Probability Game Report

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The Game Set

You will need:

- Two six sided dice.
- One of each card ranging from Ace to King from a standard deck of cards (1-10 + Face Cards).

Instructions for Players

! This game is a singleplayer game, meaning it must be played alone.

I. Draw one card, and roll your two dice at once.

II. Check the "*Rules*" section for the value of each die face and card.

III. Add up the values of the two die you rolled and the card you drew.

IV. A total of 15 or greater counts as a victory.

V. A total of 14 or less counts as a defeat.

Game Description

Overview

This is a game of chance, meaning victories and defeats will occur based almost entirely on probability, and rarely on player skill.

The game primarily involves theoretical and experimental probabilities, as well as binomial distribution, as the outcomes are either wins or losses.

The events are independent, due the fact that the sample space does not change, and the die and cards have independent probabilities each roll and draw. Both mutually exclusive and non-mutually exclusive events affect the game, because you cannot both win, and lose, at the same time, but many combinations of dice rolls and cards are possible.

Rules

- The game is played as single round matches, where rolling your two dice and drawing a card is a whole match. At the end of a match, you may play again, or choose not to.
- The six faces of your two dice
- maintain their basic values of each face, ranging from one to six for each face. For example, if you roll one of your dice and it lands on a 5, it is worth 5 towards your total.
- The Ace card is worth 1.
- The Jack card is worth 11.
- The Queen card is worth 12.
- The King card is worth 13.
- The only room for strategy is the style in which you roll your dice.

- Beginners should just focus on the basics of not tossing the dice off or away the playing surface, as well as improving their mental addition.
- Advanced players may wish to experiment with dice rolling tactics in order to maximize success.

List of required materials:

- A pair of dice.
- 13 Cards from a standard deck

Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

Probability Analysis

Theoretical Distribution for Sums of Two Dice and Thirteen Cards

There are 78 total possible sums when two dice are rolled at once and one card is drawn from cards numbered 1-13 out of a standard deck.

First Die	Cards													2nd Die
	A 1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J 11**	**Q 12**	**K 13**	
1	3	4	5	6	7	8	9	10	11	12	13	14	15	**1**
2	5	6	7	8	9	10	11	12	13	14	15	16	17	**2**
3	7	8	9	10	11	12	13	14	15	16	17	18	19	**3**
4	9	10	11	12	13	14	15	16	17	18	19	20	21	**4**
5	11	12	13	14	15	16	17	18	19	20	21	22	23	**5**
6	13	14	15	16	17	18	19	20	21	22	23	24	25	**6**

Theoretical Distribution Continued

Possible Sums x	Probability P(x)	Expected Sums (x)P(x)	Wins(√) Losses(×)
3	1/78	3/78	×
4	1/78	4/78	×
5	2/78	10/78	×
6	2/78	12/78	×
7	3/78	21/78	×
8	3/78	24/78	×
9	4/78	36/78	×
10	4/78	40/78	×
11	5/78	55/78	×
12	5/78	60/78	×
13	6/78	78/78	×
14	6/78	84/78	×
15	6/78	90/78	√
16	5/78	80/78	√
17	5/78	85/78	√
18	4/78	72/78	√
19	4/78	76/78	√
20	3/78	60/78	√
21	3/78	63/78	√

22	2/78	44/78	√
23	2/78	46/78	√
24	1/78	24/78	√
25	1/78	25/78	√
		(x)P(x) = 1092/78 = 14	

The expected sum of 2 dice and one drawn card in this game for any given match is 14, which means you can expect the average sum of two dice and one card to be 14.

The probability of winning in any given match is P(winning).

P(winning)

= P(15) + P(16) + P(17) + P(18) + P(19) + P(20) + P(21) + P(22) + P(23) + P(24) + P(25)

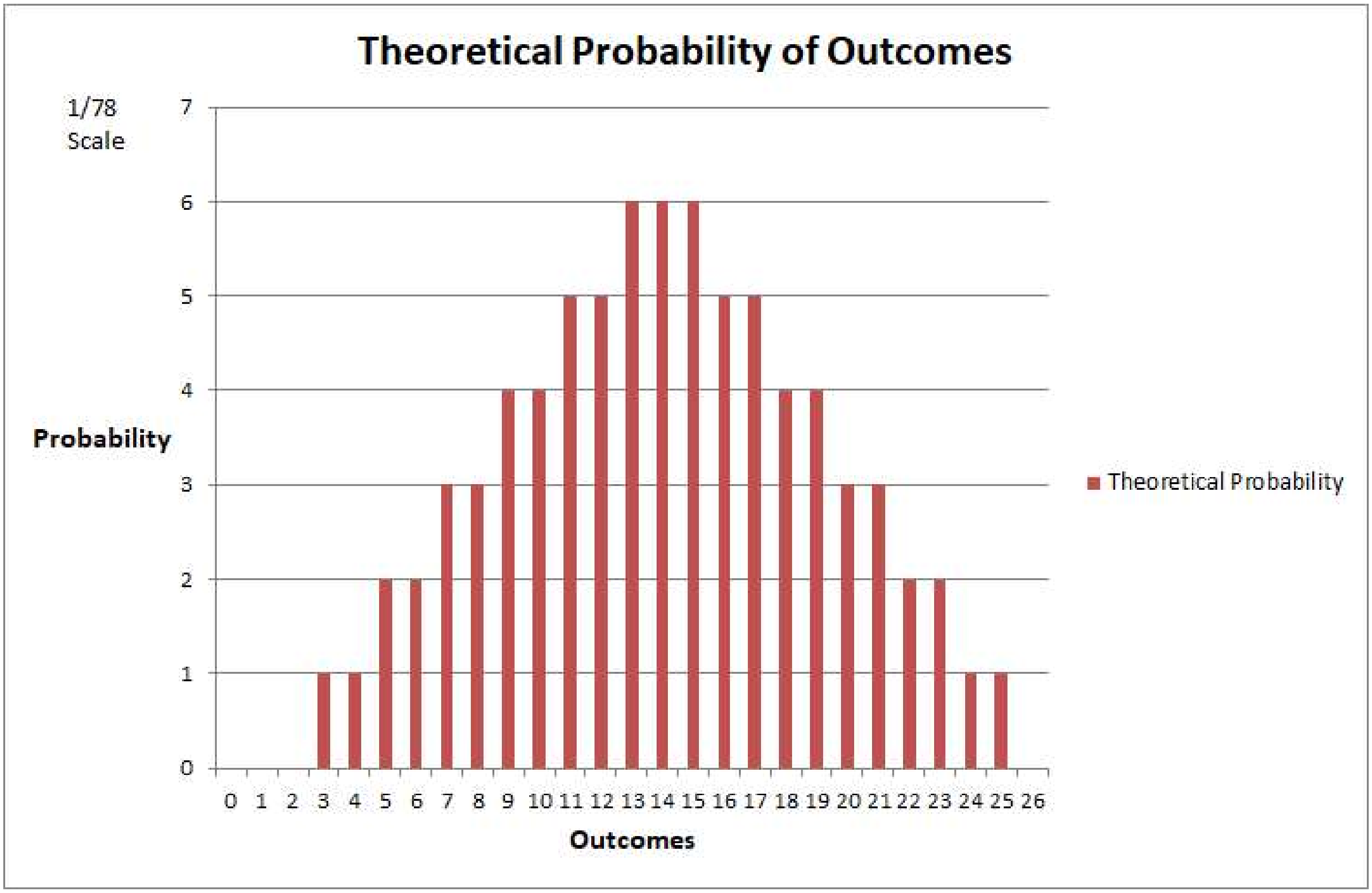
P(winning) = 6/78 + 5/78 + 5/78 + 4/78 + 4/78 + 3/78 + 3/78 + 2/78 + 2/78 + 1/78 + 1/78

= 36/78

= approximately 0.462

= approximately 46.2%

The probability of winning this game is the sum of the winning probabilities, which equals approximately 0.462, or approximately 46.2% respectively, this means that you have approximately a 46.2% chance of winning each match you play. As clearly shown, the game is theoretically unfair towards the player by a minuscule degree.



As we can see, there is definitely a pattern. Sums of 3,4, 24, and 25 each have the lowest chance of happening, 1/78, in this game due to the extremely limited number of ways that these sums can happen as per the tables above this chart. Sums of 13, 14, and 15 have the highest chance of occurring, 6/78, due to the many number of ways that these sums can manifest as also shown by the tables previously mentioned.

Experimental Data: Sample of 100 Real World Matches

Possible Sums x	Number of Times Occurred in 100 Experimental Trials	Experimental Chances Of Winning	Player Victory (√)	Player Defeat (×)
3	0	0		×
4	0	0		×
5	0	0		×
6	2	2/100		×
7	2	2/100		×
8	0	0		×
9	4	4/100		×
10	1	1/100		×
11	8	8/100		×
12	6	6/100		×
13	11	11/100		×
14	9	9/100		×
15	7	7/100	√	
16	8	8/100	√	
17	10	10/100	√	
18	10	10/100	√	
19	8	8/100	√	
20	5	5/100	√	
21	0	0	√	
22	6	6/100	√	
23	3	3/100	√	
24	0	0	√	
25	0	0	√	

Calculating Experimental Probability for Sums

To collect data for our game, we played 100 matches and recorded our findings.

Sums	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Number of Times Sum Occurred	0	0	0	2	2	0	4	1	8	6	11	9	7	8	10	10	8	5	0	6	3	0	0

To calculate the experimental probability of each sum, we used the number of times a sum happened out of the total number number of matches played.

We calculated these using the formula below:

P(sum) = (# of times occurred / # of matches played)(100)

Here are some examples of experimental probability calculations done for each of the possible sums:

P(13) = (11 / 100)(100) = 11 %

P(7) = (7 / 100)(100) = 7 %

To calculate the experimental probability of winning:

$$P(\text{experimental probability of winning}) = \left[\sum \text{each } P(\text{sum}) \right](100)$$

$$= \left[P(15) + P(16) + P(17) + P(18) + P(19) + P(20) + P(21) + P(22) + P(23) + P(24) + P(25) \right](100)$$

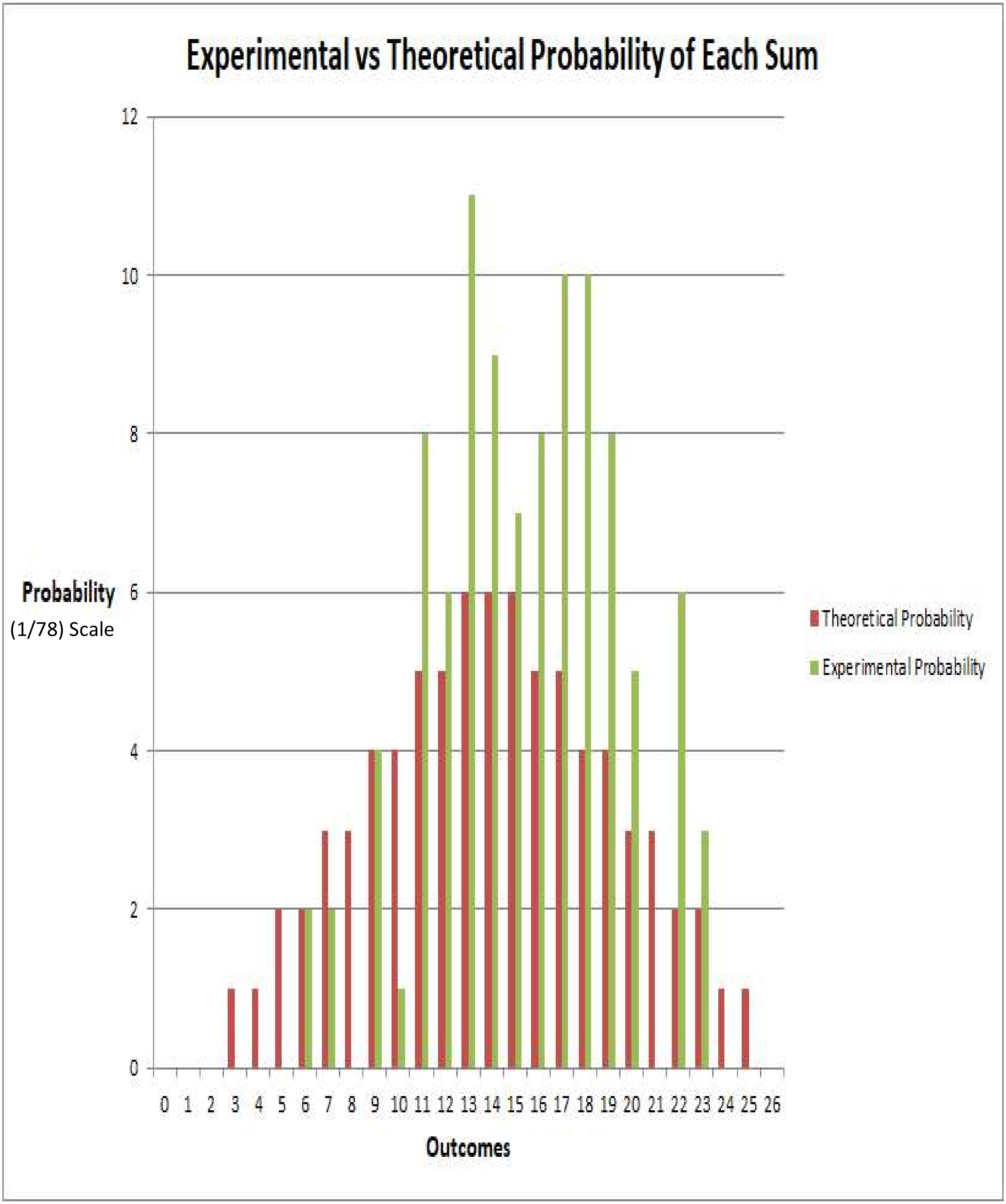
$$= \left[7/100 + 8/100 + 10/100 + 10/100 + 8/100 + 5/100 + 0 + 6/100 + 3/100 + 0 + 0 \right](100)$$

$$= \left[57/100 \right](100)$$

$$= \left[0.57 \right](100)$$

= 57%

This means that the experimental probability of winning each each match is 57% for the player.



As can be seen in this chart, there is a large difference between the two probability distributions.

Conclusions

The Experimental vs. Theoretical Probability of Each Sum graph shows which sums happened more often than expected in our game. The sums of 13, 14, 17, and 18 happened most often. This result is not surprising, as there are many ways for each of these to happen with two dice and a card, as shown by our tables and charts. On the other hand, we definitely did not expect certain sums to not show up much in our experimental data compared to our theoretical projections, such as 10 and 12. The second most often sums were 11, 15, 16, and 19., which manifested experimentally a lot more overall than our theoretical data would have had us think.

When observing the experimental distribution, the sums of 3, 4, 5, 8, 21, 24, and 25 show probabilities of zero, which is not unexpected for the most part, as most of these have relatively low theoretical probabilities, however, the sums of 8 and 21 manifesting zero times is odd, considering the moderate theoretical probability of them occurring. The theoretical probabilities of the other sums above would have us believe that these sums should have happened slightly more than they did in real world testing.

Ultimately, the game was projected to be quite balanced and restrained via the theoretical data, but the experimental data paints a different picture. The victorious sums occurred a lot more than expected, creating a skew towards victory, and certain defeat sums did occur way more than we could have foreseen, such as the sums of 11, 13, and 14. This spells a volatile situation for the player, but with a tendency leaning towards victorious conditions, as reflected in our total experimental probability of winning out of 100 matches, as well as the spread of the green data in the chart.