BALLISTIC RESEARCH LABORATORIES
REPORT NO. 405

THE INITIAL VELOCITIES OF FRAGMENTS FROM BOMBS, SHELL, GRENADES

R. W. Gurney

ABERDEEN PROVING GROUND, MARYLAND

September 1943

BALLISTIC RESEARCH LABORATORIES

Abstract

To assess the efficiency of a projectile, it is often required to predict the initial velocities of the fragments from a knowledge of the dimensions of the metal casing and the character and quantity of explosive. Between a grenade containing $1\frac{1}{2}$ ounces of high explosive and a bomb containing 3000 pounds of high explosive the difference in scale is so great, that it is a question whether my simple scheme will apply over the whole range. A theory is put forward, making the following assumption, that the contribution to the total kinetic energy made by the detonation of unit mass of explosive is independent of the size of the projectile. In a large bomb the explosion gases have actually more kinetic energy than the fragments. A simple expression is found to agree with the experimental data fairly well over the whole range from $C/M = 0.06$ to $C/M = 5.6$.

1. Introduction

To compare the efficiency of a fragmentation of different types of projectiles, one needs to know the velocities of the fragments at suitable distances from the explosion. For this purpose one needs to know both the retardation in air and the initial velocity v_0 of the fragments. We are concerned here with the question, whether the value of v_0 for any projectile can be predicted from the dimensions of the metal casing and from the character and quantity of the explosive contained in it.

We have data on projectiles containing from one pound to 3000 pounds of high explosive. Preliminary tests have also been made on grenades containing only $1\frac{1}{2}$ or 2 ounces of TNT. The range from $1\frac{1}{2}$ ounces to 3000 pounds being so great, it is a question whether my simple scheme will be found to apply for the whole set of results.

It is well known that, though a cylindrical shell is detonated from one end, both the nose-spray and the tail-spray are feeble compared with the side-spray. It is in the radial velocities of fragments from the cylindrical walls that we are interested.

2. Preliminary Calculations

Let C be the mass of explosive and M the mass of the metal casing containing it (when treating a cylindrical projectile we consider mass per unit length). We discuss a metal casing with walls of uniform thickness, and assume first the following simple picture of fragmentation:

Before the metal casing breaks into fragments, it expands to some extent. Let the radius at the moment of fragmentation be a, and let the density of the explosion gases at this moment be ρ. The metal casing is everywhere moving outwards with radial velocity v_0, which at this moment becomes the velocity of the metallic fragments (the same for all). Now v_0 is also the radial velocity of that part of the explosion gases which are in contact with the metal casing. On the axis of the cylinder the radial velocity of the gases is zero. Elsewhere the gases are moving outwards with velocity intermediate between zero and v_0. We shall take the velocity at any point to be proportional to the distance from the axis of the cylinder (or from the center of a spherical grenade) - that is

$$v = \frac{r}{a} \cdot v_0 \ .$$

Consider now different types of projectiles, shell, bombs and grenades all containing the same explosive, say TNT.

We try the assumption that the contribution to the kinetic energy made by the detonation of unit mass of this explosive is the same in all types of projectile. Let this contribution per unit mass of explosive be E. Equating the total kinetic energy to EC we have for a cylinder per unit length[1]

$$EC = \frac{1}{2} \cdot \sum_i m_i \cdot v_0^2 + \frac{1}{2} v_0^2 \int_0^a 2\pi r \rho \frac{r^2}{a^2} dr \tag{2.1}$$

and for a spherical casing

$$EC = \frac{1}{2} \cdot \sum_i m_i \cdot v_0^2 + \frac{1}{2} v_0^2 \int_0^a 4\pi r^2 \rho \frac{r^2}{a^2} dr \ . \tag{2.2}$$

[1] The length of the circumference U of a circle is related to the radius r and diameter d by:

$$U = 2\pi r = \pi d \ .$$

Note that $\rho = \frac{m}{V}$.

Both these expressions reduce to the simple form (see Appendix)

$$v_0 = \sqrt{2ER} \tag{2.3}$$

where R is the function:

for a cylinder

$$R = \frac{C}{M + \frac{C}{2}} = \frac{\frac{C}{M}}{1 + \frac{1}{2}\frac{C}{M}} \tag{2.4}$$

and for a sphere

$$R = \frac{C}{M + \frac{3C}{5}} = \frac{\frac{C}{M}}{1 + \frac{3}{5}\frac{C}{M}} \quad . \tag{2.5}$$

For small values of C/M we see that R is approximately equal to C/M. Hence for small values of the charge-weight ratio[2], the value of v_0 varies as $\sqrt{C/M}$. On the other hand, for very large values of C/M, such as are found in large bombs, we see that R tends asymptotically to the value 2 for a cylinder, and to the value $\frac{5}{3}$ for a sphere.
The quantity E has the dimension of Energy per unit mass. Therefore $\sqrt{E}$ has the dimensions of a velocity. In fact, v_0 is equal to $\sqrt{E}$ when $R = \frac{1}{2}$. We conclude then, that for large values of C/M the value of v_0 tends asymptotically to the value $2\sqrt{E}$, or to the value $\sqrt{10E/3}$ for a sphere.
For simplicity v_0 was assumed to have the same value for all the fragments. Even for cylinders with walls of uniform thickness there is always some spread in the initial velocities, at any rate when the cylinder is short. We may therefore take v_0 to be a mean of the initial velocities of all the fragments which contribute to the total kinetic energy, i.e. the smallest fragments may be excluded since they make a negligible contribution to the total kinetic energy.

2 The conventional charge-weight ratio C/W takes into account the whole mass of metal in the projectile, while our C/M takes into account only the metal in the walls of the casing. If Θ is the ratio of the external to the internal diameter, and ρ_e and ρ_m are the densities of the explosive and metal, respectively, we have for cylindrical walls

$$\frac{C}{M} = \frac{\rho_e}{(\Theta - 1)\,\rho_m} \quad .$$

3. Previous Calculations of Schwarzschild and Sachs

Measuring the initial velocities of the leading fragments from large bombs, Schwarzschild and Sachs[1] found that v_0 appeared to increase very slowly with C/W, and proposed the relation

$$v_0 = q \cdot \left(\frac{C}{W}\right)^{0.22} \tag{3.1}$$

which is inconsistent with the observed fact, that for small projectiles v_0 varies as rapidly as $\sqrt{C/M}$. We are able to remove the discrepancy, since for large values of C/M the velocity given by equation (2.3) varies as slowly as that given by equation (3.1).

[1] M. Schwarzschild and R. G. Sachs, "Properties of Bomb Fragments", BRL No. 347, 7 Apr 1943

4. Discussion of Results reached so far

The expression (2.3) may be written in the form

$$v_0 = v_l \cdot R^{1/2} \tag{4.1}$$

where the parameter v_l depends on the particular explosive used. It is difficult to know to what extend we ought to expect the velocities of fragments in the side-spray of a shell or bomb to agree with equation (2.4). But **FIG. 1** and **FIG. 2** show that for projectiles containing TNT, using the value $v_l = 8000$ feet/sec, the formula fits the experimental data fairly well over the whole range from $C/M = 0.06$ to $C/M = 5.62$.
It seems then that the basic assumption may be correct that for a series of projectiles containing different quantities of the same explosive, the contribution made to the total kinetic energy by the detonation of each unit mass of explosive is the same.
The reason why for large values of C/M the initial velocity fails to increase as rapidly as $\sqrt{C/M}$ is clear. In a shell with a relatively thick and heavy casing, nearly the whole of the kinetic energy is possessed by the metal casing, as it breaks up into fragments. But for projectiles with C/M greater than 2 there is actually more energy in the kinetic energy of radial motion of the explosion gases inside the bomb than in that of the metal casing which contains them. This severely limits the value of v_0 for the fragments.
In deriving equation (2.4) and (2.5) it was assumed, that ρ was constant, and that inside the projectile the radial velocity v of the explosion gases was proportional to r. It may be that this assumption overestimates the amount of kinetic energy of radial motion retained by the explosion gases. If this is so, the numerical factor $\frac{1}{2}$, which occurs in the denominator of equation (2.4) should be replaced by a somewhat smaller value, such as 0.45. At the same time the value of v_l in equation (4.1) would have to be reduced. Experimental data on initial velocities are at present too scanty to decide this point. But with the present data no significant improvement is obtained by replacing $\frac{1}{2}$ by a different factor.
The expressions (2.1) and (2.2) are intended to express the fact that under optimum conditions of detonation a certain fraction of the chemical energy of explosion is converted into kinetic energy, other details being important.

The integral is to be taken to a radius a. And it was stated, that this was the radius of the casing at the moment of rupture (suggesting that this might depend on the strength of the metal forming the casing). This remark, however, was introduced only for the sake of simplicity. The kinetic energy of the metal should depend only on its mass. In the fragmentation of simulated shell at Bruceton, described below, steel casings of varying degrees of hardness were tried, ranging from Brinell 105 to 500. No significant effect of hardness on the initial velocities was found.

5. Remarks to the characteristic Value E

For each explosive the initial velocities will be determined by the characteristic value of E. We have seen that for TNT the value of the constant v_l is in the neighborhood of 8000 feet/sec. We have then

$$\begin{aligned} \sqrt{2E} = v_l &= 8000 \text{ feet/sec} \\ &= 2.44 \cdot 10^5 \text{ cm/sec} \; . \end{aligned} \tag{5.1}$$

Whence

$$\begin{aligned} E &= 3 \cdot 10^{10} \text{ ergs per gram} \\ &= 715 \text{ cal per gram} \; . \end{aligned} \tag{5.2}$$

The report 03RD 1510 gives calculated values for a quantity W, which is not directly comparable with E. This W is the "reversible work per gram of products of adiabatic expansion from the adiabatic constant volume explosion state to a pressure of one bar". For TNT of density 1.59 g/cm^3 the value given is

$$\begin{aligned} &= 3.72 \cdot 10^{10} \text{ ergs per gram} \\ &= 890 \text{ cal per gram} \; . \end{aligned} \tag{5.3}$$

6. Comparison of Empiricism and Theory

The available data for TNT-filled projectiles are as follows:

(a)

In the experiments by Division 8 of the N.D.R.C.[1] steel cylinders filled with TNT or other explosive were used. The cylinders had an internal diameter of 2" in and a uniform thickness of wall. The velocities of fragments were measured at a distance of about nine feet by means of the velocity camera. The values for different thicknesses of steel casing filled with TNT are given in **table 6.1**, attention being paid only to the large and medium fragments for which the retardation in air will be negligible. As the number of fragments recorded was small, the probable error is large.

Table 6.1.
Velocities of fragments of different kinds of steel casings. The higher value of 2870 feet/sec in the second column from left was obtained when the experiment was repeated a month later (Interim report May - June)

Wall Thickness	$\frac{1}{2}$"	$\frac{3}{8}$"	$\frac{5}{16}$"	$\frac{3}{16}$"	$\frac{1}{8}$"
C/M	0.165	0.231	0.286	0.500	0.775
v_0, ft/sec	2600 2870	3240	3800	5110	6108

(b)

The 4000-lb bomb AN-M56, filled with TNT. The diameter of the central cylindrical part of this bomb was 34.25 inch, with a thickness of steel casing 0.31 inch. These values give C/M equal to 5.62. The velocities of some of the leading fragments only were measured. These had a mean value

[1] Interim Reports of Division 8. April - June 1943

of 10300 feet/sec. The mean velocity of all the large fragments must be somewhat less than this. We may take 9800 feet/sec as a value more suitable for comparison with velocities obtained from other projectiles.
Further data for bombs, including TNT-filled, will soon be available at he Ballistic Research Laboratory.

(c)

For the 105-mm Howitzer shell M1 and for the 75-mm shell M48 the velocities in the side-spray have been estimated from the change in angle of projection with change in residual velocity of the shell. The charge of TNT in these two shell had the value 4.93 pounds and 1.56 pounds, respectively. The total weights of the unfuzed shell are 30.625 pounds and 12.50 pounds, respectively. The thickness of the cylindrical walls, as in most modern shell, is variable. Before we could predict the resultant distribution of fragment velocities, we should have to answer the question, to what extend the wall acts as a whole during rupture. Instead of a complex theory, however, what is needed here is a formula by which the average fragment velocity can be rapidly estimated, when the total weight of the unfuzed shell, and the charge are given. If the ratio of this two quantities is taken as C/M (instead of the correct quantities) and v_0 is calculated from equation (2.4) and (4.1), setting $v_l = 8000$, as before, one obtains the points plotted for the 105-mm and 75-mm shell in **FIG. 2**. It will be seen that these plots lie on the straight line as well as, or better than, the neighboring points for the N.D.R.C. shell of constant wall thickness.
The reason for this may be as follows. There is a theoretical objection to drawing the line through the origin, since this implies that an exceedingly small charge will be sufficient to rupture a heavy casing, and give the fragments an initial velocity. An expression of the form

$$v_0 = v_l \cdot \left(R^{1/2} - \text{constant}\right) \tag{6.1}$$

is more acceptable and seems to fit the facts better. For the practical purpose of estimating the v_0 for shell similar to the 75-mm and 105-mm, it seems, however, unnecessary to use an expression containing an additional new constant.

(d)

A British report[2] records measurements of fragment velocities for a 40-mm Bofors shell, which is interesting as its C/M is exceptionally low. The

[2] A.C. 3432
N. A. Tolch and R. Gurney, BRL Memo Report No. 207

velocity was found to 650 metres/sec, or 2130 feet/sec. The charge of TNT was 56.4 grams, and the weight of casing excluding the brass cap and copper band was 820 grams. The ratio of these quantities is only 0.069. Taking this ratio as C/M, as in the case of the other shell, the point plottet in **FIG. 2** is obtained.

(e)

Although the initial velocities of fragments from grenades containing high explosive have not been measured, there is some indirect evidence that the expression (4.1) gives a correct estimate for grenades containing $1\frac{1}{2}$ to 2 ounces of TNT. Calculations made on this assumption were in good agreement with direct panel tests.

7. Results of the Comparison

A knowledge of the initial velocities of fragments is a first step toward the desired knowledge of the velocity at any distance around the explosion. For a fragment of mass m this may now be obtained from the expression

$$v = v_l\, R^{\frac{1}{2}} \cdot e^{-0.013\,\mathsf{s/m}^{\frac{1}{3}}} \tag{7.1}$$

where R is obtained from equation (2.4), m is expressed in grams, s is expressed in feet, and v is given the value appropriate to the explosive used. For TNT v_l has the value 8000, while for some other explosives recent experiments give values up to 20 per cent higher[1]. The measurements, however, are not yet very consistent.

For cylindrical TNT-filled casings of constant wall thickness, the expected values of v_0 may be read from **table 8.1**.

In the range of C/M less than 0.3 the values of v_0 have been adjusted to agree with the N.D.R.C. results plottet in **FIG. 2**.

1 Sachs and Schwarzschild, Properties of Bomb Fragments, Ballistic Research Laboratory, Aberdeen Proving Ground, Report No. 347

8. Final Conclusion

In conclusion it may be mentioned that the fragmentation of an infinitely long cylinder, detonated from one end, was discussed by G. I. Taylor[1], and an expression was obtained for the fragment velocities. It was assumed that the radial motion of the explosion gases was small compared with the longitudinal motion. And the result were not intended to apply to a projectile from which the end-sprays are feeble compared with the side-spray.

Table 8.1.
Fragment velocities from cylindrical walls of uniform thickness. Column 1 gives the ratios of the external to the internal diameter. In calculating C/M the density of metal was taken to be 4.9 times the density of the explosive.

d_e/d_i	C/M	v_0
1.02	5.05	9560
1.04	2.50	8430
1.06	1.65	7610
1.1	0.97	6460
1.2	0.464	4910
1.3	0.296	4000
1.4	0.213	3400
1.5	0.163	2900
1.6	0.131	2580
1.7	0.108	2340

[1] Taylor G. I., Analysis of the explosion of a long cylindrical bomb detonated at one end. Scientific Papers of G. I. Taylor, Vol. III No. 30. Cambridge University Press, 1963, p 277-286.

A. Appendix

Solving equation (2.1) for a cylinder per unit length.

We set $\frac{1}{2} \cdot \sum_i m_i \cdot v_0^2 = \frac{1}{2} \cdot M v_0^2$ and solve the integral:

$$EC = \frac{1}{2} M v_0^2 + \frac{1}{2} v_0^2 \cdot 2\pi\rho \frac{1}{a^2} \cdot \int_0^a r^3 dr \quad . \tag{A.1}$$

With

$$f(x) = x^n \text{ and } F(x) = \frac{x^{n+1}}{n+1}$$

and remembering that

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

one gets

$$EC = \frac{1}{2} M v_0^2 + \frac{1}{4} v_0^2 \cdot \pi\rho \frac{1}{a^2} \cdot a^4 \quad , \tag{A.2}$$

This gives, by multiplying both sides with 2

$$2EC = M v_0^2 + \frac{1}{2} v_0^2 \pi\rho a^2 \quad . \tag{A.3}$$

Setting

$$\pi\rho a^2 \equiv C$$

and solving for v_0^2, we get

$$v_0^2 = \frac{2EC}{\left(M + \frac{1}{2}C\right)} \quad . \tag{A.4}$$

Using equation (2.4) and taking the square root, we finally get

$$v_0 = \sqrt{2ER} \quad . \tag{A.5}$$

Solving equation (2.2) for a spherical casing.

We set $\frac{1}{2} \cdot \sum_i m_i \cdot v_0^2 = \frac{1}{2} \cdot M v_0^2$ and solve the integral:

$$EC = \frac{1}{2} M v_0^2 + \frac{1}{2} v_0^2 \cdot 4\pi\rho \frac{1}{a^2} \cdot \int_0^a r^4 dr \ . \tag{A.6}$$

With

$$f(x) = x^n \text{ and } F(x) = \frac{x^{n+1}}{n+1}$$

and remembering that

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

one gets

$$EC = \frac{1}{2} M v_0^2 + \frac{1}{5} v_0^2 \cdot 2\pi\rho \frac{1}{a^2} \cdot a^5 \ , \tag{A.7}$$

Knowing that the volume of a sphere is $\frac{4}{3}\pi r^3$, expanding the r.h.s. of equation (A.7) with $\frac{3}{4} \cdot \frac{4}{3}$ and setting

$$C \equiv \frac{4}{3}\pi\rho a^3$$

as the mass of the explosive charge, we get

$$EC = \frac{1}{2} M v_0^2 + \frac{3}{5} \frac{v_0^2}{2} C \ , \tag{A.8}$$

This gives, by multiplying both sides with 2

$$2EC = M v_0^2 + \frac{3}{5} v_0^2 C \ . \tag{A.9}$$

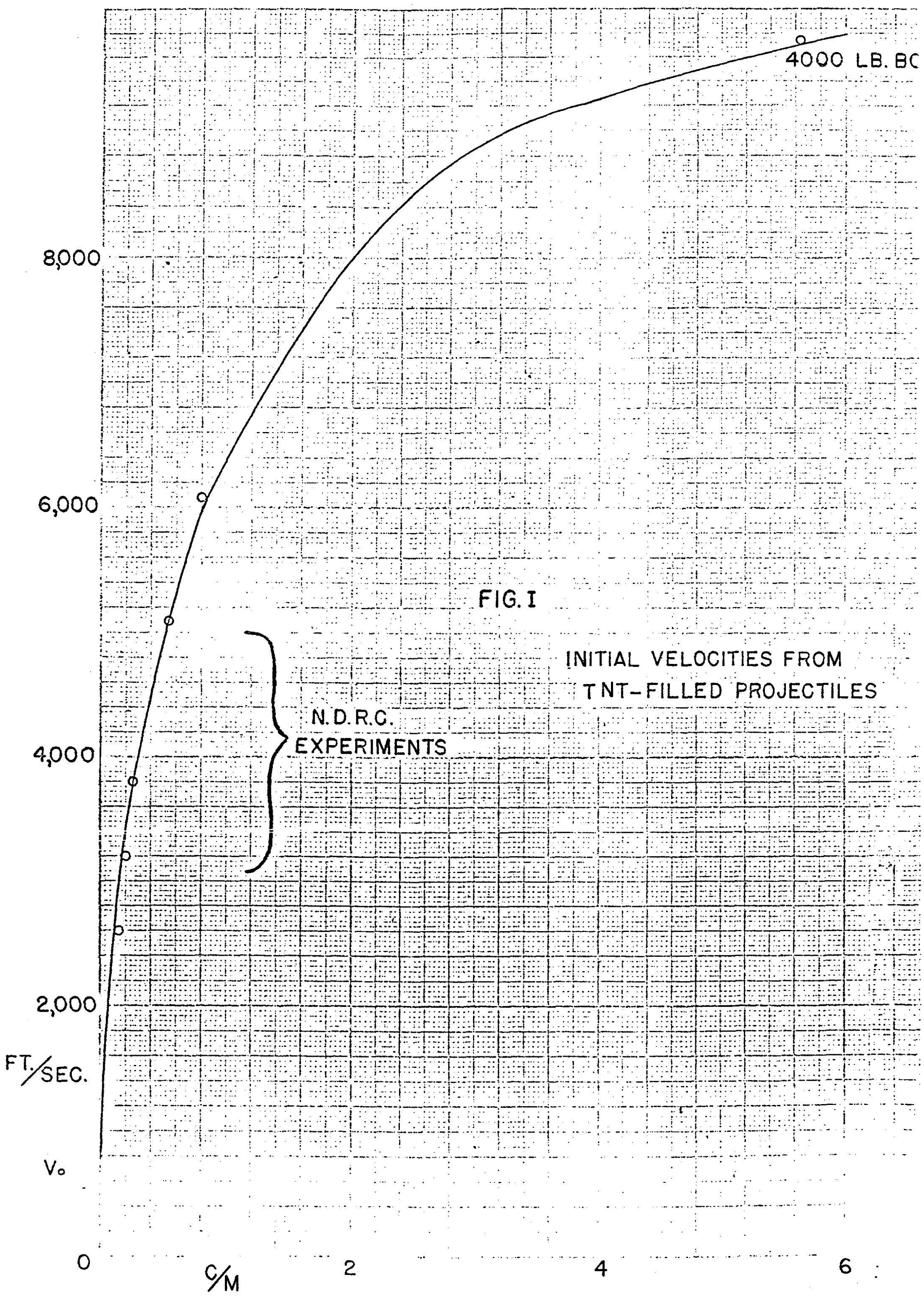
Solving for v_0^2, we get

$$v_0^2 = \frac{2EC}{\left(M + \frac{3}{5}C\right)} \ . \tag{A.10}$$

Using equation (2.5) and taking the square root, we finally get

$$v_0 = \sqrt{2ER} \ . \tag{A.11}$$

Figures


4000 LB. BC
8,000
6,000
FIG. I
INITIAL VELOCITIES FROM
TNT-FILLED PROJECTILES
N.D.R.C.
EXPERIMENTS
4,000
2,000
FT./SEC.
V₀
0
C/M
2
4
6

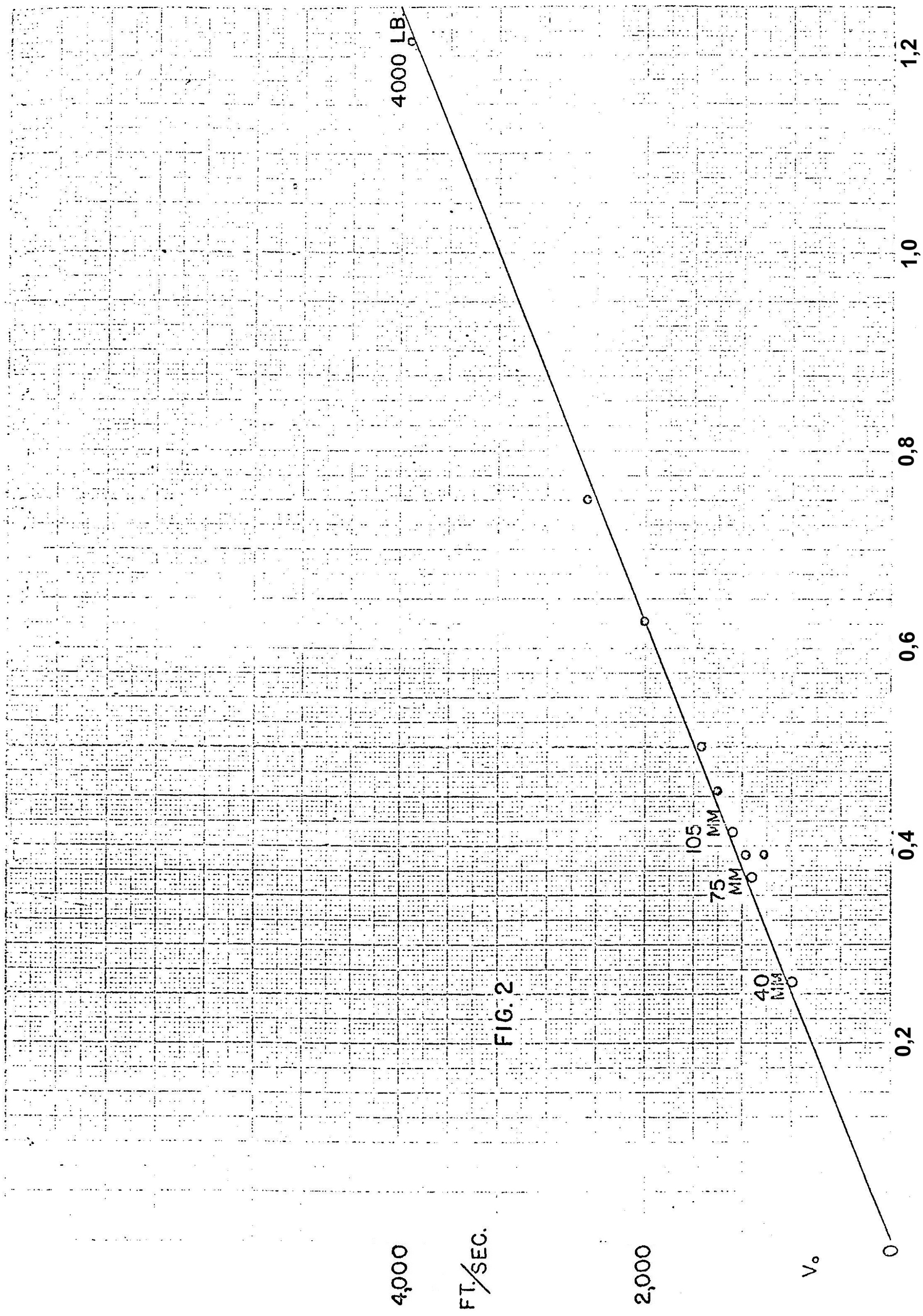


FIG. 2

List of Symbols

C	The mass of the explosive charge	kg
M	The mass of the accelerated shell or fragments	kg
m	mass in general	kg
v_0	initial velocity of accelerated fragments at the moment of detonation	m/s
v	velocity of the explosion gases	m/s
ρ	density of the explosion gases	kg/m^3
V	Volume in general	m^3
a	the radius of the casing at the moment of rupture	m
r	the radius of the casing or projectile, respectively	m