Theoretical Assignment DeepBayes Summer School 2019 (deepbayes.ru)

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Problem 1

The random variable ξ has Poisson distribution with the parameter λ. If $\xi = k$ we perform k Bernoulli trials with the probability of success p. Let us define the random variable η as the number of successful outcomes of Bernoulli trials. Prove that η has Poisson distribution with the parameter $p\lambda$.

Solution

Use definition

$\eta = \sum_{i=1}^{\xi} \{b_i = 1\}$ where $b_i \sim Ber(p)$ and $\xi \sim Poi(\lambda)$.

Then

$$
\begin{aligned}
p(\eta = k) &= \sum_{n=0}^{\infty} p(\eta = k, \xi = n) \\
&= \sum_{n=0}^{\infty} p(\eta = k | \xi = n) p(\xi = n) \\
&= \sum_{n=k}^{\infty} p(\eta = k | \xi = n) p(\xi = n)
\end{aligned}
$$

As everything is (assumed) independent, conditioned on $\xi = n$, $\eta \sim B(n,p) \implies$

$$p(\eta = k | \xi = n) = \binom{n}{k} p^k (1-p)^{n-k}$$

ξ is Poisson-distributed,

$p(\xi = n) = \frac{\lambda^n e^{-\lambda}}{n!} \implies$

$$
\begin{aligned}
p(\eta = k) &= \sum_{n=k}^{\infty} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \frac{\lambda^n e^{-\lambda}}{n!} \\
&= \sum_{n=k}^{\infty} \frac{(\lambda p)^k}{k!} \frac{[\lambda(1-p)]^{n-k}}{(n-k)!} e^{-\lambda} \frac{e^{-\lambda p}}{e^{-\lambda p}} \\
&= \frac{(\lambda p)^k e^{-\lambda p}}{k!} \sum_{n=k}^{\infty} \frac{[\lambda(1-p)]^{n-k}}{(n-k)!} e^{-\lambda(1-p)} \\
&= \frac{(\lambda p)^k e^{-\lambda p}}{k!} \sum_{n=0}^{\infty} \frac{(\hat{\lambda})^n e^{-\hat{\lambda}}}{n!} \\
&= \frac{(\lambda p)^k e^{-\lambda p}}{k!}
\end{aligned}
$$

It follows that η is Poisson-distributed with parameter $p\lambda$.

Problem 2

A strict reviewer needs t1 minutes to check assigned application to DeepBayes summer school, where t_1 has normal distribution with parameters $\mu_1 = 30$, $\sigma_1 = 10$. While a kind reviewer needs t_2 minutes to check an application, where t_2 has normal distribution with parameters $\mu_2 = 20$, $\sigma_2 = 5$. For each application the reviewer is randomly selected with 0.5 probability. Given that the time of review t = 10, calculate the conditional probability that the application was checked by a kind reviewer.

Solution

$$
\begin{aligned}
p(r = kind | t = 10) &= \frac{\frac{1}{\sigma_{kind}}}{\frac{1}{\sigma_{kind}} + \frac{1}{\sigma_{strict}}} \\
&= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{10}}
\end{aligned}
$$

It is given that p(r=strict) = p(r=kind) = 0.5, $p(t|r = strict) \sim N(30,10)$ and $p(t|r = kind) \sim N(20,5)$.

$$
\begin{aligned}
p(r = kind|t = 10) &= \frac{p(t = 10, r = kind)}{p(t = 10)} \\
&= \frac{p(t = 10|r = kind)p(r = kind)}{p(t = 10|r = kind)p(r = kind) + p(t = 10|r = strict)p(r = strict)} \\
&= \frac{p(t = 10|r = kind)}{p(t = 10|r = kind) + p(t = 10|r = strict)}
\end{aligned}
$$

When calculating normal pdf densities, in both cases $\frac{(t-\mu)^2}{2\sigma^2} = 2 \implies$

It follows that the probability the application was reviewed by kind reviewer is $\frac{2}{3}$.