

Pu Pauli Exclusion Principle leh Quantum Mechanics

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Thuhma

Energy band hi a awm a ni tih chu kan hre theuh ang a. Quantum Mechanics avanga lo awm a ni ber a. Mahse a lo awm theih dan hi awmze nei taka hriatchian erawh a har reuh fu mai. Awlsam taka energy bands lo insiam dan kan pawm theih nan Pu *Pauli exclusion principle* hmangin han chhui chho ang hmiang...

Classical picture atangin aw...

Metal chhungah hian positive *nucleus* leh *core electron* nei ta ila. Chungte chu awmhmun neiin an awm a, an che sawn lo a. Mahse *valence electron* kan tih mai thin hi chu an che sawn thei a. An duh duhnaah an awm mai a ni. Chu chu sea of electron te pawh an ti thin.

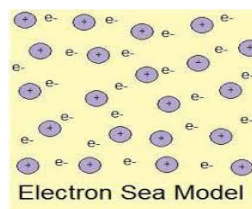


Figure 1: Drude model

Metal (copper emaw iron emaw) ah chuan *electric field* \vec{E} han pe ta la. Electron chu charge nei a nih avangin chakna (force) an lo nei anga, hetiangin:

$$\vec{F} = -e\vec{E} \quad (1)$$

Chuan electron chu a chunga force avang khian a kal chak (*accelerate*) tulh tulh dawn ta a nih chu. Hetiangin:

$$\vec{a} = \frac{-e\vec{E}}{m} \quad (2)$$

A nih chuan *electric field* kan pekna direction opposite chiahin *electron* chu a a kal (*accelerate*) dawn tihna a ni. Hei hi a chhan chu *negative charge* an nih vang a ni a, an accelerate hian an *velocity* pawh a chak tulh tulh zel a ni.

Awle, electron hian *initial velocity* v_i nei ta sela, hun t a lo ral chuan a *velocity* chu a hnuai ami hi a lo ni ang:

$$v = v_i - \frac{-e\vec{E}}{m}t \quad (3)$$

Electron hian figure 1-a kan entir *positive ions* a sut hma chu chak takin a *accelerate* a. Mahse hun engemaw ti hnuah *ions* chu a su ta a. A sut veleh a velocity a lo danglam ta a. A chhan chu *direction* leh *initial velocity* hran neiin a chet leh tawh vang mai a ni. *Ions* a sut hma kha chuan a *velocity* kha a chak tulh tulh si a, hei pawh hi a chhan chu a chunga *velocity*

expression kan entirna equation-ah khian *time* chiah a danglam thei a. Time hi *continuous variable* a nih tlat vang a ni. Hetiang hian *classical picture* hmangin metal-a *electron* chet dan chu an hrilhfiah thin a ni. Hei hi *drude model* pawh an ti bawk a.

Conduction hi a hma pawhin miin an hre daih tawh. Faraday leh Benjamin Franklin a awm lai atang daih tawhin *experiment* pawh miin an ti teuh tawh a. Mahse *quantum mechanics* chuan a aia Chiang zawk leh awmze nei zawkin a hrilhfiah leh zel a ni.

Particle in a box kha mawle...

Quantum mechanics kan zir hmasak berin '*Particle in a box*' tih kan zir vek kha kan la hre theuh ang a. Khita copper wire emawa *positive ions* leh *core electron* awmnaa electron che tla vel *drude model* hmanga kan sawifiah khi *electron*, three dimensional well (particle in a box) chhunga awm ang chiahin kan ngai thei a. Copper wire chu *3 dimension* a ni a. *Conduction* siamtu electron chu '*particle*' well chhunga awm kan tih tak maia chu a ni dawn tihna a lo ni.

A nih chuan particle hi potential well chhunga awm chuan an energy chu a *quantize* thin tih kan hriat kha. A chhan chu *boundary condition* leh *normalization condition* vang kha a ni a. Electron kha metal surface hnuai a awm '*localized*' tihna a ni a. Chuan *electron* chu *potential well*-ah a awm tlat a. Wire atangin a lo chhuak ngai dawn lo tihna a ni. Chumi awmzia chu: *photoelectric effect* emaw *thermionic effect* emaw avangin electron chuan metal chu a lo chhuahsan dawn lo tihna a nih chu. A nih chuan electron chu metal chhungah a awm reng a, *electron* chu *three dimensional well* chhunga awm reng ang chiah a ni tihna a ni. Mahse chuti ang tan chuan energy chu *quantized* tlat si a. Energy level tam tak kan nei thin a ni.

Sawifiah tum tak tak tawh ang

Tunah hian electron chu *three dimensional well* chhunga awm angin ngaihtuah tak tak ta ila. A awlsam zawk nan *degeneracy* nei lo ni ta bawk sela. A nih chuan cube (three dimensional) kan ngaihtuah chu a side a inang lo vek tihna a nih chu. Chuan energy level chu nondegenerate level a ni dawn tihna a ni. Energy level inkar zelah chuan awl (*space*) kan nei zel dawn a lo ni a. Hei tak hi *continuous* si lo, mahse *quantized* kan tih fo – a *quantum mechanics* lai tak chu a ni. Electron chu *energy level* ah chiah an awm thei a. An inkarah hian a awm thei ngai miah lo tihna a ni.

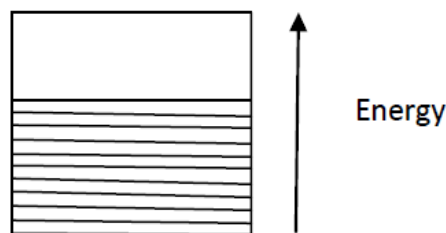


Figure 2: Energy level diagram

A chunga *diagram* ah khian level hnuai ber hi energy nei hniam ber a ni ang a, chuan a san dan indawtin energy chak zawng a tam zel dawn tihna a ni. A nih chuan line ka rinte khian energy level khat zel an entir tihna a ni chu. Tin, khita *line* chin chauh khi energy hlutna *electron* chuan an nei thei bawk a.

Chuan tunah hian electron tam tak lo nei ta ila, *electron*-te chu a mal tetein energy levelah khian han dah diat diat dawn ta ila. *Zero Kelvin* ni bawk ta sela, electron chu energy hniam ber – *ground state*-ah dah i tum vek ang tiraw? Mahse hei hi thil tih miah loh tur a ni tlat mai. Classical taka ngaihtuah chuan *ground state*-ah awm vek tur ni awm tak a ni a.

Quantum number palite sawifiah phawt ang...

Electron hi three dimensions chhunga awm anga kan ngaihtuah chuan *quantum number* pathum – n , l leh m_l a nei tih kan hria a. Chuan heng *quantum number* 3te hi Pu *Schrodinger* picture atanga lo lang kha a ni. Chuan, *spin quantum number* – m_s pawh kan nei leh a. Mahse hei chu *Dirac* picture atanga lo lang a ni thung. *Quantum number* hmasa pathum khian *electron* awmna (*spatial part*) a hrilhfiyah a ni. Chuan a tawp ber khi chuan *electron spin* chungchang a hrilhfiyah thung a. Hetiangin kan ziak thei anga

$$\Psi = R_{m,l} \Theta_{l,m_l} \varphi_{m_l} \Xi_{m_s} \quad (4)$$

Khita $R_{m,l}$, Θ_{l,m_l} leh φ_{m_l} te khian *electron* energy an hrilhfiyah a. Ξ_{m_s} erawh khi chuan *electron spin* a sawifiah thung a, *energy* tam lam leh tlem lamah engmah nghawng a nei lo thung a ni.

Pu Pauli exclusion principle chu...

Tunah chuan *quantum mechanics* chhunga pawimawh lutuk mai, *Pauli exclusion principle* kan tih mai – *fermions* 2te'n *quantum number* inang pali an nei thei miah lo tih kha kan ngaihtuah chhuak tawh ang. *Electron* chu energy level-ah chuan han dah ta la. Energy hniam berah pakhat han dah keuh la. Electron dang 1 i dah leh thei em? Khi laia kan ziak $R_{m,l}$, Θ_{l,m_l} leh φ_{m_l} khian *electron* energy a hril a. A nih chuan energy level pakhat atan chuan hemi quantum number 3te hi a inang reng dawn tihna a lo ni. Mahse vanneihthlak tak maiin Ξ_{m_s} hian hlutna pahnih a nei hlauh mai a, chu chu – *spin up* leh *spin down* kha a ni. Electron energy level pakhatah *quantum number* pathum inang an neih vek chuan *spin quantum number* chiah hi energy level khatah kan tidanglam thei tihna a lo ni dawn a ni. Energy level pakhat zelah *electron* 2 zel kan dah thei dawn tihna a nih chu. Kan hrethiam em aw...

Electron pahnihte quantum number hmasa 3te chu inang an nei anga mahse spin quantum number erawh 'up' leh 'down' an nei dawn tihna a lo ni. Energy level pakhatah electron 2 kan dah tawh chuan electron dang dah han tum viau mah ila a theih tlat tawh lo a ni. A chhan chu quantum number hmasa 3te kha a inang vek tawh si a, *spin quantum number* chi 2 chiah kan nei thei bawk si a, 2 hnuah a dang kan dah belh chuan quantum number 4 inang kan nei dawn tihna a lo ni. Mahse chu chu a theih tlat si lova. Energy level inangah chuan electron 2 chiah kan nei thei tihna a nih chu. Chuan *spatial part* chiah kan neih chuan *wave function* pawh inang an nei dawn tihna a ni. Mahse *spin* part lam kan telh chuan wave function inang

lo state hrana awm kan nei dawn tihna a lo ni. Chu chuan *Pauli Exclusion principle* dan a zawm dawn chiah baw k a.

Zero Kelvin-ah chuan *Classical mechanics* hmangin electron zawng zawng kha energy hniam berah an awm vek tur a ni a. Mahse chutiang chu a ni tlat lo. *Quantum mechanics* avangin ground state-ah emaw 2 a lo awm tawh chuan a chungah 2 chiah kan dah leh thei a, chutiang zelin kan chho zel ang. Energy level pakhat zelah electron 2 zel kan nei dawn tihna a ni chu. Chuan energy level sang ber kan thlen hma chuan electron chu kan level tinah 2 zel kan dah anga. Electron kan neih tawp ber kan dahna level (tawp ber) kan thlen hunah kan duh taw k ang. Chuta level sang ber chu *Fermi level* an tih chiah kha a ni leh ta baw k a.

Heta *Pauli exclusion principle* powerful zia kan hmu a ni tiraw... Electron hi an lo in hal (*repel*) ve thin tihna a ni. Mahse heta electron repulsion kan tih hi electrostatic repulsion a ni lova. Quantum mechanics avang liau liau a awm a ni (nakinah a rem chan chuan hemi topic hi kan la zia k ang chu).

Calculation tlem ti lawk ila...

Chuan electron 3 dimension chhunga awm energy chu hetiangin kan nei a:

$$E = \frac{m^2 \hbar^2}{2\pi L^2} (n_x^2 + n_y^2 + n_z^2) \quad (5)$$

Awle tunah chuan a chungah cube (3 dimension) sei zawng kan tih L khi 1cm lo ni ta se. Chuan electron energy chu 1eV ni ta baw k sela. Heta 1eV hi metal chhunga *electron* energy zat ang vel chiah chiah kha a ni a. Tunah chuan a hnuai ami hlutna hi zawng dawn ang hmiang:

$$(n_x^2 + n_y^2 + n_z^2) = \frac{2\pi L^2 E}{m^2 \hbar^2} \quad (6)$$

Equation number (6) ding lampang hlutna kan hre ta vek a. Ngun taka kan chawh chuan hetiang zat hi kan hmu dawn a ni: 8.8×10^{13} . A nih chuan ground level chungah energy level tam lutuk mai, 2.06×10^7 lai kan nei dawn tihna a ni. Hei hi 1eV energy nei electron energy level awm zat kan neih kha a ni a. A turu khawp mai. Level tam lutuk kan nei ta a, tunah chuan level 2 inkar thui zawng han zawng leh chhin ta ila:

$$\Delta E = \frac{m^2 \hbar^2}{2\pi L^2} \{(n_x + 1)^2 + n_y^2 + n_z^2\} - (n_x^2 + n_y^2 + n_z^2) \quad (7)$$

$$\Delta E = \frac{m^2 \hbar^2}{2\pi L^2} (2n_x + 1) \quad (8)$$

Mahse n_x hi a len em avangin 1 khi kan paih anga. Chuan energy level 2 inkar chu:

$$\Delta E = \frac{m^2 \hbar^2}{\pi L^2} n_x \quad (9)$$

Tunah chuan $(n_x^2 + n_y^2 + n_z^2)$ tluk pui kan hriat sa atang khan n_x value kan zawng leh anga. Hetah hian a awlsam zawk nan n_x , n_y leh n_z te chu inangah kan ngai mai anga:

$$n_x^2 + n_y^2 + n_z^2 = 3n_x^2 = 8.8 \times 10^{13} \quad (10)$$

A nih chuan n_x chuan 2.06×10^7 vel a ni dawn tihna a ni. Awle tunah chuan energy level 2te inkar zau zawng khi energy level hnuai ber atanga a chung ber inkar nen kan khai khin dawn a ni. Hetiangin:

$$\frac{\Delta E}{E} = \frac{\left(\frac{m^2 \hbar^2}{\pi L^2} n_x\right)}{\left\{\frac{m^2 \hbar^2}{2\pi L^2} (n_x^2 + n_y^2 + n_z^2)\right\}} \cong \frac{2n_x}{3n_x^2} \cong \frac{1}{n_x} \cong 10^{-7} \quad (11)$$

Electron energy E chu hmun 1,00,00,000-ah han then hrang ta la chuan a hmun khat chu energy level 2te inhlat zawng, ΔE a lo ni dawn tihna a lo ni. Chutiang chu a energy diagram kan siam chuan line 1,00,00,000 zet inkar chu a te em em mai dawn a, inzawm put angin a lang mai dawn a. A chhan chu an inkar a tet em avang a nih ber mai chu. A nih chuan energy level kan siam chu inzawm anga ngaiin *continuum* hmangin kan ziak thei a, chu chu *energy bands* an tih kha a ni a. Mahse *magnifying glass* hmangin emaw kan en chuan an lo inzawm lo a ni tih kan hre thei ang. *Core shell* lam kan ngaihtuah chuan *energy level* ho khan an inkar a hnaih em avangin *continuous energy spectrum* siam angin an lang tlat thung si a ni. Chu *continuous energy spectrum* tak mai chu *Energy bands* kan tih chu a ni ta mai a ni. Vanglaini Chanchinbu pawh hi hla deuh hlek atanga kan en chuan thu inziak hi a inzawm put a mahse a bul hnaih chuan a hrang leh tlat si a ni. Chutiang chu a ni e.

Kan hriat theuh angin *Zero Kelvin* ah khan *electron-in energy level* a luah khan *Fermi level* thleng chiah a luah thei a. A nih chuan electron-in a luah miah loh *band: empty band* pawh kan nei thei tihna a ni. Chuan *electric field* han apply ta ila engtin nge a awm ang le? *Electron*, energy level hnuai ber amite kha an che chhuak thei ang em? Kan hriat tawh angin *electric field* kan apply chuan *electron* khan energy an nei (gain) thin kha a ni a. Energy level hnuai ber emaw a inkar ami emaw te kha an che sawn thei dawn tlat lo a ni. A chhan chu eng dang vang ni lovin energy level tinah khan electron 2 theuh an awm tawh tlat a ni. A nih chuan electron, energy level sang bera awmte chauh kha a chungah an chho thei ang. A tirah chuan *Fermi level*-a awm emaw a hnuai vela awm emaw chiah kha an che sawn dawn tihna a lo nih chu. Chutiang zelin a hnuai amite tan hmun awl an siam zel anga electron che thei chu an lo tam zel dawn a lo ni. Chu'ng electron-te chuan conduction current an siam dawn tihna a lo nih chu.

Titawp ang le...

A sei leh ta viau mai a. BSc lama zirlaite tana ziak a ni a. An lo chhiar chuan an hlawkpui viau ka ring. Mahse evangin nge band gap a awm tih te kan la sawifiah thleng rih lo a. Tin, energy band kan hmuh thin tamtakte chu a kual nguai zel a, engvanga kual nge a nih tihte pawh remchangah kan la sawi chhunzawm ang chu. A tangkaipui turte chuan in tangkai pui ngei ka beisei.

