

GREEN'S FUNCTION

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Thuhma

Secondary, Higher leh Graduate zirlai bakah Master zirlai thlengin an hlawkpuui theih tur, Physics lam hawi (ka thiam ve chhunte), Mizo Tawng ngeia ziah leh dah that hi a tha ka ti a. Chuvangin tun tum chu tlema *level* sang deuh hlek, mahse pawimawh em em – Green's Function hi hrilhfiah tum dawn chhin dawn a ni. Mathematics a tam (*heavy*) deuh hlek a mahse calculus inhriathiam chuan a harsa lutuk lo ang.

Thupui-ah lut nghal hmiah mai ang aw...

Hetiang zawng hian a nihna (*concept*) hriat a awl deuh: hun a danglam ang zela chet dan danglam thei *particle* ngaihtuah ta ila; mahse chu chu a tirah a che lo tur a ni (*Initially at rest* tihna a nih chu). Chu chu hun a danglamin a chak dan pawh danglam thin thilin han tidanglam ta ila. A nih chuan chu *system* chu *time – dependent force*, $F(t)$ avangin a danglam dawn tihna a nih chu. Vawilekhkatah chakna han pe thut ta la, mahse i chakna pek chu reilote chhung (*impulsive force*) a ni tur a ni. Hun t' a ralin chu *impulse force* chuan *unit momentum change* pe ta se. Hun dang t a lo ral meuh chuan chu *particle* kal thui dan $x(t)$ chu Green's function kan tih mai – $G(t, t')$ hian a hril dawn a ni.

Hun reilote $\Delta t'$ chhung leka chakna kan pek, $F(t')$ hian nawrtu (*impulsive*), $F(t') \Delta t'$ a nei tihna a ni a. Hemi hi engtik lai pawha khaihlak miah lo (*continuously*) a particle nawr kaltu angah kan ngai thei tihna a nih chu. Anih chuan particle kal thui dan chu khi laia nawrtu zawng zawng khi hun tiam chin neia khaikhawm (*integrate*) in kan hmu thei mai dawn a lo ni. Mathematics takin ti hian ziak ta ila:

$$x(t) = \int_{t_0}^t G(t, t') F(t') dt' \quad (1)$$

Mahse khi equation khian tiam chin (*boundary conditions*) a nei tur a ni. Chu chu $t = 0$ a nih chuan $x(t_0)$ hian *zero* a tluk tur a ni.

Problem pakhat enchhin ang hmiang

A hnuiaia *differential equation* hi en ta ila:

$$m \frac{dv}{dt} + \alpha v = F(t) \quad (2)$$

$F(t)$ hi particle tiche tu, chakna petu kha a nia, tin αv hi a lo dangtu – *viscous force* a ni bawk a ni. Chuan particle hi chakna pek hma chu a che lo angah ngai ta bawk ila. A nih leh eng tin nge a kal chak dan – *velocity* kan zawn tak ang le? Tih dan awlsam tak i rilrua lo lang chu ka hre mai. A chung ami khi *ordinary differential equation* a ni a, pawl 12 zir tan chuan awlsam takin a chawh theih ang. Mahse chutianga chawk lovin Green's function kan hriat

chuan awlsamte in kan chawk chhuak thei tlat a ni. A nih leh Green's Function hmanga chawh chu engnge a that bikna tiin kan inzawt nghal maithei. Chu tak mai chu nakinah duhthawh takin kan la sawi lang dawn a ni.

Khilaia Green's Function chu:

$$G(t, t') = \frac{1}{m} e^{-\alpha(t-t')/m} \quad (t \geq t') \quad (3)$$

A nih chuan *equation* (1) hmangin particle kal chak dan chu hetiang hian kan chawk chhuak mai dawn a ni:

$$v(t) = \int_0^t \frac{1}{m} e^{-\alpha(t-t')/m} F(t') dt' \quad (4)$$

Mahse hriat reng tur chu Green's Function hi chakna kan pekah a inngat lo a; mahse *system* chhunga a chetdan phungah a inngat zawk a ni.

Green's Function pawimawhna chu

Pu VanlalGreen-a khan he mathematics hmanrua tangkai lutuk hi a hmuhchhuah dan tak chu - *Electric* leh a kaihhnawih vel a zir chianna lamah a ni. Tin, a awmzia tak pawh hi *electrostatics* (mahse *bounded regions* a ni tur a ni) lamah hian hriatfiah a awl ber bawk. A hrilhfiah dan chuan *unit point charge* hi \mathbf{r}' ah awm ta sela. Chu *point charge* chuan chakna/ theihna (*potential*) a pe chhuak anga. Chu *potential* ngei mai chu Green's Function kan tih chu a ni ta a ni. Chuan *point charge* chu hun reilote chhunga chakna petu (*impulsive force*) angah kan ngai leh ang.

Awlsam tea hrilhfiah dan leh hman dan tlanglawn ber hrethiam tur chuan *operator* kan neih a ngai phawt a. Mahse chu *operator* chu *linear* a ni tur a ni. *Linear operator* entirna tha deuh mai chu Laplacian ∇^2 kha a ni a. Chuan hriat leh tur dang chu *function* kha *operator*-in a tihchet khan tiam chin (*boundary conditions*) a nei bawk tur a ni. A nih chuan Green's function chu enge ni ta ang le tiin han inzawt ta ila. A chhanna chu he equation solution hi a ni mai dawn tihna a nih chu:

$$\mathcal{L}G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (5)$$

Mahse a chung ami khian tiam chin a nei tur a ni. \mathcal{L} hi *differential operator* kha a ni a. A nih chuan a chung ami khi *differential equation* a ni tihna a nih chu. Anih chuan a chunga kan sawifiah ho leh equation hmanga kan dahte nen khian a la inang vek tihna a ni. Equation dinglama *Dirac delta function* khian field lo chhuahna (*source*) ai a awh a. Chu chu a hmaa kan sawi ang khan hun reilote chhunga chakna pe thei chakna (*impulsive force*) a entir a ni. Delta function hmanga kan dah chhan ringawt pawh hi thui tak a sawi theih.

Delta function (mahse *function* a nih chiah lo, physics-ah chuan function kan ti mai a) awmzia hre turah ka ngai che a. Dimension pathum nei chinah hian *charge* inzawmkhawm hi *charge muk* dan (*charge density*) angin kan dah mai thin a nih kha. Chu *charge muk* zawng chu kan khaikhawm (integrate) chuan *charge* tam lam kan nei leh mai dawn tihna a lo ni. Hetiangin:

$$Q = \int d^3\mathbf{r} \rho(\mathbf{r}) \quad (6)$$

Anih leh tunah hian *charge* pakhat chauh hmun pakhatah (*single point charge*) nei ta ila. Chu chu q ni sela, chuan a awmna hmun pawh \mathbf{r}' ni bawk ta se. Anih chuan chu lai hmunah chuan charge muk zawng (*charge density*) chu enge ni ta ang le? $\mathbf{r} = \mathbf{r}'$ ah lo chuan *zero* a ni vek dawn tihna a nih chu. A chhan chu hmun dangah kha charge kha a awm lo miau a. Chuti a nih chuan kan sawi lai mek hmun zau lo tak mai – tereuhte (*point*) ah khan charge chu a awm tlat si a ni. *Charge* awmna hmun len zawng (*volume*) chu a tet em avangin (*infinitely small*) a muk zawng (*charge density*) chu chu lai hmunah chuan a lian/sang tham (*infinite*) viau dawn tihna a ni. Chutiang a nih avang chuan awlsam takin *delta function* hmangin kan dah mai thin a ni. Hetiangin:

$$\rho(\mathbf{r}) = q\delta^3(\mathbf{r} - \mathbf{r}') \quad (7)$$

Duh chuan charge chauh ni lovin rihzawng muk lam (*mass density*) pawh hetiang hian kan dah thei a ni.

Problem awl tak ngaihtuah chhin ila :

Physics law chuan *electric field* darh dan (*divergence*) hi *charge* muk lam (*charge density*) ah a innghat a ti a. Chu chu Poisson's equation an tih kha a ni. Hetiangin bul tan ta ila:

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho(\mathbf{r}) \quad (8)$$

Hei pawh kan nei leh a:

$$\vec{E} = -\nabla V(\mathbf{r}) \quad (9)$$

A nih chuan equation (8) leh (9) atang khian Poisson equation kha kan lo nei ta a:

$$-\nabla^2 V(\mathbf{r}) = \frac{1}{\epsilon_0} \rho(\mathbf{r}) \quad (10)$$

Charge muk zawng chu *delta function* hmangin kan dah thei a ni tih kan ziak tawh a. Chuvangin equation (10) khi tidanglam leh ta ila:

$$-\nabla^2 V(\mathbf{r}) = \frac{1}{\epsilon_0} q\delta^3(\mathbf{r} - \mathbf{r}') \quad (11)$$

Emaw hetiang hian:

$$-\nabla^2 V(\mathbf{r}) = \frac{1}{\epsilon_0} q\delta^3(\mathbf{r} - \mathbf{r}') \quad (12)$$

A chunga equation khi equation (5) nen khaikhin ta ila. *Operator* chu *Laplacian* (*negative laplacian* a ni zawk mah) a ni a. Anih chuan Green's function chu charge awm avanga thiltihtheihna (potential) lo chhuaka kha a ni tihna a nih chu! A nih chuan Green's function chu:

$$G(\mathbf{r}, \mathbf{r}') = \frac{\epsilon_0}{q} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \quad (13)$$

tihna a ni. Green's function kan hriat chuan awlsam takin *Poisson equation* chu kan chawk tawh mai dawn a ni. Mahse -chawk chhuak turin tiān chin nei (*boundary conditions*) pahnih a ngai thin a. Chungte chu: *Dirichlet boundary condition* leh *Neumann boundary condition* kha a ni a. *Dirichlet boundary condition*-ah hi chuan *potential* kha a pawn lam chin (*boundary*) zelah a nihna zat a lo pe ang che. Chuan *Neumann boundary condition*-ah ve thung chuan pawn lam chin atanga *degree sawmkua-a* a danglam zel dan (*derivative normal to surface*) a pe thung ang.

Quantum mechanics lamah hian

Quantum mechanics lamah hlei hlei hian Green's function hi a pawimawh a. Quantum Field Theory pawh hi a tel lo chuan a harsa viau ang le! Tunah hi chuan quantum mechanics bul tanna pawimawh tak, time-independent Schrodinger equation chawh chhuah nan han hmang chhin dawn teh ang:

Time-independent Schrodinger equation kha hetiang hi a ni a:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (14)$$

Chu chu hetiang hian kan ziak danglam thei a:

$$(\nabla^2 + k^2)\psi(\mathbf{r}) = K \quad (15)$$

Khitah khian k tlukpui chu $\frac{\sqrt{2mE}}{\hbar}$ a ni a. K erawh chuan $\frac{2m}{\hbar^2}V(\mathbf{r})\psi(\mathbf{r})$ a tluk thung a ni.

Tunah chuan a hmaa kan tih angin equation (15) na khi hmun pakhat atanga chakna (*point source*) lo kal avanga insiam ni ta sela, hetiang hian kan ziak leh mai dawn a ni:

$$(\nabla^2 + k^2)G(\mathbf{r}, \mathbf{r}') = \delta^3(\mathbf{r} - \mathbf{r}') \quad (16)$$

Tichuan a chunga equation hi kan chawk chhuak tawh mai dawn a ni. Pu Green-a ngaihtuah chhuah tho – Green's function ni lo ***Green's theorem*** hman khan kan chawk chho ang a (Green theorem leh Green's function hi ngaih pawlh loh tur a ni). Mahse mathematics kan bel phah leh zual nan *Fourier transform* hmanga Green's function zawn lam hi thlur bing ta zawk ila. Mathematics chawh lai hi chuan Green's function hi $G(\mathbf{r}, \mathbf{r}')$ tia ziak lovin $G(\mathbf{r})$ tiin ka ziak dawn a ni. Chumi awmzia chu nihdan phung pangngai (*general*) takin ka ziak tihna a ni a. *Point charge* awmna kha ka ziak lang lo tihna a ni mai.

$$G(\mathbf{r}) = \frac{1}{(2\pi)^3/2} \int e^{i\mathbf{l}\cdot\mathbf{r}} f(\mathbf{l}) \delta^3 \mathbf{l} \quad (17)$$

Chuan *delta function* pawh *Fourier transformation* hmang hian kan ziak ve leh thei a:

$$\delta^3(\mathbf{r}) = \frac{1}{(2\pi)^3/2} \int e^{i\mathbf{l}\cdot\mathbf{r}} \delta^3 \mathbf{l} \quad (18)$$

Tichuan equation (17) leh (18) na khi equation (16) ah khian han dah la, hei hi a lo chhuak ang:

$$f(\mathbf{l}) = \frac{1}{(2\pi)^{3/2}(k^2 - \mathbf{l}^2)} \quad (19)$$

A nih chuan Green's function chu hemi hi a ni mai dawn tihna a ni:

$$G(r) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{l}\cdot\mathbf{r}} \frac{1}{(k^2 - \mathbf{l}^2)} \delta^3 \mathbf{l} \quad (20)$$

Mahse hei hi kan chawk te leh ang a. Chawk te lem lo pawhin equation (4) na ami ang khian a dikna chu kan dah thei tho mai. Equation (20) na chawk te tur chuan \mathbf{r} hi che lo angah kan ngai ang a. Chuan *contour integration – Cauchy integral formula* hmang khan kan chawk chhuak mai dawn a ni. A sei deuh dawn avangin mahniin ti mai ta ila, a chhuak ka lo ziak tawp mai ang. Mahse *contour* khar dan kha fimkhur takin tih erawh a ngai ang.

Equation (20) tlukpui chu hei hi a ni a:

$$G(r) = -\frac{e^{ikr}}{4\pi r} \quad (21)$$

Green's function chu tui a lungte kan thlaka tui fawn bial (*waves*) lo insiam chhuak zung zung ang mai a ni tihna a nih chu. A tira kan sawi dan nen khan a la inang reng a ni tiraw... Mahse hei hi chu tlemin a danglam deuh hlek a. Ngun taka zir chian a ngai thung. A nihna takah chuan a chunga Schrodinger equation kan chawh mek hi chakna nei (*potential*) kan va tihbuai avanga lo sawhkhawk (*scattering problem*) kha a ni zawk mah a. *Quantum mechanics*-ah khan *plane wave* pangngai hawi zawng bik (*direction*) nei (x emaw, y emaw, z emaw) khan *potential* nei kha a va sut khan a lo sawhkhawk kha *spherical wave* angin a awm vek thin a nih kha. Anih chuan *plane wave* anga lo insawh let lo in hawi zawng (*direction*) tam tak neiin a darh chhuak dawn tihna a nih chu. Chu tak mai chu Pu Green-a function hi a lo ni leh ta chiah a ni. Equation number (21) khi a ni a. Mak ve khawp mai!

Mahse kan chawh chhuah (*solution*) ah chuan *free particle* (*kinetic energy* chiah nei kha) pawh kan telh ve a ngai dawn a. Chutiangah chuan K tlukpui chu zero a lo ni ang a. Equation chawh awlsam tak – *homogenous equation* kha kan nei leh mai dawn a lo ni, chumi tan chuan. A nih chuan hetiang hian kan nei thei anga:

$$\psi(\mathbf{r}) = \psi_o(\mathbf{r}) + \int G(\mathbf{r}, \mathbf{r}') \frac{2m}{\hbar^2} V(\mathbf{r}') \psi_o(\mathbf{r}') d\mathbf{r}'^3 \quad (22)$$

Emaw hetiangin:

$$\psi(\mathbf{r}) = \psi_o(\mathbf{r}) - \frac{m}{2\pi\hbar^2} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} V(\mathbf{r}') \psi_o(\mathbf{r}') d\mathbf{r}'^3 \quad (23)$$

Equation (23) na hi kan equation chawh chhuah chu a ni ta a ni. Mahse khi laia kan chawhchhuah khi ngun taka kan en chuan *integral chhungah khian wave function* hriat chian loh, la chawh chhuah ngai a la awm fo a nia. Kan equation chawh tur nihphung (*physical system*) kan hriatchian chuan thil awm dan pangngai tak a ni tlat mai! Khi laia wave function $\psi_o(\mathbf{r})$ lo lan chhan chu: lo sawhkhawk tu (*source of scattered wave, K*) kha lo nawr hmasa tu (*incident wave*) a awm loh chuan a bo hmiah (*not exist*) zel vang a ni.

A sei lutuk dawn e...

Duh aiin a sei leh ta tlat mai. Chhuah man pek a ngai zawk dawn a ang leh ta! Mahse Sir Rsa te, Sir Maenga te, Sir Lk-a te, Sir Zova te leh mi dang dang te hi he chanchinbu enkawltu an nih miau avangin inthlahrung lo deuhin kan lo ti leh nghek mai a ni. Mahse thianpa Ruata of STAM office chuan sei a ti dawn lo hrim hrim tih ka chiang.

Eng pawh nise Green's function ka hriatthiam dan kan lo ziah ve kher chhan hi eng dang vang ni lovin a pawimawh em vang a ni. Physics pangngai-ah hian Classical Field Theory atanga Quantum Field Theory thlengin hmun a chang a. Zirlaite tibuai em em tu – Professor leh zirtirtute question set har tirtu leh hrilhfiah thiam lo anga lan tirtu a ni thin!

Physical law zawng zawng hi *differential equation* hmanga ziah emaw entir emaw a ni mai zel a. *Newton's law* chu hun (*time*) leh hmun (*position*) danglamna hmangin *differential equation* hmanga ziah a ni a. *Maxwell equation* pawh hi *magnetic* leh *electric field*, hun leh hmun a danglam dan anga ziah a ni mai thin. A nih chuan chung ho zawng zawng chu equation (5) na ami ang vek khian kan dah thei tihna a nih chu. *Newton's law*-ah chuan chakna (*force*) kha delta function hmangin kan dah mai anga. Chutiang zelin *Electrodynamics* lamah chuan *charge* emaw *current* emaw muk zawng (*density*) kha *delta function* angah kan chan tir leh mai dawn a ni.

A tawpna tak tak!

Green's function pawimawhzia hi engineering leh mathematics pangngai lama zirte hlei hlei hian an hre chiang ang le! Zirbingna atana pawimawh em em – branch hran hrana pawimawhna nei kual vel vek Green's function hmu chhuak tu Pu George Green-a erawh hi chu a dam laiin tumahin an ngai ropui lo. A thih hnu-ah mi turu tak a ni tih Pu Lord Kelvin-a leh Pu Stokes-a te hovin an hre chiang ve chauh a ni. Amah hi Pu Robin Hood a tual lenna a piang leh sei lian niin zirna mumal pawh nei lo mah se kum 40 a nih hnua Cambridge-a lut tha leh a ni a. Mahse a lar loh em avangin Cambridge-ah hian a hnuhma hmuh tur a awm lo a, chhui theih pawh a ni lo. Cambridge a luh atanga kum 8naah rihsang min kai san ta a ni. A dam laiin a lar lova, chawimawi a ni lo bawk. A lungchhiat thlak khawp mai!

Mahse hnam fing – sapho na na chu an lo dangdai khawp mai. Kum 1993, Zofate'n Chanchin Tha thlen champhaphak ropui tako kan lawm dawn kum chiah khan ropui takin Westminster Abbey, London-ah amah hriatrengna hun hman a ni a. He mi kum hi a pian atanga kum 200na chiah a ni bawk. Hemi hunah hian amah hriatrengna lung chu, Sapho (English)in physics lama an chhuanvawr: Pu Newton-a te, Pu Faraday-a te, Pu Maxwel-a te leh amah ngaisang em em tu leh a lehkhabu ziah hmuchhuak tu – Pu Kelvin-a te lung phunna hmunah, amah an chhuanzia leh ngaihsanzia tihlannan phun a ni ve ta a ni!

