text{ cm}}$

$n = \frac{l_{c.r}}{r} = 4 \Rightarrow \boxed{l_{c.r} = 60\text{ cm}}$

Take 1unit = 10 cm.



For triangle :

* length of line OA = Velocity of the crank OA.
* The length of the line AM represents velocity of the connecting rod AB.
* The length of the line OM represents velocity of slider / Piston.

For Polygon (OANPO):

* The length of the line OA represents the radial (or) centripetal Acceleration.
* The length of the line AN represents the length of the connecting rod.
* The length of the line NP represent tangential acceleration. CIR
* The length of the line OP represents the acceleration of slider / Piston.

For velocity Δ^{le} OAM

AM = 1.2 cm

OM = 1.4 cm

OA = 1.5 cm

FOR parallelogram:

$OP = 1.1 cm$

$r = 1.5 cm = 0.15 m$

$\omega = 52 rad/s$

$V = r\omega = 0.15 \times 52$

$= 7.8 m/s$

V = length of the graph × scale

$\Rightarrow 7.8 = (DA) \times (x)$

$7.8 = 1.5 \times x$

$x = 7.8 / 1.5 = 5.2 cm$

$\boxed{x = 5.2 cm}$

Take $OM = 1.31 cm$

V_{om} = length on the graph × scale.

Velocity of slider (V_{om}) $= 1.4 \times 5.2 = 7.28 cm$

$a = r\omega^2 \Rightarrow a = \frac{15}{100} \times (52)^2 = 405.6 m/s^2$

$\Rightarrow a$ = length on polygon × scale

$\Rightarrow a = OA \times x$

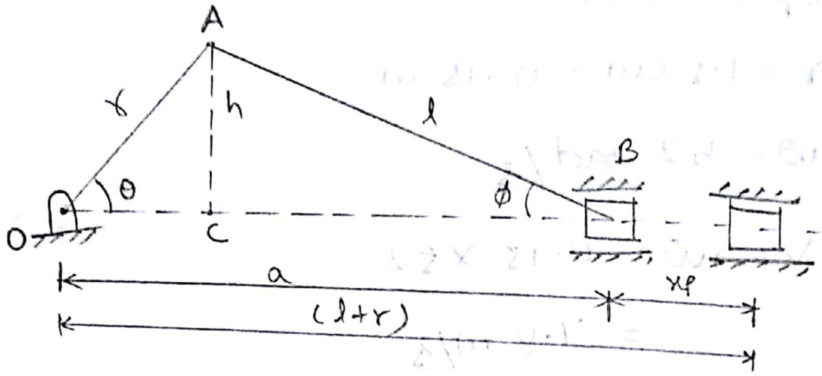
$\Rightarrow 405.6 = (1.5) \times (x)$

$\Rightarrow x = \frac{405.6}{1.5} = 270.4 cm$

$a_{op} = OP \times x = 1.1 \times 270.4 = 297.44 cm/s^2$

Acceleration of slider (a_{op}) $= 297.44 cm/s^2$.

Velocity & Acceleration of Piston & C.R:-



$$x_p = (l+r) - a$$

$$= (l+r) - (r\cos\theta + l\cos\phi)$$

ΔAOC
ΔABC

$$h = r\sin\theta = l\sin\phi$$

$$\sin\phi = \frac{r\sin\theta}{l}$$

Obliquity ratio $\left(\because \eta = \frac{l}{r}\right)$

$$\boxed{\sin\phi = \frac{\sin\theta}{\eta}} \rightarrow (1)$$

$$x_p = (l+r) - r\cos\theta - l\cos\phi$$

$$= (\eta r + r) - r\cos\theta - \eta r\cos\phi$$

From eq. (1),

$$\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{\eta^2}}$$

$\left(\because \cos^2\phi + \sin^2\phi = 1\right.$
$\cos^2\phi = 1 - \sin^2\phi$
$\left.\cos\phi = \sqrt{1-\sin^2\phi}\right)$

$$\cos\phi = \frac{\sqrt{\eta^2 - \sin^2\theta}}{\eta}$$

$$x_p = r\left[(\eta+1) - \cos\theta - \cancel{\eta}\cdot\frac{\sqrt{\eta^2 - \sin^2\theta}}{\cancel{\eta}}\right]$$

$$\boxed{x_p = r\left[(1-\cos\theta) + (\eta - \sqrt{\eta^2 - \cancel{\sin^2\theta}})\right]}$$

$\sin\theta << 1$

$$= r[(1-\cos\theta) + (\eta - \sqrt{\eta^2})]$$

$$\left(\begin{array}{c} \sin\theta <<<< 1 \\ l >>>> r \\ \eta = \frac{l}{r} \\ \eta >>>>> \sin\theta \end{array}\right)$$

$$\boxed{x_p = r(1-\cos\theta)}$$

S.H.M (simple Harmonic motion)

$$x_p = r[(1-\cos\theta) + (\eta - \sqrt{\eta^2 - \sin^2\theta})] \rightarrow f(\theta)$$

velocity, $V = \frac{dx_p}{dt} = \frac{dx_p}{d\theta} \times \frac{d\theta}{dt} \rightarrow \omega$

$$\boxed{\frac{d}{dx} a^n = n \cdot a^{n-1}}$$

$$= \frac{d}{d\theta}[r(1-\cos\theta) + (\eta - \sqrt{\eta^2 - \sin^2\theta})] \cdot \omega$$

$$= r\omega[0 - (-\sin\theta) + 0 - (\tfrac{1}{\cancel{2}} \times (\eta^2 - \sin^2\theta)^{-1/2} (-\cancel{2}\sin\theta\cos\theta))]$$

$$= r\omega\left[\sin\theta + \frac{2 \times \sin\theta\cos\theta}{2 \times \sqrt{\eta^2 - \sin^2\theta}}\right]$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{\eta^2 - \sin^2\theta}}\right]$$

$$\boxed{V_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\eta}\right]} \quad (\because \eta^2 >>>> \sin^2\theta)$$

$$a_p = \frac{dv_p}{dt} = \frac{dv_p}{d\theta} \times \frac{d\theta}{dt} \rightarrow \omega$$

$$= r\omega^2\left[\frac{d}{d\theta}\left(\sin\theta + \frac{\sin 2\theta}{2\eta}\right)\right]$$

$$= r\omega^2\left[\cos\theta + \frac{\cancel{2}\cos 2\theta}{\cancel{2}\eta}\right]$$

$$\boxed{a_p = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{\eta}\right]}$$

Velocity and Acceleration of C.R (Connecting Rod)!

$$h = r\sin\theta = l\sin\phi$$

$$\sin\phi = \frac{r\sin\theta}{l}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$\cos\phi \frac{d\phi}{dt} = \frac{\cos\theta}{n} \cdot \frac{d\theta}{dt}$$

$$\cos\phi \, \omega_{CR} = \frac{\cos\theta}{n} \cdot \omega$$

$$\Rightarrow \omega_{CR} = \frac{\cos\theta . \omega}{\cos\phi . n}$$

$$\left(\cos\phi = \frac{\sqrt{n^2 - \sin^2\theta}}{n}\right)$$

$$\Rightarrow \omega_{CR} = \frac{\cos\theta \cdot \omega}{\frac{\sqrt{n^2-\sin^2\theta}}{n} \cdot n}$$

$$\boxed{\therefore \omega_{CR} = \frac{\omega . \cos\theta}{\sqrt{n^2 - \sin^2\theta}}}$$

Acceleration of connecting Rod

$$\alpha_{C.R} = \frac{d\omega_{C.R}}{dt} = \frac{d\omega_{C.R}}{d\theta} \times \frac{d\theta}{dt} \rightarrow \omega$$

$$= \omega^2 \frac{d}{d\theta}\Big[\underbrace{(\cos\theta)}_{u} \times \underbrace{(n^2 - \sin^2\theta)^{-1/2}}_{v}\Big]$$

$$= \omega^2 \Big[(-\sin\theta) \times (n^2 - \sin^2\theta)^{-1/2}\Big] + \Big[(\cos\theta . \tfrac{-1}{2}(n^2 - \sin^2\theta)^{-\frac{3}{2}} \times (-\cancel{2}\sin\theta\cos\theta)\Big]$$

$$= \omega^2\Big[-(n^2 - \sin^2\theta)^{-1/2} + (\cos^2\theta . (n^2 - \sin^2\theta)^{-3/2}\Big]$$

$$= \omega^2 \sin\theta\,(n^2-\sin^2\theta)^{-3/2}\left[-(n^2-\sin^2\theta)^{-1/2}(n^2-\sin^2\theta)^{3/2} + \cos^2\theta\right]$$

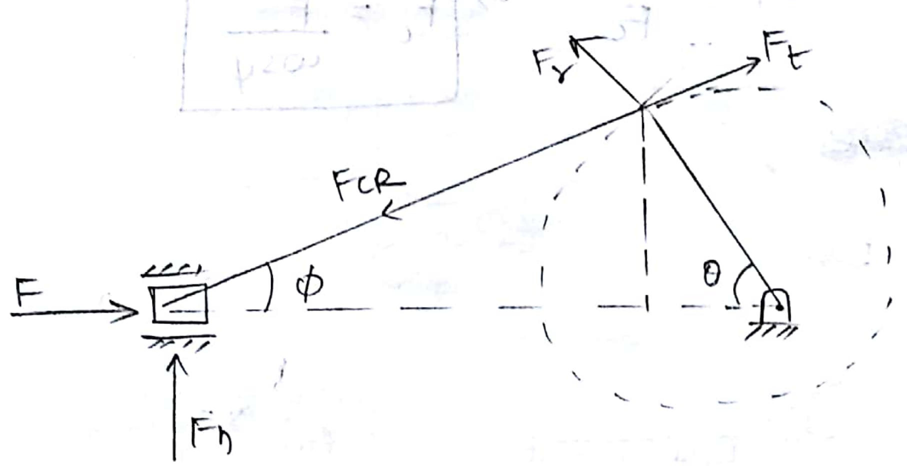
$$= \frac{\omega^2 \sin\theta}{\sqrt[3]{n^2-\sin^2\theta}}\left[-(n^2-\sin^2\theta) + \cos^2\theta\right]$$

$$= \frac{\omega^2 \sin\theta}{\sqrt[3]{n^2-\sin^2\theta}}\left[-n^2+(\sin^2\theta+\cos^2\theta)\right]$$

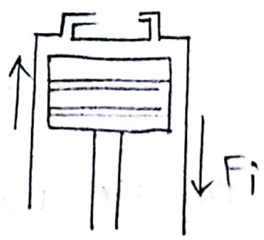
$$\boxed{\alpha_{CR} = \frac{\omega^2 \sin\theta\,(1-n^2)}{(n^2-\sin^2\theta)^{3/2}}}$$

Force Analysis of IC engines :

① Effect on Piston (or) Force effect on Piston :



$F = P_i A_i$



$$\boxed{F_p = P_1 A_1 - P_2 A_2}$$

Here, P_1 — Pressure on cover end

P_2 — Pressure on the rod end

A_1 — Area of piston on cover end

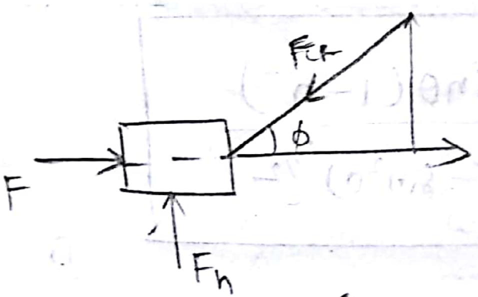
A_2 — Area of piston on rod end.

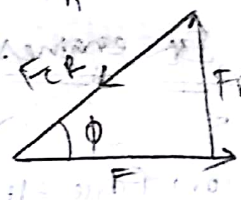
$$\boxed{F = F_p - F_i - F_f \pm mg}$$

$$F_i = m \cdot \alpha_p$$

$$\boxed{F_i = m \cdot \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$

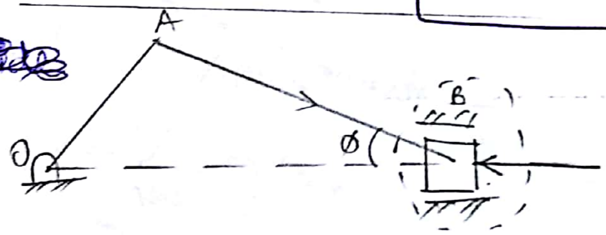
② Force on Connecting Rod :

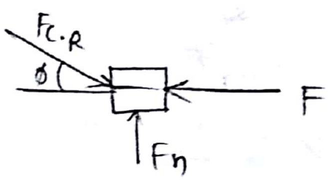


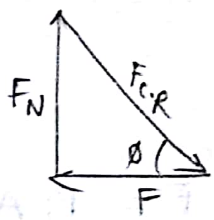


$$\cos\phi = \frac{F}{F_C} \Rightarrow \boxed{F_c = \frac{F}{\cos\phi}}$$

(OR)







$$\cos\phi = \frac{F}{F_{C.R}} \Rightarrow \boxed{F_{C.R} = \frac{F}{\cos\phi}} \rightarrow ①$$

③ Side thrust on Piston,

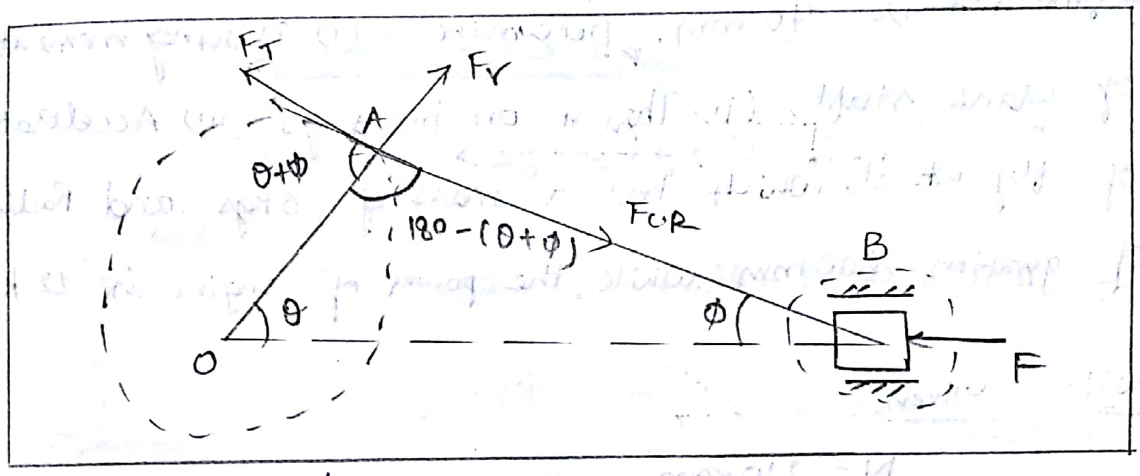
$$\sin\phi = \frac{F_N}{F_{C.R}} \Rightarrow F_N = F_{C.R}\sin\phi$$

From eq ①, $\boxed{F_N = \tan\phi \cdot F}$

(4) CRANK EFFORT:-

Crank effort is net effort or force applied at crank pin, perpendicular to crank which gives required turning movement on crank shaft.

Inorder to find the tangential force "F_T" we will be resolving force on the connecting rod '$F_{C.R}$' in plane of "F_T" (or) 'F_R'



$\phi \rightarrow$ connecting rod angle.

$\theta \rightarrow$ Crank angle.

$$F_T = F_{CR} \sin(\theta+\phi)$$

$$\boxed{F_T = \frac{F}{\cos\phi} \sin(\theta+\phi)}$$

(5) THRUST ON BEARINGS:

$$F_r = F_{C.R} \cos(\theta+\phi)$$

(6) Turning moment on crank shaft:

$$T = F_T \times r_{crank}$$

$$= \{F_{CR} \sin(\theta+\phi)\} \times r$$

$$\boxed{\therefore T = \frac{F}{\cos\phi} \times (\sin(\theta+\phi)) \times r}$$

Scanned by CamScanner

Problem-1: A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weighs 20 kgs. When the crank has turned through an angle of 30° from IDC (Inner Dead Centre). The gas pressure on the cover and crank sides are 500 kpas and 60 kpas. (1 pas = N/m^2 ; 1 megapas = N/mm^2). Dia. of piston rod is 40 mm. Determine :- (i) Turning moment of crank shaft, (ii) Thrust on bearings (iii) Acceleration of fly wheel, which has a mass of 8 kgs and Radius of gyration 600 mm while the power of engine is 22 kWatts

Sol:- Given,

$N = 210\,rpm$

$d_1 = 220\,mm = 0.22\,m$

stroke $= 440\,mm = 0.44\,m = d_{crank}$

C.R length $= 924\,mm = 0.924\,m$

$m = 20\,kg$

$\theta = 30°$

$P_1 = 500\,kN/m^2$

$P_2 = 60\,kN/m^2$

$1\,Pas = 1\,N/m^2$
$1\,watt = 1\,J/s$

Dia of piston } $d_2 = 40\,mm = 0.04\,m$

Solution,

(i) Turning moment on crank shaft:

$$T = \frac{F}{\cos\phi} \times \sin(\theta+\phi) \times r_{crank}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$n = \frac{l}{r} = \frac{0.924}{0.99}$$

$$r = \frac{0.44}{2} = 0.22 m$$

$$\boxed{n = 4.2}$$

$$\sin\phi = \frac{\sin(30)}{4.2}$$

$$\Rightarrow \phi = \sin^{-1}\left(\frac{\sin(30)}{4.2}\right)$$

$$\boxed{\phi = 6.83}$$

$$F_p = P_1A_1 - P_2A_2 = \left(P_1 \times \frac{\pi}{4}(d_1^2)\right) - \left(P_2 \times \frac{\pi}{4}(d_1^2 - d_2^2)\right)$$

$$F_p = \left\{(500\times10^3)\left(\frac{\pi}{4}\times(0.22)^2\right)\right\} - \left\{(60\times10^3)\left(\frac{\pi}{4}\times((0.22)^2-(0.04)^2)\right)\right\}$$

$$F_p = 19006.63555 - 2205.398043$$

$$\boxed{F_p = 16792.7237 \text{ N}}$$

$$F_i = m a_p$$

$$= m r \omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= (20)(0.22)(22^2)\left[\cos 30 + \frac{\cos(2\times30)}{4.2}\right]$$

$$\omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi(210)}{60}$$

$$\boxed{\omega = 22 \frac{rad}{s}}$$

$$\boxed{\cancel{F_i = 9508 \text{ N}}}$$

$$\boxed{F_i = 2097.81 \text{ N}}$$

$$F = F_p - F_i = 16792 - 2097.81 = 14694.91 \text{ N}$$

$$\boxed{F = 14694.91 \text{ N}}$$

$$\boxed{F = 14694.91 \text{ N}}$$

$$T = \frac{F}{\cos\phi} \times \sin(\theta+\phi) \times r$$

$$T = \left(\frac{14694.91}{\cos(6.83)}\right) \times \left(\sin(30+6.83)\right) \times 0.22$$

$$\boxed{T = \cancel{1951.32} \text{ N-m}}$$

$$\boxed{T = 1951.32 \text{ Nm}}$$

(ii) Thrust on bearings,

$$F_r = \frac{F}{\cos\phi} \cos(\theta+\phi)$$

$$F_r = F_c \cdot \cos(\theta+\phi)$$

$$F_r = \frac{14694.91}{\cos(6.83)} \cos(30+6.83)$$

$$\boxed{F_r = 11846.2 \text{ N}}$$

(iii)

$$T = I\alpha$$

$$T = \left(\text{Turning moment}\right) - \left(\text{Turning moment due to its inertia}\right)$$

$P = 22$ kW $\Rightarrow P = \frac{2\pi NT}{60}$

$\Rightarrow P = \omega T = 22 \times T_i$

$\Rightarrow 22 \times 10^3 = 22 \times T_i$

$$\boxed{T_i = 1000 \text{ N-m}}$$

$$T = 1951.32 - 1000$$

$$\boxed{T = 951.32 \text{ N-m}}$$

$T = I\alpha$

$I = mk^2 = 8 \times (0.6)^2 = 2.88$

$$\boxed{I = 2.88 \ kg\text{-}m^2}$$

$951 = 2.88 \times \alpha$

$\alpha = \frac{951}{2.88} = 330.31$

$$\boxed{\alpha = 330.31 \ rad/s^2}$$

Problem-2: The crank & connecting rod of a vertical petrol engine running at 1800 rpm are 60mm and 270 mm respectively. The diameter of piston is 100mm and the mass of the reciprocating parts is 1.2 kg. During the expansion stroke, when the crank has ~~(turning)~~ turned 20° from the TDC, the gas pressure is 650 KPas. Determine (i) net force on piston (ii) net load on gudgeon ~~(butt joint)~~ pin (F_{CR}) (iii) Thrust on cylinder walls ~~valves~~, (iv) speed at which gudgeon pin load is reverse in direction

Sol:- Given,

$N = 1800 \ rpm$

$r = 60mm = 0.06m$

$l = 270mm = 0.27m$

$d = 100 \ mm = 0.1m$

$m = 1.2kg$, $P = 650 \ KN/m^2$.

$\theta = 20°$.

Net force,

$$F = F_p - F_i + mg$$

$$F_p = p \times \frac{\pi}{4} d^2$$

$$= (650 \times 10^3) \times \frac{\pi}{4} (0.1)^2$$

$$= 5.10\ KN = 5100 N$$

$$\omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times (1800)}{60}$$

$$\omega = 60\pi$$

$$\omega = 188.4\ \frac{rad}{s}$$

$$F_i = m \cdot \alpha$$

$$= m \cdot r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{\eta}\right)$$

$$= (12)(0.06)(188.4)^2 \left(\cos(20) + \frac{\cos(2 \times 20)}{4.5}\right)$$

$$F_i = 2836.5\ N$$

$$\eta = \frac{0.27}{0.06} = \frac{l}{r}$$

$$\eta = 4.5$$

$$mg = (1.2) \times (9.81) = 11.76\ kg\frac{m}{s^2}$$

$$= 11.76\ N$$

$$F = 5100 - 2836.5 + 11.76$$

$$\boxed{F = 2275.26\ N}$$

(ii) Net load on gudgeon pin (C.R)

$$F_c = \frac{F}{\cos\phi}$$

$$\sin\phi = \frac{\sin\theta}{\eta}$$

$$\phi = 4.35$$

$$F_c = \frac{2275.26}{\cos(4.35)}$$

$$\boxed{F_c = 2281.8\ N}$$

(iii) Thrust on cylinder walls

$$F_n = F \tan\phi$$

$$F_n = 2275.26 \times \tan(4.35)$$

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$$\boxed{F_n = 173.07\,N}$$

(iv) Speed at which gudgeon pin load is reversed in direction.

$$F = F_p - F_i - mg \quad (\text{Here } F = 0)$$

$$0 = 5100 - \left[mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right) \right] - (9.81 \times 1.2)$$

$$0 = 5100 - \left[(1.2)(0.06)\omega^2 \left(\cos 20 + \frac{\cos 40}{4.5} \right) \right] - 11.76$$

$$0 = 5100 - 0.0799\,\omega^2 - 11.76$$

$$\omega^2 = \frac{5088.228}{0.07991}$$

$$\omega = 252.3380 \text{ rad/sec}$$

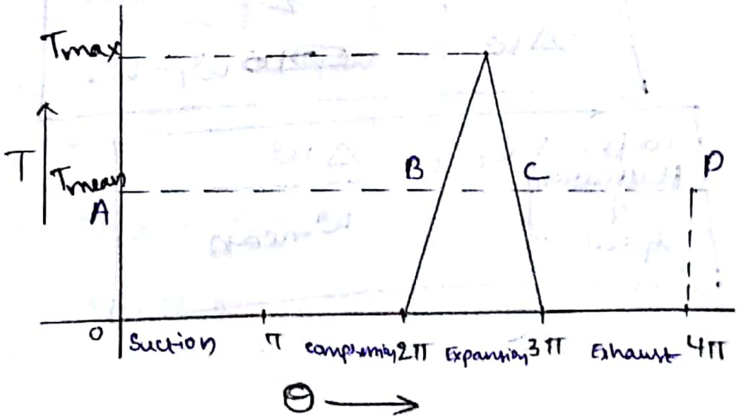
$$\Rightarrow \frac{2\pi N}{60} = 252.3380$$

$$\boxed{N = 2409.6 \text{ RPM.}}$$

CO I - 2

Fly wheel:

The objective of the fly wheel to reduce the fluctuations in speed with respect to speed arising due to the fluctuations in the available energy.



Mean Torque (Theoritical (or) Imaginary Torque):

When you apply same torque throughout the cycle which would develop same energy per cycle. As per that of actual energy per cycle.

$$\boxed{\text{Work done} = [\underbrace{T_{mean}}_{\text{Denoted as}} (T_m)] \times \theta}$$

where,
$\theta = 4\pi$ (4-stroke)
$\theta = 2\pi$ (2-stroke)

Flywheel design :-

The kinetic energy for a flywheel rotating with a max. and min. speeds of ω_1 & ω_2 can be given as $(KE)_{max}$ ~~cod~~ $= \frac{1}{2} I \omega_1^2$; $(KE)_{min} = \frac{1}{2} I \omega_2^2$

$C_s \rightarrow$ Coefficient fluctuation of speed.

w.k.T, $KE = \frac{1}{2} m v^2$

$$\boxed{KE = \frac{1}{2} I \omega^2}$$

$(KE)_{max} = \frac{1}{2} I \omega_1^2$ $\quad$ ω_1 — max. speed rad/s

$(KE)_{min} = \frac{1}{2} I \omega_2^2$ $\quad$ ω_2 = min. speed

$$(\Delta KE) = (KE)_{max} - (KE)_{min}$$

$$= \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$\boxed{\begin{aligned} \omega_{mean} &= \frac{\omega_1 + \omega_2}{2} \\ \Delta\omega &= \omega_1 - \omega_2 \end{aligned}}$$

$$\boxed{\text{Coeff. fluctuation of speed} \; C_s = \frac{\Delta\omega}{\omega_{mean}}}$$ **

$$\Rightarrow (\Delta KE) = \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$\boxed{\Delta KE = I \cdot \omega_{mean} \cdot \Delta\omega}$$

$$\Rightarrow \Delta KE = I \cdot \omega_{mean} \cdot (C_s \cdot \omega_{mean})$$

$$\boxed{\Delta KE = I \cdot \omega_m^2 \cdot C_s}$$

ΔKE – fixed

ω_m – fixed

$$\Delta\omega \propto \frac{1}{I}$$

$I \uparrow \quad \Delta\omega \downarrow$

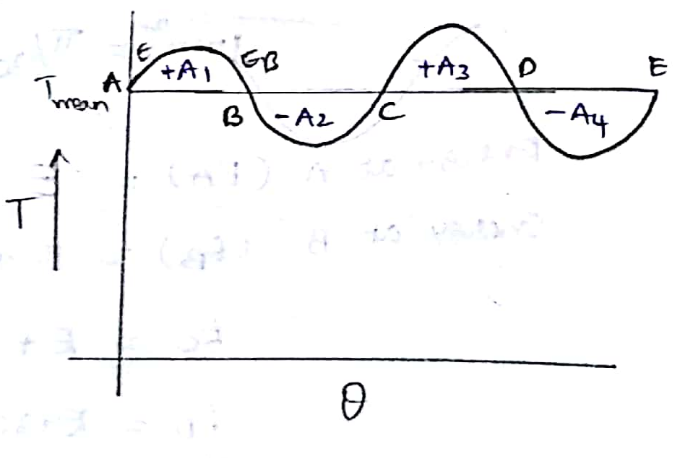
→ In order to reduce the fluctuation ~~speed~~ in speed $\Delta\omega$, the mass moment of inertia should be high.

→ $\boxed{I = mk^2}$, By increasing 'k', the mass moment of inertia increases without any increase in mass 'm'.

For multi cylinder Engine

(ΔE) is nothing but (ΔKE)

$$\boxed{\Delta E = E_{max} - E_{min}}$$



$E_A = E$, $E_B = E + A_1$,

$E_C = E + A_1 - A_2$,

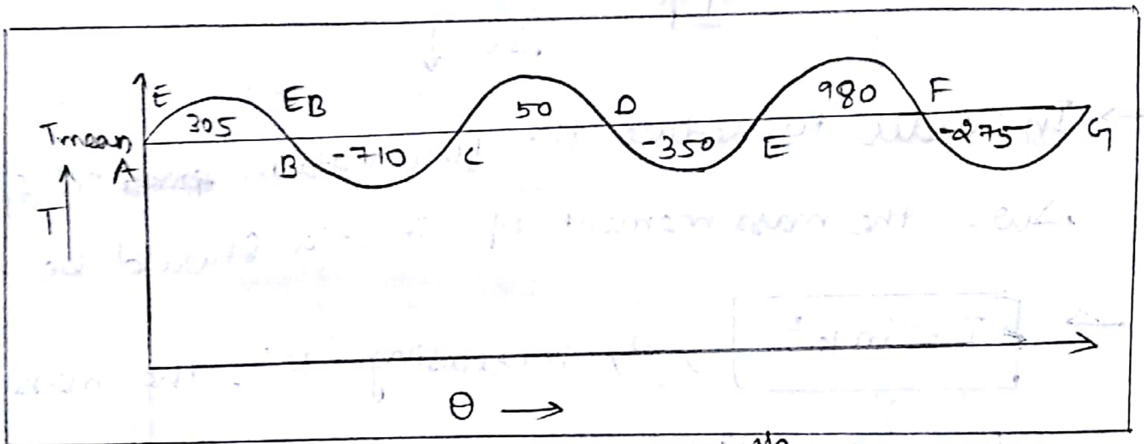
$E_D = E + A_1 - A_2 + A_3$,

$E_E = E + A_1 - A_2 + A_3 - A_4$

Where; $A_1, -A_2, A_3, -A_4$ are the areas of wave.

Problem - 1:

The turning moment diagram for a petrol engine, is drawn to a vertical scale of 1mm to 6 N-m. And a horizontal scale of 1mm to 1°. The turning moment repeats itself after every half revolution of engine. The area's above and below the mean torque line are 305, −710, 50, −350, 980, −275 mm^2, The rotating parts amount ~~to~~ a mass of 40 kg and a radius of gyration 2m. Determine co-efficient of fluctuation of speed, if the speed of engine is 1500 rpm.



Rough

(mm^2) $\to 6\ N\text{-}m \times \frac{\pi}{180}$

$J = N\text{-}m$

$1\ mm = 6\ N-m$

$1\ mm = 1°$

Area $\Rightarrow$ $1\ mm^2 = 6 \times 1 \times \frac{\pi}{180}$

$1\ mm^2 = \pi/30 = 0.104\ N-m$

Energy at A (EA) = E

Energy at B (EB) = E + 305 $\longrightarrow$ E_{max}

$E_C = E + 305 - 710 = E - 405$

$E_D = E + 305 - 710 + 50 = E - 355$

$E_E = E + 305 - 710 + 50 - 350 = E - 705 \to E_{min}$

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$E_F = E + 305 - 710 + 50 - 350 + 980 = E + 275$

$E_G = E + 305 - 710 + 50 - 350 + 980 - 275 = E.$

$$\Delta E = E_{max} - E_{min}$$

$$= E + 305 - (E - 705)$$

$$= E + 305 - E + 705$$

$$\Delta E = 305 + 705 = 1010 \text{ mm}^2$$

$$\left(\because 1\text{mm}^2 = 0.104 \text{ N-m}\right) = 1010 \times 0.104 \text{ N-m}$$

$$\boxed{\Delta E = 105 \text{N-m}}$$

W.K.T, $\Delta E = I.(\omega_{mean})^2 . C_s$ $\rightarrow kg.m^2$ $\boxed{\because I = mk^2 = kg.m^2 \text{ (units)}}$

$N = 1500$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi * 1500}{60} = 50\pi = 157 \text{ rad/s}$$

$$\Rightarrow 105 = \overset{kg}{(40)}\ \overset{m^2}{(2^2)} \times (157)^2 \times C_s$$

$$\Rightarrow C_s = \frac{105}{(40)(2^2) \times (157)^2}$$

$$C_s = 2.66 \times 10^{-5}$$

$$\boxed{\left.\begin{array}{l}\text{Coefficient}\\ \text{fluctuation}\\ \text{of speed}\end{array}\right\} C_s = 0.0266 \times 10^{-3}}$$

Problem-2:

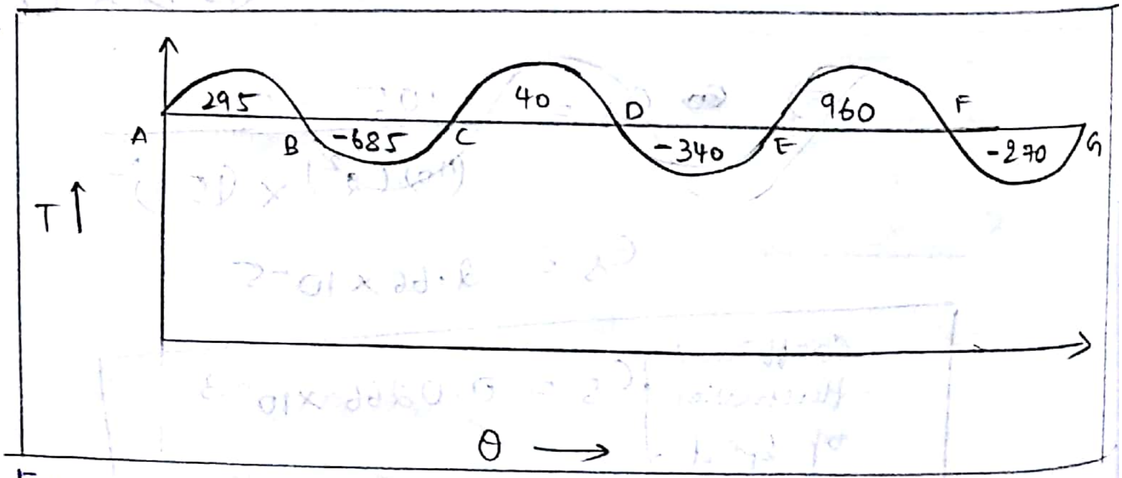
The turning moment diagram for a Petrol engine is drawn to a scale ~~of~~ on Y-axis 1mm = 5 N-m and on X-axis 1mm = 1°. The turning moment diagram repeats itself at every half revolution of engine. And the areas above and below the mean turning moment line 295, 685, 40, 340, 960 and 270 mm^2. Determine the mass of a 300 mm diameter wheel rim, with coefficient of fluctuation of speed 0.3 % ($\frac{0.3}{100}$) And engine runs at 1800 rpm.

$I = mR^2$ — cylinder

$I = \frac{1}{2} mR^2$ — circular ring

$I = \frac{3}{2} mR^2$ — sphere.

Sol:-



Energy at A (E_A) = E

$E_B = E + 295 \longrightarrow E_{max}$.

$E_C = E + 295 - 685 = E - 390$

$E_D = E + 295 - 685 + 40 = E - 350$

$E_E = E + 295 - 685 + 40 - 340 = E - 690 \rightarrow E_{min}$

$E_F = E + 295 - 685 + 40 - 340 + 960 = E + 270$

$E_G = E + 295 - 685 + 40 - 340 + 960 - 270 = E.$

$\Delta E = E_{max} - E_{min}$

$= (E + 295) - (E - 690)$

$\Delta E = 985 \ mm^2$

$\Delta E = 985 \times 5 \times 1 \times \frac{\pi}{180}$

$\boxed{\Delta E = 85.6 \ N\text{-}m.}$

W.K.T, $\Delta E = I.(\omega_{mean})^2 . C_s$

$\omega = \frac{2\pi N}{60} = \frac{2\pi * 1800}{60} = 60\pi = 188.4 \ rad/s$

$\Rightarrow 85.6 = I * (188.4)^2 * \frac{0.3}{100}$

$\Rightarrow I = \frac{85.6 \times 100}{(188.4)^2 \times 0.3}$

$\Rightarrow I = 0.803 \ kg\text{-}m^2$

W.K.T for cylinder $\Rightarrow$ $I = mr^2$

$r = \frac{300}{2} = 150 \ mm$

$\boxed{r = 0.150 \ m}$

$\Rightarrow 0.803 = m(0.150)^2$

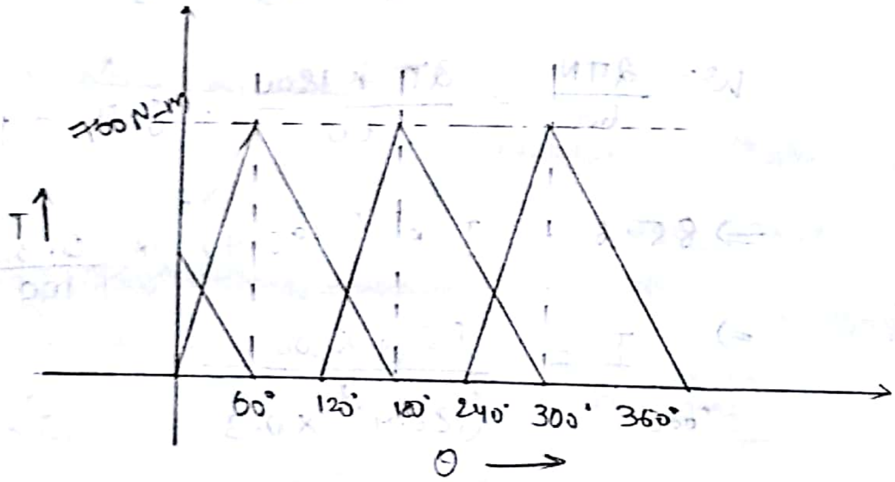
$m = \frac{0.803}{(0.15)^2} = 36 kg$

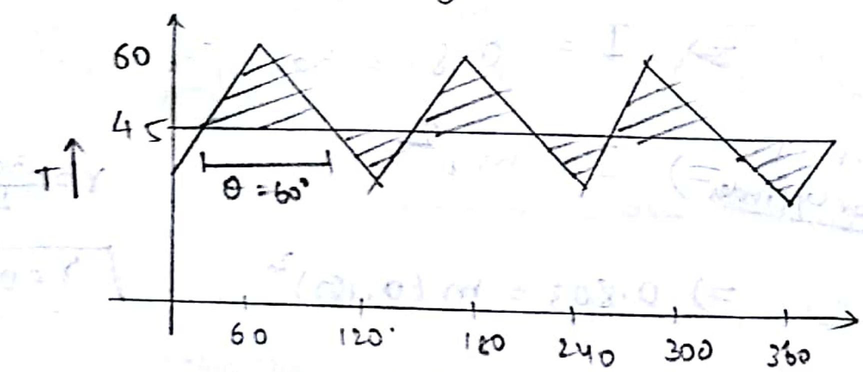
$\boxed{m = 36 \ kg}$

Problem-3:

A three cylinder, Single acting engine has its cranks at 120°. The turning moment diagram of each cycle is a triangle for the power stroke with a maximum torque of 60 N-m at 60° after the dead centre of corresponding crank. There is no torque on the return stroke. The engine runs at 400 rpm. Determine the (i) Power developed, (ii) Coefficient of fluctuation of speed (C_s) if mass is 10kg and radius of gyration 88mm (iii) maximum angular acceleration.

Sol:-





$P = T\omega$

Tmean.

$$W.D/cycle = 3 \times \frac{1}{2}\left(180 \times \frac{\pi}{180} \times 60\right)$$

$$= 3 \times 60 \times \frac{\pi}{2} = 90\pi \text{ N-m}$$

$$W \cdot D/\text{cycle} = T_{mean} \times \theta .$$

$$T_{mean} = \frac{W \cdot D/\text{cycle}}{2\pi} = \frac{90\pi}{2\pi} = 45 \text{ N-m}$$

$$P = T\omega .$$

$$= 45 \times \frac{2\pi \times 400}{60} = 1884 \text{ W} = 1.8 \text{ KW}$$

$$\Delta E = I \omega^2 C_s$$

$$I = mk^2 ; \quad \omega = \frac{2\pi N}{60}$$

$$\Delta E = \frac{1}{2} (T \times \theta)$$

$$= \frac{1}{2} (15 \times 60 \times \frac{\pi}{180})$$

$$= \frac{1}{2} (15 \times \frac{\pi}{3})$$

$$= 7.85 \text{ N-m}$$

$$\Rightarrow 7.85 = m \cdot k^2 \times \left(\frac{2\pi N}{60}\right)^2 \times C_s$$

$$7.85 = (10)(0.088)^2 \times (41.88)^2 \times C_s$$

$$\boxed{C_s = 0.058}$$

Coefficient of fluctuation of energy (C_E):

It is the ratio of maximum fluctuation of energy to the work done per cycle.

$$C_E = \frac{\Delta E}{W \cdot D/\text{cycle}} .$$

Date: 10.08.2018. CO-2

Balancing:

For (two) rotating components, if the axis of rotation does not match with each other then there will be unwanted forces due to the eccentricity prevailed.

The balancing of rotating components can be classified into static and dynamic balancing.

Static balancing:

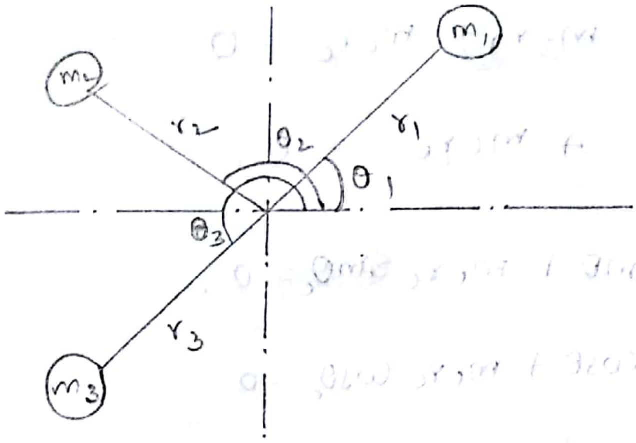
If all forces remain in one plane and pass through one point is called static balance

Dynamic Balancing:

If a momentum exists due to the static forces (or) non-concurrence.

- Balancing
  - Rotary
    - Static
      $\Sigma F = 0$
    - Dynamic
      $\Sigma F = 0$
      $\Sigma M = 0$.
  - Reciprocating

Static Balancing :



Let m_1, m_2, m_3 are the masses (centre of gravity) acting at a distances r_1, r_2, r_3 from the centre of rotation. Due to this rotation of masses the system will experience centrifugal forces due to which net force $\Sigma F = 0$

$$m_1 r_1 \omega_1^2 + m_2 r_2 \omega_2^2 + m_3 r_3 \omega_3^2 = 0$$ (where $\omega_1, \omega_2, \omega_3$ are angular velocities)

↳ Statically balanced

$$m_1 r_1 \omega_1^2 + m_2 r_2 \omega_2^2 + m_3 r_3 \omega_3^2 = F \neq 0$$

↳ Statically unbalanced

In the above eq. represents the unbalanced forces where resultant is not equal to zero ($F \neq 0$). To make it statically balance we have to add (or) subtract the mass from the system (m_c) at a distance (r_c):

$$m_1 r_1 \omega_1^2 + m_2 r_2 \omega_2^2 + m_3 r_3 \omega_3^2 + m_c r_c \omega_c^2 = 0$$

↳ Statically balanced.

Now consider $\omega_1 = \omega_2 = \omega_3 = \omega_c = \omega$.

$$\Rightarrow (m_1 r_1 + m_2 r_2 + m_3 r_3) + m_c r_c = 0$$

$$\Rightarrow \Sigma mr + m_c r_c = 0.$$

(For vertical component) $\Sigma mr \sin\theta + m_c r_c \sin\theta_c = 0$.

(For Horizontal component) $\Sigma mr \cos\theta + m_c r_c \cos\theta_c = 0$

$$m_c r_c \sin\theta_c = -\Sigma mr \sin\theta \rightarrow (1)$$

$$m_c r_c \cos\theta_c = -\Sigma mr \cos\theta . \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \boxed{\tan\theta_c = \frac{-\Sigma mr \sin\theta}{-\Sigma mr \cos\theta}}$$

Squaring and Adding (1) & (2)

$$(m_c r_c)^2 [\cancelto{1}{\sin^2\theta + \cos^2\theta}] = (-\Sigma mr \sin\theta)^2 + (-\Sigma mr \cos\theta)^2$$

$$(m_c r_c)^2 = (-\Sigma mr \sin\theta)^2 + (-\Sigma mr \cos\theta)^2$$

$$m_c r_c = \sqrt{(-\Sigma mr \sin\theta)^2 + (-\Sigma mr \cos\theta)^2}.$$

<u>Problem-1</u>: A circular disc mounted on a shaft carries three attached masses of 4 kg, 3 kg, 2.5 kg at radial distances of 75, 85, 50 mm is. At the angular positions of 45°, 135°, 240° respectively. The angular positions were measured counter clockwise from the reference of X-axis. Determine the amount of counter mass required at a radial distance of 75 mm to make it statically balanced.

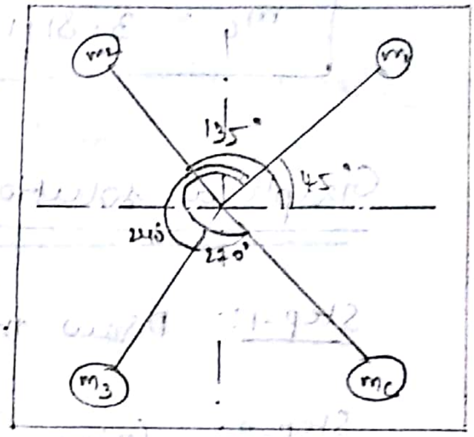
Sol:- **Given,**

$m_1 = 4\,kg$ $\quad r_1 = 75\,mm$ $\quad \theta_1 = 45°$

$m_2 = 3\,kg.$ $\quad r_2 = 85\,mm$ $\quad \theta_2 = 135°$

$m_3 = 2.5\,kg.$ $\quad r_3 = 50\,mm$ $\quad \theta_3 = 240°.$

$r_c = 75\,mm$



Formulae:

$$\tan\theta_c = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta}$$

$$m_c r_c = \sqrt{(\Sigma mr\sin\theta)^2 + (-\Sigma mr\cos\theta)^2}$$

Solution:

$$-\Sigma mr\sin\theta = -[m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3]$$

$$= -[(4\times75\times\sin45) + (3\times85\times\sin135°) + (2.5\times50\times\sin240°)]$$

$$= -284.19$$

$$-\Sigma mr\cos\theta = -[m_1 r_1\cos\theta_1 + m_2 r_2\cos\theta_2 + m_3 r_3\cos\theta_3]$$

$$= -[(4\times75\times\cos45) + (3\times85\times\cos135) + (2.5\times50\times\cos240°)]$$

$$= -[-30.6801]$$

$$= 30.6801$$

$$\tan\theta_c = \frac{-284.19}{30.68}$$

$$\theta_c = \tan^{-1}(-9.26303).$$

$\theta_c = -83.8384.$ $\longrightarrow$ clock wise so −ve

$$\theta_c = 360 - 83.834$$

From positive X-axis, $\boxed{\theta_c = 276.2}$ $\longrightarrow$ counter clockwise +ve.

$$m_c r_c = \sqrt{(-284.19)^2 + (30.6)^2}$$

$$m_c = \frac{285.832}{75}$$

$$\boxed{m_c = 3.8111 \text{ Kg.}}$$

Graphical solution:

Step-1: Draw the parent diagram with geometry.

Step-2: Give nomenclature ~~nomiglature~~ for ~~e~~every masses.

Step-3: Take a suitable scale i.e., (1 cm = X Newtons)

$1\ cm = \underline{X}\ \frac{(m \times r)}{kg - mm}$

Step-4: Draw the force polygon for the Product of mass x radius ($m \times r \rightarrow kg.mm$)

Step-5: Using set square, draw the Projection of the first ~~Bo~~ (~~force~~) mass.

Step-6: Follow the same procedure for different masses given in the question (Don't draw for complimentary mass).

Step-7: Join the last point to the first point (aa').

Step-8: The length of the closing line of polygon gives you the product of $m \times r$,

Step-9: Measure the angle at which the closing line of polygon is oriented w.r.to +ve x-axis in counter clockwise direction.

Problem - 21

* The rotor has following Properties:

$m_1 = 3\ kg$ $r_1 = 30\ mm$ $\theta_1 = 30°$

$m_2 = 4\ kg$ $r_2 = 20\ mm$ $\theta_2 = 120°$

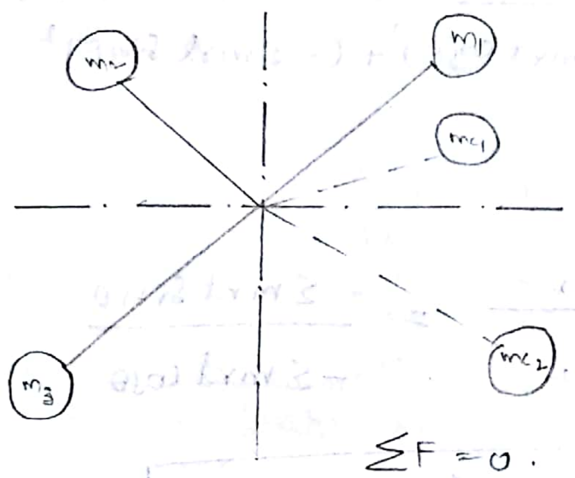
$m_3 = 2\ kg$ $r_3 = 25\ mm$ $\theta_3 = 270°$

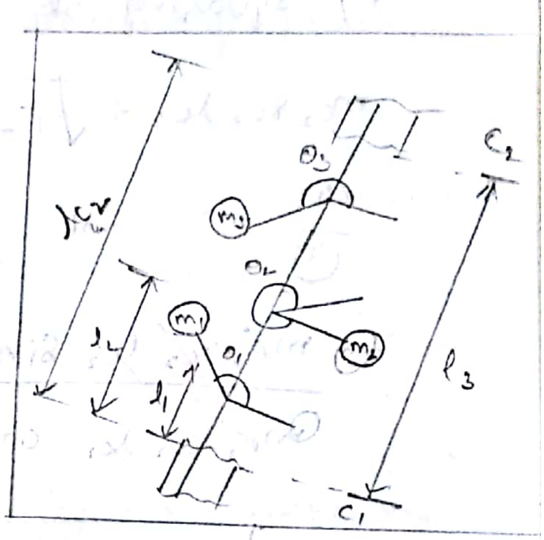
Find the amount of counter mass of a radial distance 35 mm for the static balance.

The ... the following ...

Find the amount of ... of standard deviation 25.40 ... the static ...

Dynamic Balancing :

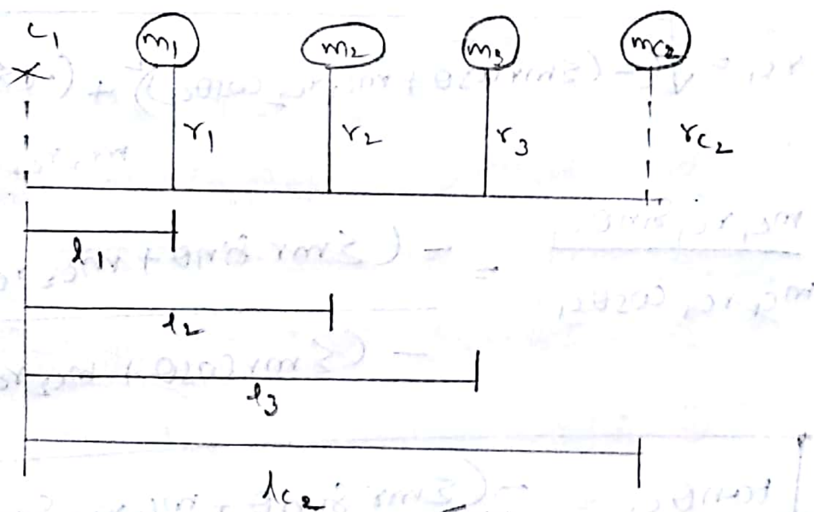




$$\Sigma F = 0.$$

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_{c_1} r_{c_1} + m_{c_2} r_{c_2} = 0$$

$$\boxed{\Sigma mr + m_{c_1} r_{c_1} + m_{c_2} r_{c_2} = 0}$$



$$\Sigma M = 0.$$

$$m_1 r_1 l_1 + m_2 r_2 l_2 + m_3 r_3 l_3 + m_{c_2} r_{c_2} l_{c_2} = 0.$$

$$\Sigma mr\cos\theta + m_{c_1} r_{c_1} \cos\theta_{c_1} + m_{c_2} r_{c_2} \cos\theta_{c_2} = 0 \longrightarrow (1)$$

$$\Sigma mr\sin\theta + m_{c_1} r_{c_1} \sin\theta_{c_1} + m_{c_2} r_{c_2} \sin\theta_{c_2} = 0 \longrightarrow (2)$$

$$\Sigma mrl + m_{c_2} r_{c_2} l_{c_2} = 0$$

$$\Sigma mrl\cos\theta + m_{c_2} r_{c_2} l_{c_2} \cos\theta_{c_2} = 0 \longrightarrow (3)$$

$$\Sigma mrl \cancel{\cos\theta}\, \sin\theta + m_{c_2} r_{c_2} l_{c_2} \sin\theta_{c_2} = 0 \longrightarrow (4)$$

By squaring & adding ③ & ④,

$$m_{c_2} r_{c_2} l_{c_2} = \sqrt{(-\Sigma mrl\cos\theta)^2 + (-\Sigma mrl\sin\theta)^2}$$

$\frac{④}{③}$

$$\frac{\cancel{m_{c_2}} \cancel{r_{c_2}} \cancel{l_{c_2}} \sin\theta_{c_2}}{\cancel{m_{c_2}} \cancel{r_{c_2}} \cancel{l_{c_2}} \cos\theta_{c_2}} = \frac{-\Sigma mrl\sin\theta}{-\Sigma mrl\cos\theta}.$$

$$\boxed{\tan\theta_{c_2} = \frac{-\Sigma mrl\sin\theta}{-\Sigma mrl.\cos\theta}.}$$

Squaring and adding ① & ②,

$$m_{c_1} r_{c_1} = \sqrt{(-(\Sigma mr\cos\theta + m_{c_2} r_{c_2}\cos\theta_{c_2}))^2 + (-(\Sigma mr\sin\theta + m_{c_2} r_{c_2}\sin\theta_{c_2}))^2}$$

$$\frac{②}{①} \Rightarrow \frac{\cancel{m_{c_1}} \cancel{r_{c_1}} \sin\theta_{c_1}}{\cancel{m_{c_1}} \cancel{r_{c_1}} \cos\theta_{c_1}} = \frac{-(\Sigma mr\sin\theta + m_{c_2} r_{c_2}\sin\theta_{c_2})}{-(\Sigma mr\cos\theta + m_{c_2} r_{c_2}\cos\theta_{c_2})}$$

$$\boxed{\tan\theta_{c_1} = \frac{-(\Sigma mr\sin\theta + m_{c_2} r_{c_2}\sin\theta_{c_2})}{-(\Sigma mr\cos\theta + m_{c_2} r_{c_2}\cos\theta_{c_2})}}$$

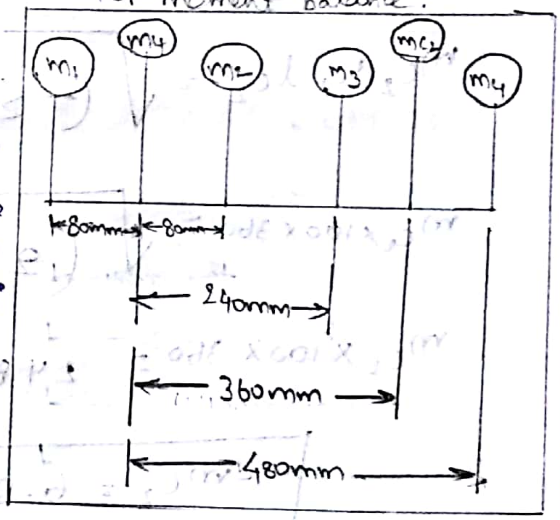
Note :- In a dynamic balance question with 2 counter masses atleast 1 counter mass should be taken as Reference Point.

Problem-1: A rotor has following Properties

Mass	Magnitude (Kg)	Radius (mm)	Angle (°)	Axial distance from 1st mass
1.	9 Kg	100	0°	
2.	7 Kg	120	60°	160 mm
3.	8 Kg	140	135°	320 mm
4.	6 Kg	120	270°	560 mm

If the shaft is balanced by two counter masses located at 100 mm radii and revolving in planes mid way of planes 1 & 2 and mid way of planes 3 & 4. Determine the magnitude of the masses and respective angular positions.

For moment balance.



Sol:- $m_1 = 9\,Kg$ $\quad r_1 = 100\,mm$ $\quad \theta = 0°$

$m_2 = 7\,Kg$ $\quad r_2 = 120\,mm$ $\quad \theta_2 = 60°$

$m_3 = 8\,kg$ $\quad r_3 = 140\,mm$ $\quad \theta_3 = 135°$

$m_4 = 6\,kg$ $\quad r_4 = 120\,mm$ $\quad \theta_4 = 270°$

$r_{c_1} = r_{c_2} = 100\,mm$

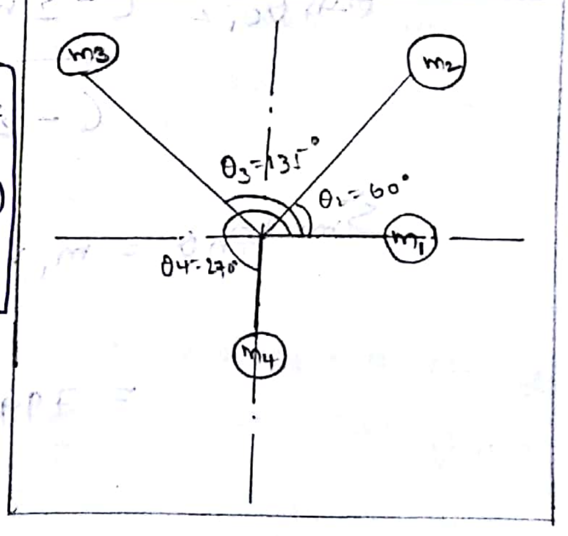
$\Sigma F = 0.$

$(\Sigma M)_{m_{c_1}} = 0$

$$\left[\begin{array}{l}(m_2 \times 80 \times 120) + (m_3 \times 240 \times 140) + \\ (m_{c_2} \times 360 \times 100) + (m_4 \times 480 \times 120) \\ -(m_1 \times 100 \times 80)\end{array}\right]$$

$$\tan\theta_{c_2} = \frac{-\Sigma mrl\sin\theta}{-\Sigma mrl\cos\theta}.$$

For Force balance.



$$\Sigma mrl \sin\theta = -(9\times100\times80\times\sin(0)) + (7\times80\times120\times\sin(60)) + (8\times240\times140\times\sin(135)) + (6\times480\times120\times\sin(270))$$

$$= -97332.79008$$

$$\Sigma mrl \cos\theta = -(9\times100\times80\times\cos(0)) + (7\times80\times120\times\cos(60)) + (8\times240\times140\times\cos(135)) + (6\times480\times120\times\cos(270))$$

$$= -228470.3$$

$$\tan\theta_{c_2} = \frac{-(-97332.79008)}{-(-228470.3)}$$

$$\boxed{\theta_{c_2} = 23^\circ}$$

$$m_{c_2} r_{c_2} l_{c_2} = \sqrt{(-\Sigma mrl\cos\theta)^2 + (-\Sigma mrl\sin\theta)^2}$$

$$m_{c_2}\times100\times360 = \sqrt{(228470.3)^2 + (97332.79008)^2}$$

$$m_{c_2}\times100\times360 = 248339.1834$$

$$\boxed{m_{c_2} = 6.89\ kg}$$

$$\tan\theta_{c_1} = \frac{(-\Sigma mr\sin\theta + m_{c_2} r_{c_2}\sin\theta_{c_2})}{(-\Sigma mr\cos\theta + m_{c_2} r_{c_2}\cos\theta_{c_2})}.$$

$$\Sigma mr\sin\theta = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4.$$

$$= 799.4$$

$$\Sigma mr\cos\theta = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4.$$

$$= 528.04$$

$$m_{c_2} r_{c_2} \sin\theta_{c_2} = 269.21$$

$$m_{c_2} r_{c_2} \cos\theta_{c_2} = 631.41$$

$$\tan\theta_{c_1} = \frac{-(799.4 + 269.21)}{-(528.04 + 634.1)}$$

$$\boxed{\theta_{c_1} = 42.06}$$

$$m_4 r_4 = \sqrt{\left(-(\Sigma mr\cos\theta + m_{c_2} r_{c_2}\cos\theta)\right)^2 + \left(-(\Sigma mr\sin\theta + m_{c_2} r_{c_2}\sin\theta)\right)^2}$$

$$m_4 \times 100 = \sqrt{(528.04 + 634.1)^2 + (799.4 + 269.21)^2}$$

$$\boxed{m_4 = 15.78\ kg}$$

---

1) Equilibrium Law:

A body can be in equilibrium with two forces only when the two forces have same magnitude, opposite in direction, co-linear (same line) in action.

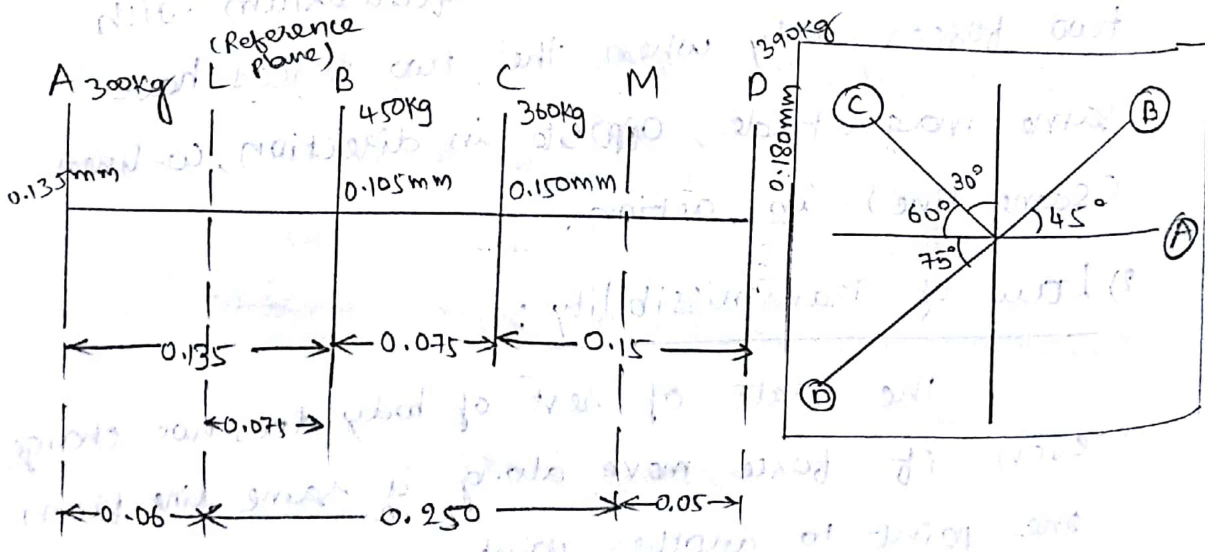
2) Law of Transmissibility:

The state of rest of body does not change even if force move along a same line from one point to another point.

Law of Superposition :

The state of rest of body does not change if we add to (or) subtract from a system of equilibrium forces from any system in equilibrium law.

Problem : Four masses m_1, m_2, m_3, m_4 are 300 kg, 450 kg, 360 kg, 390 kg are attached to a shaft, these masses are revolving at radii 135 mm, 105 mm 150 mm, 180mm respectively. In planes measured from A and 135 mm, 210mm, 360mm, respectively. The angles measured anti clockwise are m_1 to m_2 45°, m_2 to m_3 75°, m_3 to m_4 135° and the distance between the planes L and M in which the balancing masses are placed is 250mm. The distance between plane A and L is 60mm and plane M and D is 50mm. If the Balancing masses revolve at a radius of 36 mm. Find their magnitudes and angular positions.

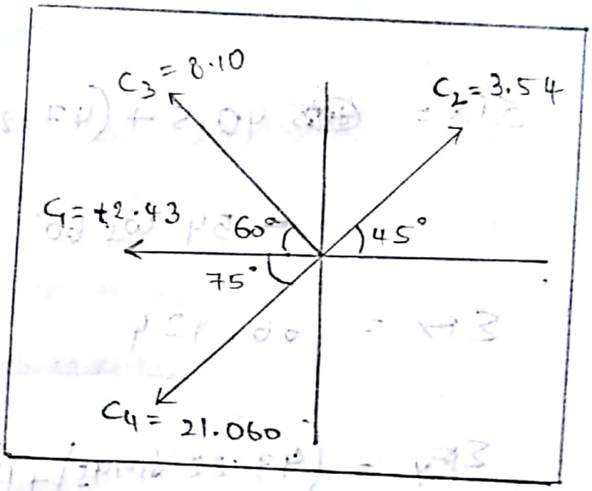


plane	m	r	mr	l	mrl
A	300	0.135	40.5	−0.06	−2.43
L (RP)	mL	0.036	0.036 mL	0	0
B	450	0.105	47.25	0.075	3.54
C	360	0.150	54	0.15	8.10
M	Mm	0.036	0.036 Mm	0.25	0.009
D	390	0.180	70.2	0.3	21.060

Couple vectors:

$$\Sigma C_x = (3.54 \cos 45) - (8.10 \cos 60) - 2.43 - (21.06 \cos 75°)$$

$$\Sigma C_x = -9.42$$



$$\Sigma C_y = (3.54 \sin 45) + (8.1 \sin 60°) - 21.06 \sin 75°$$

$$= -10.82$$

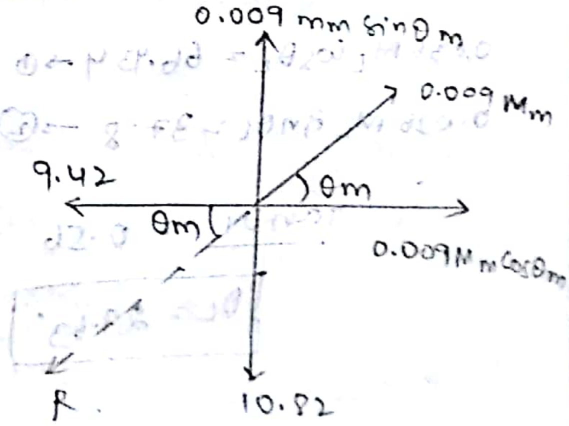
The ΣC_x and ΣC_y values are negative. So these belongs to III rd Quadrant.

$$0.009\, M_m \sin \theta_m = 10.82$$

$$0.009\, M_m \cos \theta_m = 9.42$$

$$\tan \theta_m = 1.14$$

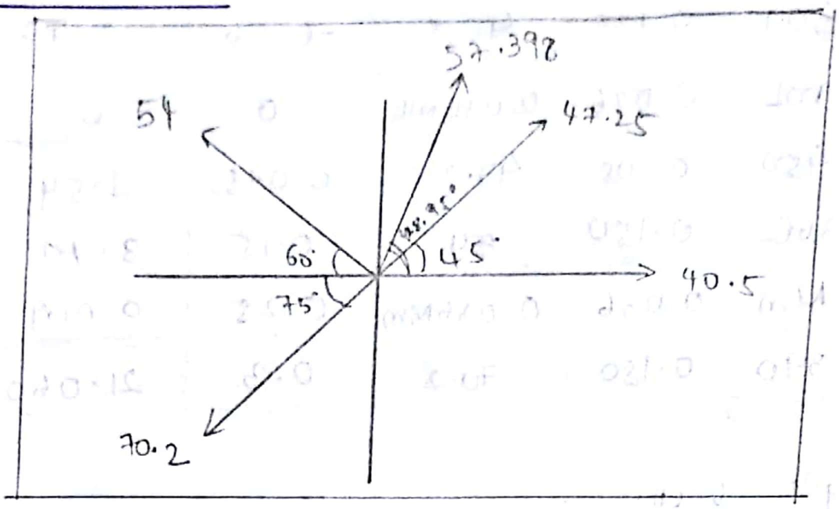
$$\boxed{\theta_m = 48.95}$$



$$① \Rightarrow 0.009\, M_m \sin(48.94) = 10.82$$

$$\boxed{M_m = 1594.41 \text{ kg}}$$

Force Vectors:



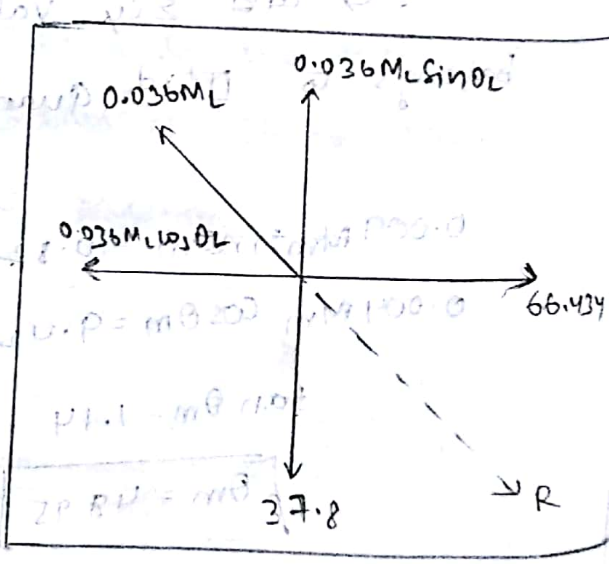
$\Sigma F_x = 40.5 + (47.25 \cos(45°)) + (57.398 \cos(48.95))$

$- 54 \cos 60° - 70.2 \cos 75°$

$\Sigma F_x = 66.434$

$\Sigma F_y = (47.25 \sin 45) + (57.398 \sin(48.95)) - (54 \sin 60°) - (70.2 \sin 75°)$

$\Sigma F_y = -37.8$



$0.036 M_L \cos\theta_L = 66.434 \rightarrow ①$

$0.036 M_L \sin\theta_L = 37.8 \rightarrow ②$

$\tan\theta_L = 0.56$

$\boxed{\theta_L = 29.63°}$

① $\Rightarrow 0.036 M_L \cos(29.63) = 66.434$

$\boxed{M_L = 2123.0005 \text{ Kg}}$

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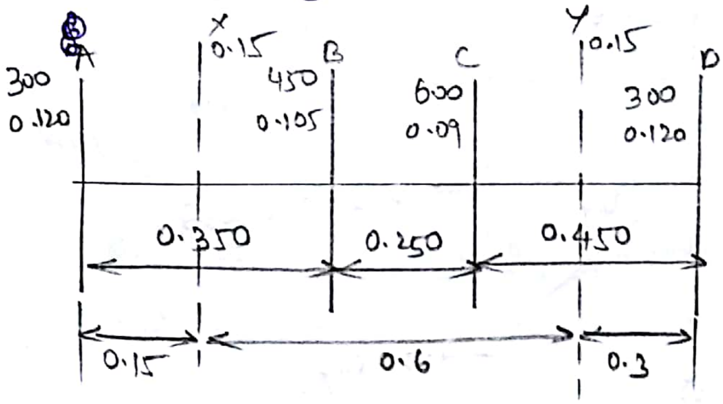
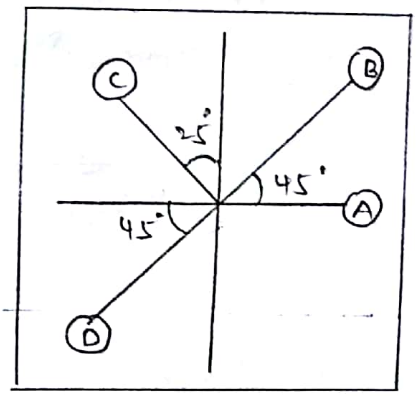
problem : A shaft carries 4 masses A, B, C and D of magnitudes 300 kg, 450 kg, 600 kg and 300 kg. And revolving at a radii 120, 105, 90 and 120 mm's respectively in planes measured from A at 350 mm, 600 mm and 1050 mm. The Angles b/w the Cranks measured Anticlockwise are A to B - 45°, B to C - 70°, C to D - 120°. The Balancing masses are to be placed in planes X and Y. The distance b/w Planes A and X is 150 mm, b/w X and Y is 600 mm and b/w Y & D is 300 mm. If the Balancing mass revolve at a radius 150 mm. Find their magnitudes and direction.





Plane	Mass (m)	Radii (r)	mr	l	mrl.
A	300	0.12	36	-0.15	-5.4
(RP) X	Mx	0.15	0.15Mx	0	0
B	450	0.105	47.25	0.2	9.45
C	600	0.09	54	0.45	24.3
Y	My	0.15	0.15My	0.6	0.09 My
D	300	0.12	36	0.9	32.4

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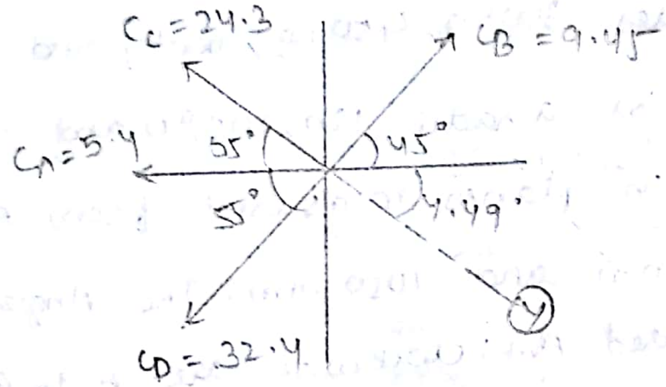
Couple vectors:

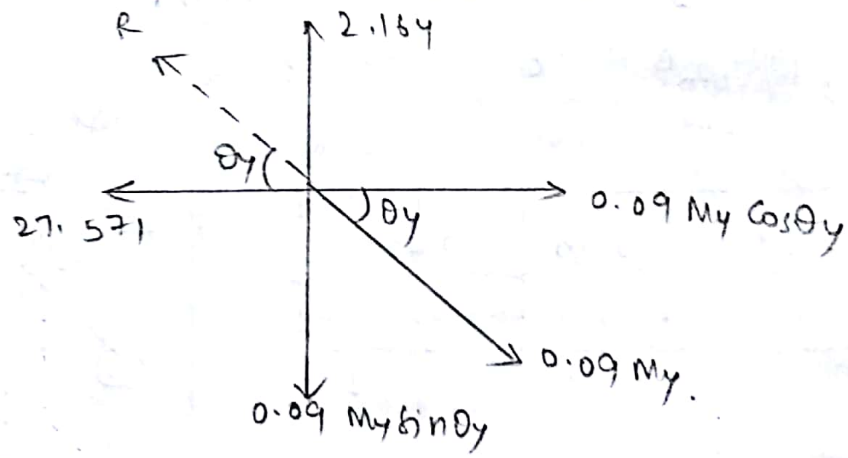


$\Sigma C_x = 9.45 \cos 45° - 24.3 \cos 65° - 5.4 - 32.4 \cos 55°$

$\Sigma C_x = -27.571$

$\Sigma C_y = 9.45 \sin 45° + 24.3 \sin 65 - 32.4 \sin 55°$

$= 2.164$



For equilibrium

$0.09 M_y \sin\theta_y = 2.16$

$0.09 M_y \cos\theta_y = 27.57$

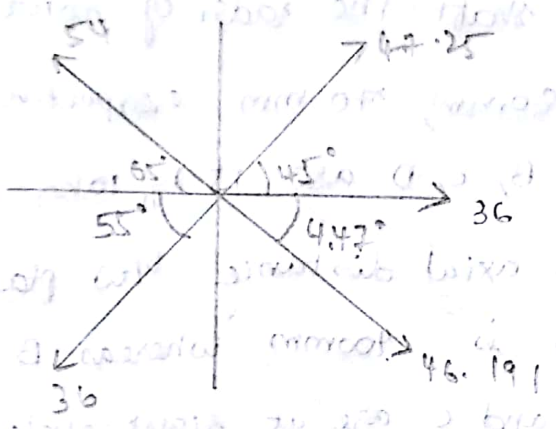
$\tan\theta_y = 0.078$

$\boxed{\theta_y = 4.47°}$

$0.09\, M_y \cdot \sin(4.47) = 2.16$

$\boxed{M_y = 307.94 \text{ kg}}$

Force vectors:



$\Sigma F_x = 47.25 \cos 45° + 36 - 54 \cos 65° - 36 \cos 55° + 46.191 \cos(4.47°)$

$\Sigma F_x = 33.41 + 36 - 22.82 - 20.64 + 46.050$

$\Sigma F_x = 71.999$

$\Rightarrow \Sigma F_y = 47.25 \sin 45° + 54 \sin 65° - 36 \sin 55° - 46.191 \sin(4.47)$

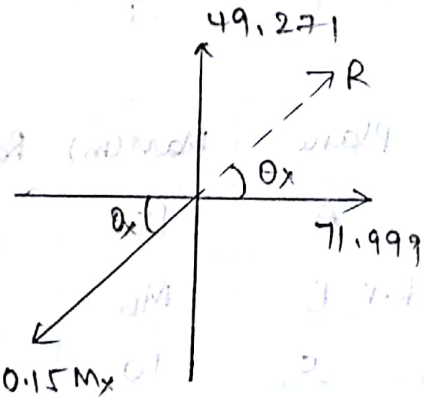
$\Sigma F_y = 49.271$

$0.15 M_x \cos\theta_x = 71.999$

$0.15 M_x \sin\theta_x = 49.271$

$\tan\theta_x = 0.68$

$\boxed{\theta_x = 34.38°}$



$0.15 M_x \cos\theta_x = 71.999$

$0.15 M_x \cos(34.38) = 71.999$

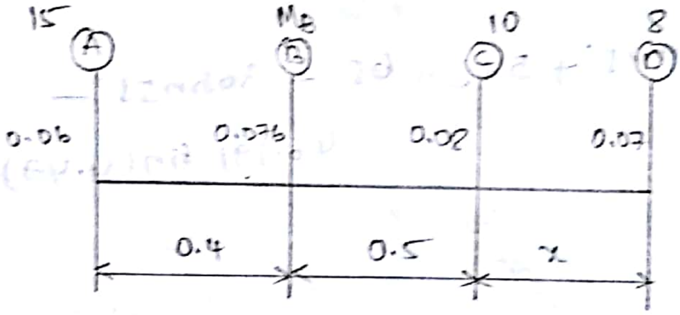
$\boxed{M_x = 581.59 \text{ Kg}}$

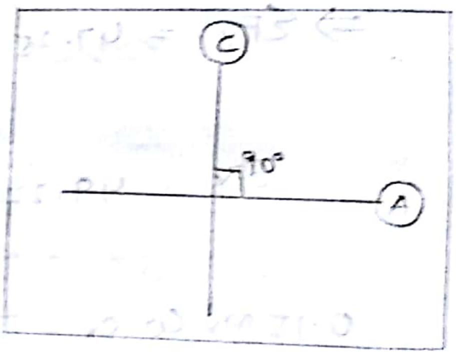
Problem-3: Four masses A, B, C, D are rigidly attached to a shaft. The radii of rotations are 60mm, 76mm, 80mm, 70 mm respectively. The magnitude of masses A, B, C, D are 15 kg, 10kg and 8 kg respectively. The axial distance b/w planes of rotations of A and B is 400mm whereas B and C is 500mm. The masses A and C are at right angles to eachother. If masses are incomplete balance. Then find,

(i) The Angles of masses B and D from A.

(ii) The Axial distance b/w planes C and D.

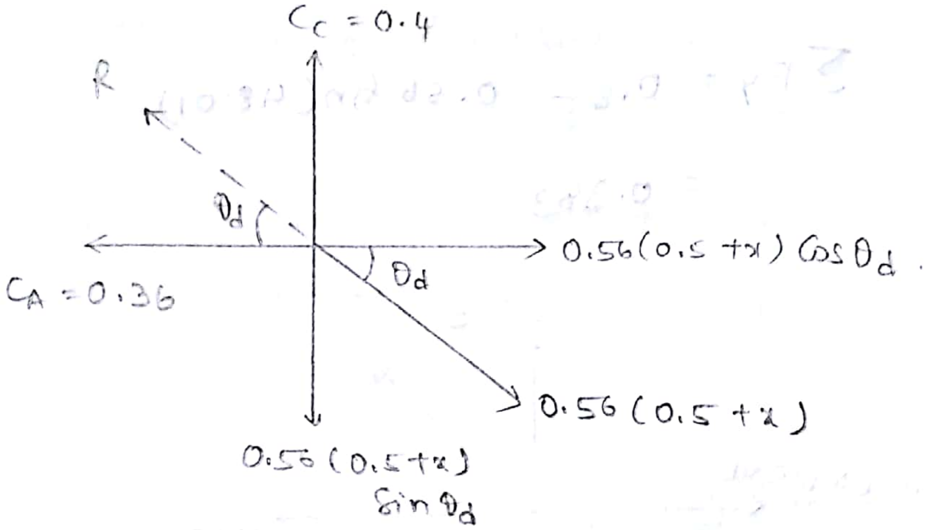
(iii) Magnitude of mass.





Plane	Mass(m)	Radii(r)	mr	l	mrl
A	15	0.06	0.9	-0.4	-0.36
R.P B	M_b	0.076	$0.076M_b$	0	0
C	10	0.08	0.8	0.5	0.4
D	8	0.07	0.56	(0.5+x)	0.56(0.5+x)

Sol:- Couple vectors:



For equilibrium,

$$0.56(0.5+x)\cos\theta_d = 0.36 \rightarrow ①$$

$$0.56(0.5+x)\sin\theta_d = 0.4$$

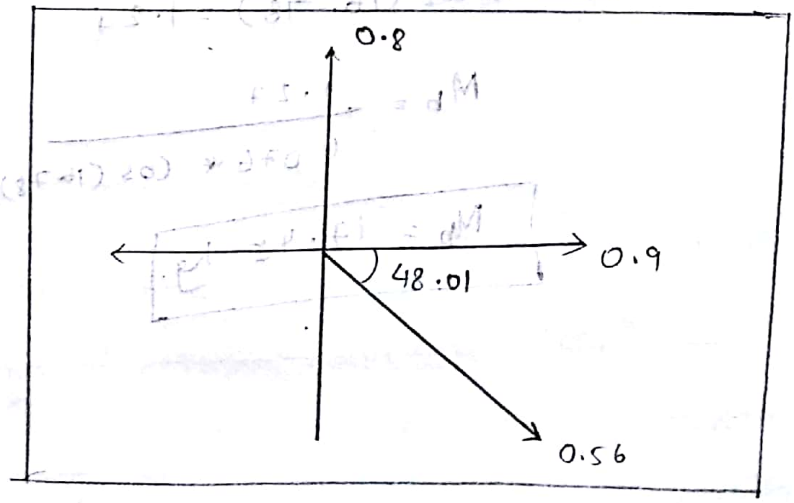
$$\tan\theta_d = 1.11$$

$$\theta_d = 48.01°$$

$$① \Rightarrow 0.56(0.5+x)\cos(48.01) = 0.36$$

$$x = 0.46 \text{ m}.$$

Force Vectors :



$$\Sigma F_x = 0.9 + 0.56\cos(48.01)$$

$$= 1.27$$

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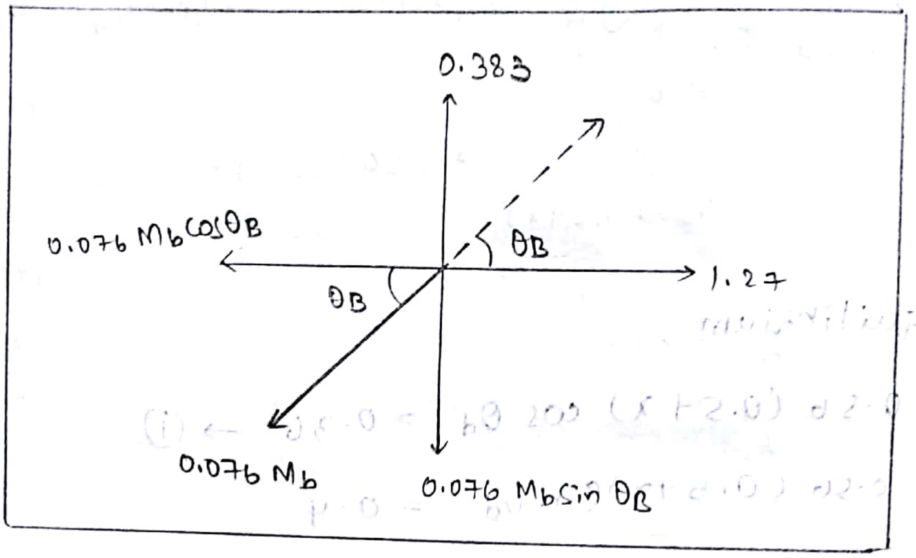
$$\Sigma F_y = 0.8 - 0.56 \sin(48.01)$$

$$= 0.383$$



$$0.076\ M_b \cos\theta_B = 1.27 \longrightarrow ①$$

$$0.076\ M_b \sin\theta_B = 0.383 \longrightarrow ②$$

$$\tan\theta_B = \frac{0.383}{1.27}$$

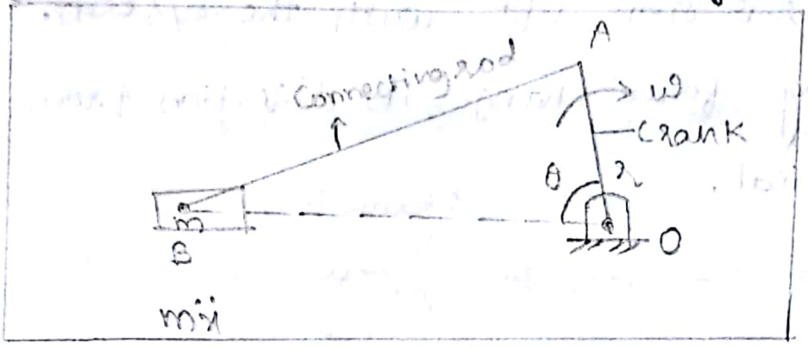
$$\boxed{\theta_B = 16.78°}$$

$$① \Rightarrow 0.076\ M_b \cos(16.78) = 1.27$$

$$M_b = \frac{1.27}{0.076 * \cos(16.78)}$$

$$\boxed{M_b = 17.45\ kg}$$

Partial Balancing of Reciprocating masses:



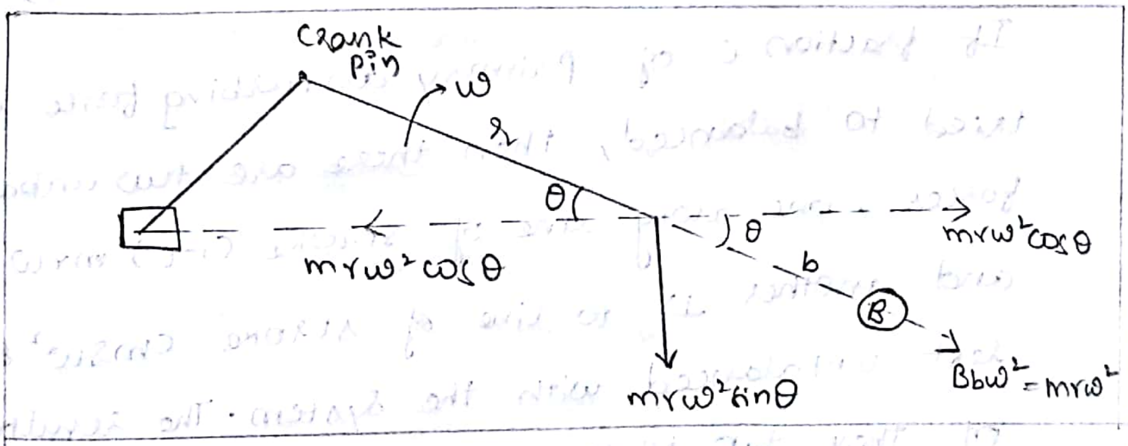
In reciprocating engines, the disturbing force along line of action of stroke is,

$$m\left[r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)\right]$$

The first term $(mr\omega^2\cos\theta)$ is primary disturbing force.

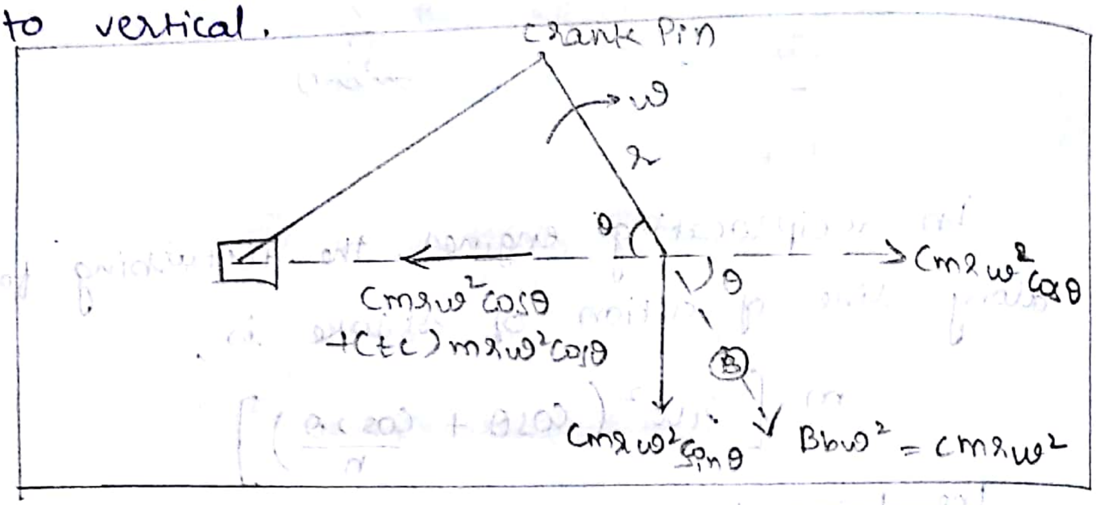
The second term $\left(\frac{mr\omega^2\cos 2\theta}{n}\right)$ is secondary disturbing force.

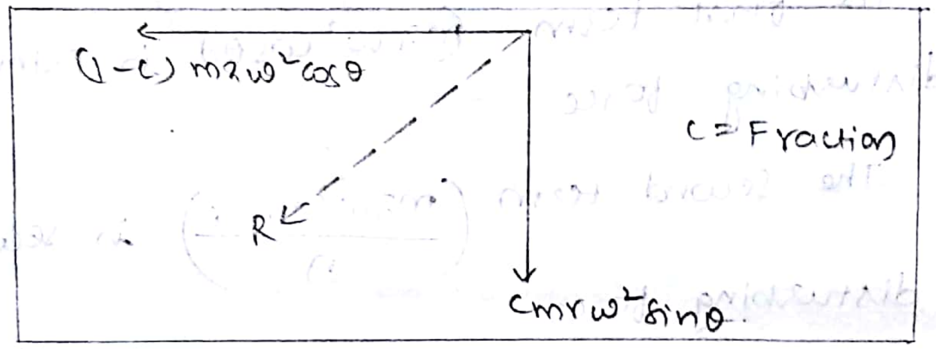
In many applications, the primary disturbing force is need to be balanced.



If we try to (~~try~~) balance horizontal primary disturbing force by attaching balancing mass B opposite to crank pin at a radius of b, then the Horizontal disturbing primary force is neutralized with $Bb\omega^2\cos\theta$ i.e, $mr\omega^2\cos\theta$ along line of stroke.

But the other vertical component $Bb\omega^2 \sin\theta$.
i.e., $mr\omega^2 \sin\theta$ left with the system. The disturbing force change its direction from Horizontal to vertical.





Residual unbalanced Force.

$$RUF = \sqrt{((1-c)\, mr\omega^2\cos\theta)^2 + (cmr\omega^2\sin\theta)^2}$$

If fraction c of primary disturbing force is tried to balanced, then there are two unbalanced forces, one along line of stroke $(1-c)\, mr\omega^2\cos\theta$ and another $\perp^r$ to line of stroke $cmr\omega^2\sin\theta$ left unbalanced with the system. The resultant of these two forces components:

$$R = \sqrt{((1-c)mr\omega^2\cos\theta)^2 + (cmr\omega^2\sin\theta)^2}$$

This unbalanced force is known as Residual unbalanced force.

problem: A single cylinder reciprocating engine has the following data,

Speed of the engine = 240 rpm

Stroke = 320 mm

Mass of reciprocating parts = 67.5 kg

If 60% of reciprocating parts are to be balanced

Find: (i) The balancing mass required at a radius of 300 mm.

(ii) Residual unbalanced force when the crank has turned $60°$.

Sol:- Given,

Speed = 240 rpm

Stroke = 320 mm

Mass, m = 67.5 kg

$2r = 320 \Rightarrow r = 160\,mm$

$\omega = \frac{2\pi N}{60}$

$\omega = 25.13\ rad/s$

(i) $c m r \omega^2 = B b \omega^2$

$(0.6)\, m r = B(0.3)$

$(0.6)(0.16)(67.5) = B(0.3)$

$\boxed{B = 21.6\ kg}$

(ii) $RUF = \sqrt{[(1-c) m r \omega^2 \cos\theta]^2 + (B b \omega^2 \sin\theta)^2}$

$= \sqrt{(1364.076)^2 + (3543.97)^2}$

$= 3797.42\ N.$

The effects of partial balancing of reciprocating masses of locomotives :

In locomotives, there are two cylinders and the cranks are 90° apart.

Because of unbalanced force components there are 3 effects in locomotives.

(i) Variation of Tractive effort (VTE)
(ii) Swaying couple (SC)
(iii) Hammer Blow (HB).

Variation of Tractive Effort :

$$\longleftarrow$$
$$(1-c)\, m r \omega^2 \cos\theta$$

$$\longleftarrow$$
$$(1-c)\, m r \omega^2 \cos(\theta+90)$$

$$TE = (1-c)\, m r \omega^2 \cos\theta + (1-c)\, m r \omega^2 \cos(\theta + 90)$$

$$TE = (1-c)\, m r \omega^2 [\cos\theta - \sin\theta]$$

The condition for maximum value is,

$$\frac{d}{d\theta}[\cos\theta - \sin\theta] = 0$$

$$-\cos\theta - \sin\theta = 0$$

$$\tan\theta = -1$$

$$\theta = 135° \text{ (or) } 315°$$

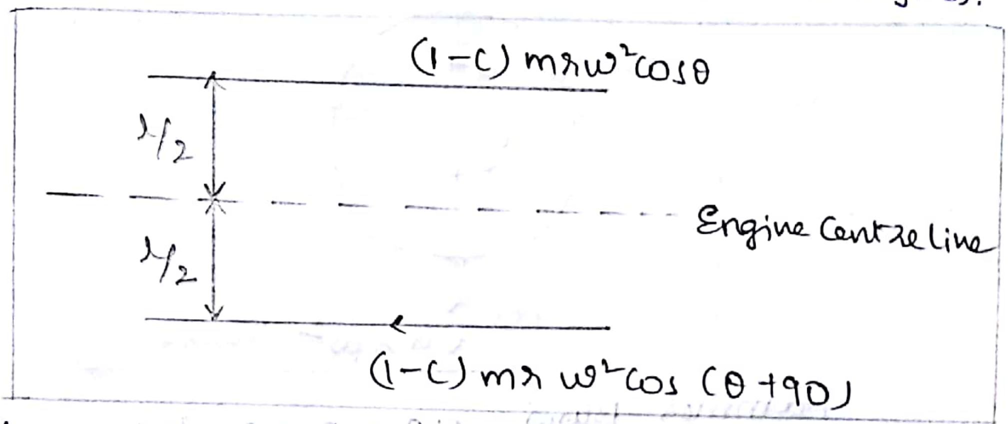
$$VTE = (1-c)\, m r \omega^2 [\cos 135 - \sin 135]$$

$$\boxed{VTE = \pm\sqrt{2}\,(1-c)\, m r \omega^2}$$

The maximum value of unbalanced primary disturbing force along the line of stroke of a locomotive is known as variation of Tractive effort.

Swaying couple:

Let 'l' be the distance between two engines.



Pitch = Distance b/w cylinders

$$\text{Couple, } C = -(1-c)\, m r \omega^2 \cos(\theta+90)\, l/2 + \left[(1-c)\, m r \omega^2 \cos\theta\, l/2\right]$$

$$C = (1-c)\, m r \omega^2\, l/2\, [\cos\theta + \sin\theta]$$

Condition for maximum value is,

$$\frac{d}{d\theta}[\cos\theta + \sin\theta] = 0$$

$$-\sin\theta + \cos\theta = 0$$

$$\tan\theta = 1$$

$$\theta = 45^\circ \text{ (or) } 225^\circ$$

$$SC = \pm\sqrt{2}\, l/2 : (1-c)\, m r \omega^2$$

$$\boxed{S.C = \pm\, l/\sqrt{2}\, (1-c)\, m r \omega^2}$$

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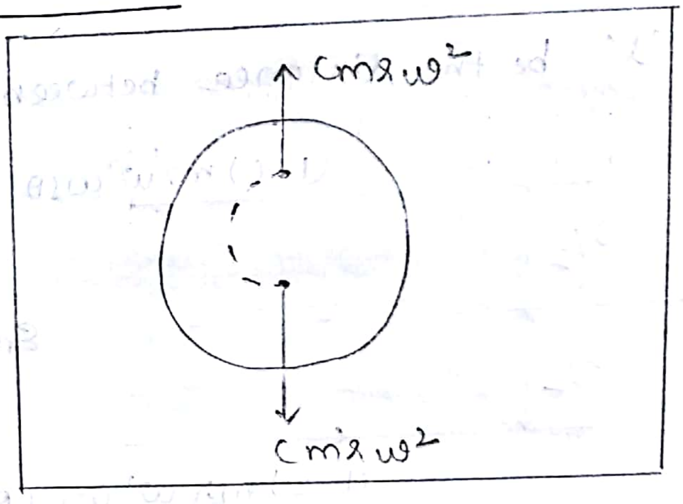
The maximum value of couple (or) moment of unbalanced primary disturbing force about engine centre line of a locomotive is known as Swaying couple.

Hammer Blow:



Hammer Blow, $HB = cmr\omega^2 \sin\theta$.

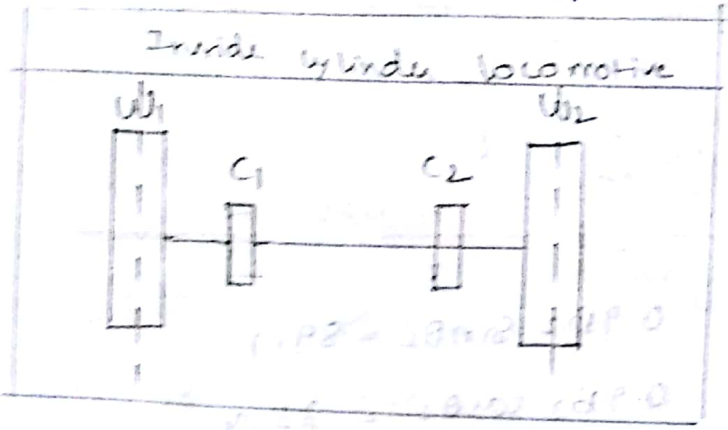
Max. value $= cmr\omega^2$

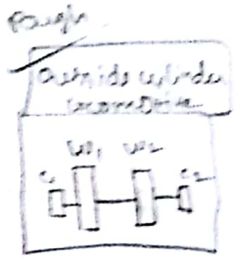
$$\boxed{HB = Bb\omega^2}$$

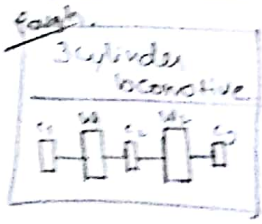
The maximum value of vertical unbalanced force perpendicular to the line of stroke is known as Hammer Blow.

Problem-1: A cranks of 2 cylinder uncoupled inside cylinder locomotive ~~are~~ are at right angles and are 300mm long. The centre lines of the cylinders are 700mm apart. The rotating mass per cylinder is 150 kg at the crank pin and the mass of the reciprocating parts is 180kg. The wheel centre lines are 1.5 m apart, the whole of the rotating and two-thirds (2/3rd) of reciprocating masses are to be balanced in the planes of driving wheels at a radius of 600mm. Find the magnitude and direction of Balancing masses, (ii) Hammer Blow, (iii) variation of tractive effect (VTE) (iv) Swaying couple (SC) when the crank speed is 150 rpm.

Sol:-

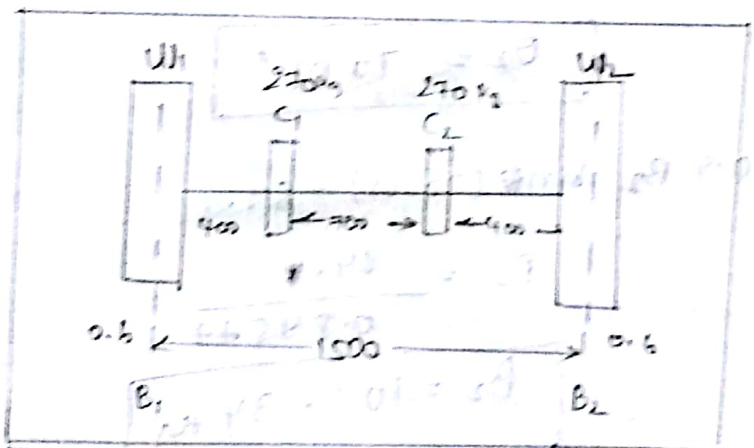


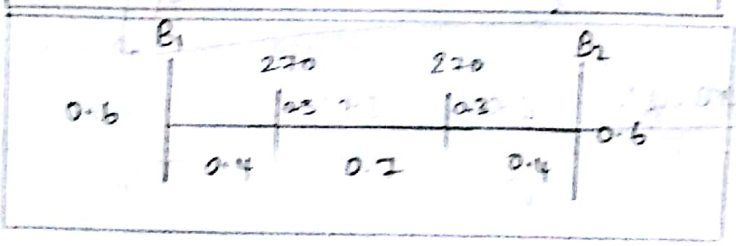




$$M = 150 + \frac{2}{3}(180)$$

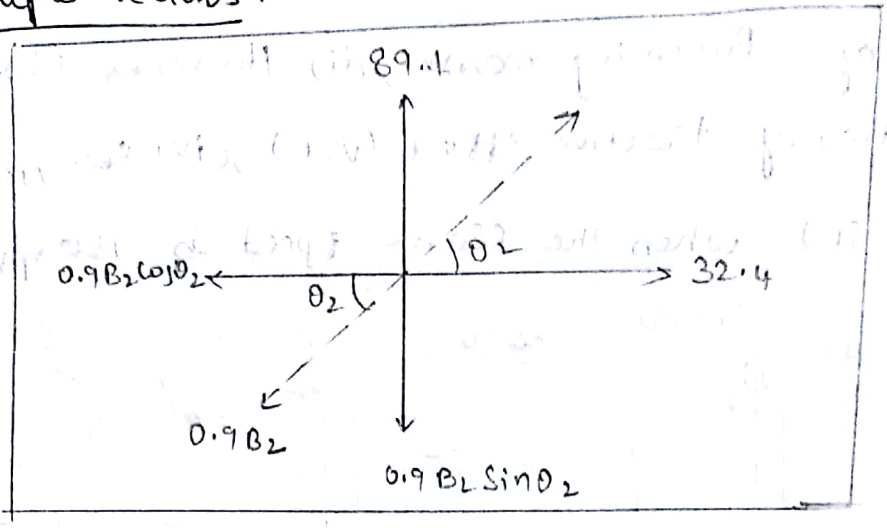
$$M = 270 \text{ kg.}$$





	Plane	m	r	mr	l	mrl
R.P	1	B_1	0.6	$0.6 B_1$	0	0
	2	270	0.3	81	0.4	32.4
	3	270	0.3	81	1.1	89.1
	4	B_2	0.8	$0.8 B_2$	1.5	$0.9 B_2$

Couple vectors :



$$0.9B_2 \sin\theta_2 = 89.1$$

$$0.9 B_2 \cos\theta_2 = 32.4$$

$$\tan\theta_2 = 89.1 / 32.4$$

$$\theta_2 = \tan^{-1}(2.75)$$

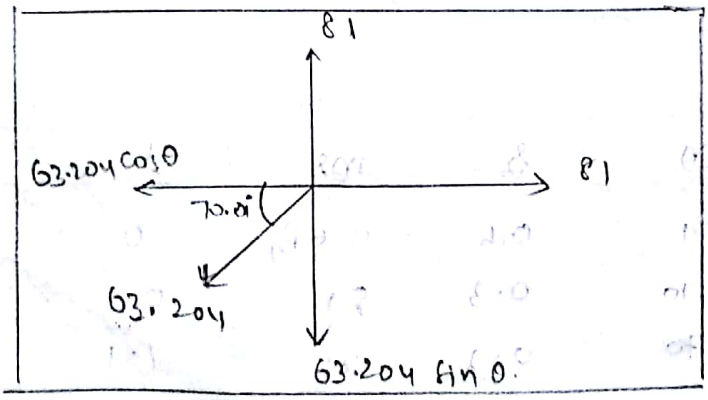
$$\boxed{\theta_2 = 70.01^\circ}$$

$$0.9\ B_2 \sin(70.01) = 89.1$$

$$B_2 = \frac{89.1}{0.84577}$$

$$\boxed{B_2 = 105.34\ kg}$$

Force vectors :



$$\Sigma F_x = 81 - 63.204 \cos(70.01^\circ) = 59.39^\circ$$

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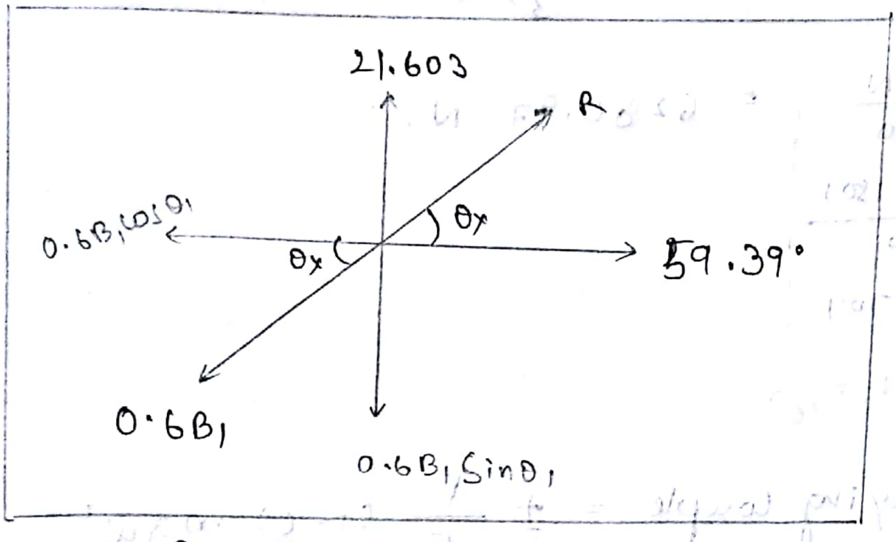
$\Sigma F_y = 81 - 63.204 \sin(70.01^\circ) = 21.603$



$0.6B_1\cos\theta_1 = 59.39$

$0.6B_1\sin\theta_1 = 21.603$

$\tan\theta_1 = \dfrac{21.603}{59.39}$

$\theta_1 = \tan^{-1}\left(\dfrac{21.603}{59.39}\right)$

$\boxed{\theta_1 = 19.98^\circ}$

$0.6B_1 \cos(19.98) = 59.39$

$\boxed{B_1 = 105.34\ \text{kg}}$

For balancing total	Balancing mass
270	105.34
180	?

Reciprocating Balancing mass (B) $= \dfrac{180 \times 105.34}{270}$

$\boxed{B = 46.82\ \text{kg.}}$

(iii) Variation of Tractive effort

$VTE = \pm\sqrt{2}(1-c)\, m r \omega^2$.

$$= \pm\sqrt{2}\left(1-\frac{2}{3}\right)(180)(0.3)(15.707)^2$$

$$\boxed{\begin{aligned} \omega &= \frac{2\pi N}{60} \\ &= \frac{2\pi(150)}{60} \\ \omega &= 15.707 \end{aligned}}$$

$= 6280.97$ N.

$kg\, m/s^2$

(iii) Swaying couple $= \pm\frac{l}{\sqrt{2}}(1-c)\, m r \omega^2$

$$= \frac{0.7}{\sqrt{2}}\left(1-\frac{2}{3}\right)(180)(0.3)(15.707)^2$$

m $\boxed{kg\, m/s^2}$

$= 2198.07$ N-m

(ii) Hammer Blow. $= B\, b\omega^2$

$$= (46.82)(0.6)(15.707)^2$$

$= \cancel{10394.27\ N}$ 6929.09 N

$kg\, m/s^2$