Schrodinger Equation

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Thuhma

Ka article pakhatah wave equation nih phung leh a nihna ka ziak teuh a. Remchangah kan la en ho ang a. Tin, en leh lo pawhin he thuziak hi ngaihnawm ti taka in chhiar theih tho ka beisei ang a. Classical particle leh quantum particle hi a awm hrang a. Statistical physics lam atanga thil a nihna zirchian dawn chuan hriat hran ngei ngei a ngai tlat a ni. Quantum Mechanics hi thlirdan (picture) chi thum an awm a, chung zinga Schrodinger picture lam hawi zirhona a ni ber ang chu, he article hi…

Classical Mechanics lamah

Classical mechanics lamah khan fundamental wave equation kan nei a, heti ang hian:

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad (1)$$

Wave function, Sine wave, $y(x,t) = A\,Sin(kx - wt)$ hian a chunga kan equation neih hi a pawm (satisfy) ve em tih han prove dawn ta ila. Function hi x leh t hmanga second derivation kan pek hnuin equation-ah khian han dah ta ila.

$$LHS = -Ak^2 Sin(kx - wt)$$

$$RHS = -A\frac{\omega^2}{v^2}Sin(kx - wt)$$

A nih chuan $v_p = \frac{\omega}{k}$ hi kan neih a ngai tihna a lo ni a. Chumi awmzia chu Sine wave khian wave equation a pawm (satisfy) tihna a lo ni. A chhan chu wave kha phase velocity hmangin a kal a ni tih a tihlan vang a ni ber a, hei hi Fourier transformation hmangin kan hriathiam tawh kha…

Wave equation atangin engah nge bul ka lo tan?

Tun tum chu thuhma pawh a bon hle mai, a hnawk nghal nuai mai a… Quantum mechanics-ah hian matter hian wave nihphung a neih avangin wave equation engemaw tal chu a neih ve ngei a ngai a. Chu chu Pu Schrodinger-a equation kan tih fo kha a lo ni ta a ni. Khilaia wave equation khi electromagnetic wave pawhin a pawm vek a, chu chu awlsamtein kan entir thei bawk a ni. Tin, fundamental principle atangin kan chawkchhuak (derive) thei a. Entirnan: Newton's law atang pawhin awlsamtein khi wave equation khi kan chawk chhuak thei thin a nih kha. Chumi awmzia chu wave equation aiin newton's law chu a fundamental zawk tihna a lo nih chu. Amaherawhchu, Schrodinger equation erawh hi chu derive theih a ni lo a. Newton's law anga bultantu – axiom anga pawm hmiah tur ang chi a ni. A hnuaiah hian schrodinger equation awlsam ber chu han ziak ta ila:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x,t)}{dx^2} = i\hbar\frac{d\psi(x,t)}{dt} \quad (2)$$

Classical wave equation kan solve nana wave function kan hman ang lo deuh $\psi(x,t) = A\exp[i(kx-wt)] = A\,[Cos(kx-wt) + i\,Sin(kx-wt)]$ ang chi hi nei ta ila. Tunah chuan wave equation hian complex number a nei a, tin kan wave function pawh hian complex component a nei ta bawk a. Heti ang hian kan nei thei ang a:

$$LHS = \frac{\hbar^2 k^2}{2m}\psi(x,t)$$

$$RHS = \hbar\omega\psi(x,t)$$

A nih chuan hei hi kan nei thei tihna a lo ni a, heti angin:

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega \quad (3)$$

A chunga equation hian enge an entir kan hre thei em aw? Kan thil zir tawh hriat chhuah kan nei em? $\hbar\omega$ hi enge ni kha? Particle energy a ni tiraw… A nih leh $\hbar k$ hi enge ni? Particle momentum a ni bawk tiraw kha… A nih chuan LHS khian free particle kinetic energy a lo ni ta tihna a ni a. Hemi hian a entir chu particle hi wave angin space-ah a awm chuan a energy neih zawng zawng chu kinetic energy a ni dawn tih mathematics takin kan entir tihna a ni a. Particle chuan Schrodinger equation chu a lo pawm ve ta reng mai tih pawh kan entir bawk a ni. Mahse helaiah hian thil concept hriat chian ngei ngei ngai a awm a. Chu chu free particle tih hi a ni. Free particle kan tih chuan kinetic energy chauh a nei tih piah lamah hian fields dang nen a inhne (interact) miah loh tih hriat bawk tur a ni a. Gravitational potential emaw eng fields mahin a interact lo a, space-ah hian a free takin a awm ve ringawt a, kinetic energy chauh a nei a. Helaiah hian schrodinger equation dikzia kan prove duh avangin ideal takin entirnan kan lo hmang mai chauh zawk a ni. Mahse heti ang particle hi a takah kan nei thei lo...

Wave function nihphung Pu Born-a sawi fiah danin

Pu Born-a khan wave function, $\psi(x,t)$ hian awmzia ril nihna a nei niin a sawi a, a hmasaberah chuan wave function conjugate leh wave function hi kan puntir a, chuan thil thui zawng tawite nen puntir bawk chuan chumi thui zawng chhunga particle kan hmuh theihna chance chu kan nei dawn a ti a, hetiangin:

$$P(x,t) = [\psi(x,t)^*\psi(x,t)]$$

Khi laia kan wave function khi han chawk dawn chhin ta ila, hei hi kan hmu anga:

$$P(x,t) = [\psi(x,t)^*\psi(x,t)] = A^*A = |A|^2$$

Probability chu x leh t ah a innghat a. Amaherawhchu, khilaiah khian $|A|^2$ chauh a tluk tihna a ni a. Chumi awmzia chu particle kan hmuhtheihna, probability density chu engahmah a innghat lo

a, chumi awmzia chu helaia quantum particle-in khi laia wave function a nei kan tih (plane wave) a chu khawiah pawh kan hmu thei dawn tihna a ni a. Hun leh hmunah a innghah loh avangin khawiah pawh khawi hmunah pawh kan hmu dawn tihna a lo ni. Chumi awmzia chu Aizawl tlangah I hmuh laiin Mumbai-ah pawh hmun hrangah I hmuh rual chiahin ka lo hmu ve dawn tihna a lo ni a, a dik thei lo a ni. Chuti ang particle chu khawvelah a awm thei tak tak lo tihna a lo ni. Helaiah hian Schrodinger equation awmzia ril zawk hrilhfiahna turin phuahchawp mai mai tihna a nih chu…

Particle, Potential energy pawh nei ngaihtuah tawh ila…

A chunga ka sawi ang khan particle hian kinetic energy ringawt a nei thei lo a. Thildang nen an inhne (interact) ve reng thin a. Chu chu eng fields pawh a ni thei a, kan hriat theih awlsam ber chu gravitational potential fields (energy) hi a ni mai awm e. A nih chuan Schrodinger equation chu a lo danglam dawn ta tihna a lo ni a, potential energy term a lo lang ve dawn tihna a lo ni. Heti ang hian:

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U(x)\right)\psi(\mathrm{x,t})=i\hbar\frac{d\psi(\mathrm{x,t})}{dt} \quad (5)$$

He equation hi ngun taka kan en chuan second order differential equation kha a ni tih kan hre theuh ang a. Chawhchhuah dan awlsam ber chu – separation of variables hmang khan a ni. A nih chuan kan wave function hi han phel te ta ila, heti ang hian:

$$\psi(\mathrm{x,t})=\psi(x)f(t)$$

A nih chuan equation (5) khi heti ang hian a lawm awm tawh ang a:

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2}+U(x)=i\hbar\frac{1}{f(t)}\frac{\mathrm{df(t)}}{dt} \quad (6)$$

Ngun takin equation lo chhuak thar ber hi han en ta ila. A dinglamah hi x ah a innghat a, tin a dinglam hi t ah a innghat bawk a. Hemi hian a entir chu he equation hi x leh t ah a innghat lo ve ve avangin he equation hi a constant tihna a ni. A nih chuan heti ang hian kan dah ang:

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2}+U(x)=i\hbar\frac{1}{f(t)}\frac{\mathrm{df(t)}}{dt}=E \quad (7)$$

Helaia E hi constant a ni. A nih chuan heti ang hian equation kan nei thei ang:

$$\frac{\mathrm{df(t)}}{dt}=-i\frac{f(t)}{\hbar}E \quad (8)$$

He equation tana solution chu hei hi a lo ni leh ang a:

$$f(t)=\exp\left(-\frac{iEt}{\hbar}\right) \quad (9)$$

Hei hi wave function-a time dependent part kha a ni a. Tunah chuan space dependent lam han thlur bing veleh ta ila, hei hi kan hmu ang a:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+\psi(x)U(x)=\psi(x)E \text{ (10)}$$

Hei hian hming lar deuh deuh pahnih a nei a. Pakhat chu Spatial part of schrodinger equation tih a ni a, a dang leh chu time independent schrodinger equation tih a ni bawk a ni. A nih chuan equation solution chu heti ang hian kan ziak thei ang a:

$$\psi(\mathrm{x},\mathrm{t})=\mathrm{A}\varphi(\mathrm{x})\exp\left(-\frac{iEt}{\hbar}\right) \text{ (11)}$$

Helaia A hi constant a ni a, tin $\varphi(\mathrm{x})$ hi equation (10) tana solution a ni. Equation (10) solution hi khi laia constant kan tih mai, energy profile-ah a innghat tlat a ni. Tunah probability density han zawng veleh chhin teh ang:

$$P(x,t)=A^{*}A\varphi(\mathrm{x})^{*}\varphi(\mathrm{x})=|A|^2|\varphi(\mathrm{x})|^2$$

Ngun taka kan en chuan probability density hi hun, t ah a innghat miah lo tih ka hmu thei ang a. Mahse hei hi a awm theih chhan chu energy, E hi a constant vang a ni. Tin, equation (11) khi stationary wave function emaw stationary states emaw tih a ni bawk a. Chumi awmzia chu helaia probability density hi hunah a innghat lo a, hun a danglamin a danglam ve lo tih a entir a ni. Tin, probability hi thil awm ngei, physical quantity a ni a, a tlukpui chu 1 a ni tih kan hriat vek ka ring a. A nih chuan heti ang hian kan ziak thei a:

$$\int_{-\infty}^{\infty}P(x,t)dx=\int_{-\infty}^{\infty}|A|^2|\varphi(\mathrm{x})|^2dx=1 \text{ (12)}$$

Helaia equation hian hming a nei a, a hming chu normalization condition an thin a.

Pussy in the well…

Tunah chuan thil chiang zawk Schrodinger equation hmang hian han chawh chhuah tum ta ila. Particle in a potential well hi kan hriat theuh ka ring a. A hnuaia kan entir ang khian a sei zawng hi ‘L’ a ni a.

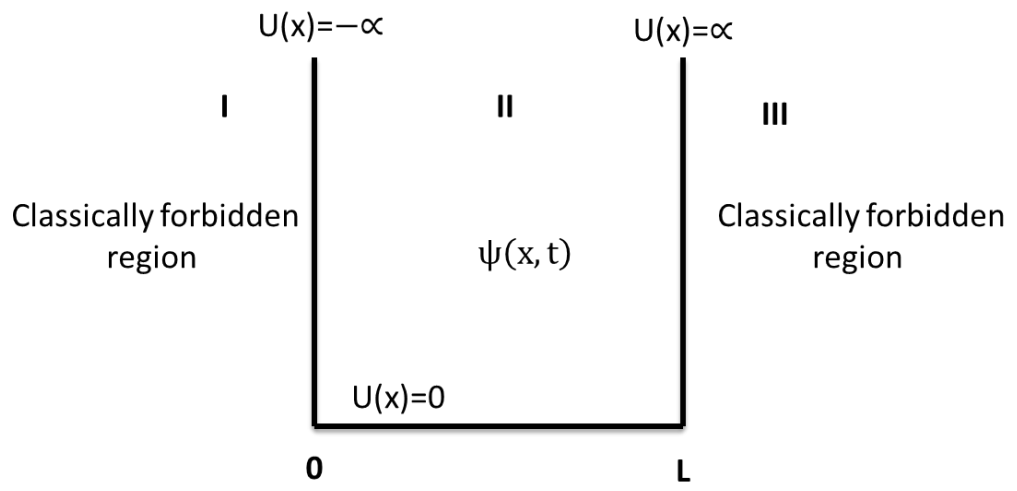

Figure 1: Potential well

Tin, a thuk zawng pawh hi teh theih loh khawpa thuk – infinite a ni a. A chhungah hian particle, electron awm ta se a chhuak thei tawh ngai dawn lo tihna a ni. Physical takin classically forbidden region-ah khi chuan probability density function chuan zero a tluk dawn tihna a lo ni a. Well chhungah hian potential energy zero a nih avangin equation (10) hi heti ang hian kan nei thei a:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + 0 = \psi(x)E$$

Emaw heti ang hian:

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m\psi(x)E}{\hbar^2} = 0 \ (13)$$

Hei pawh hi second order differential equation kha a nil eh mai a, heti hian kan chawk chhuak thei ang a:

$$\psi(x) = Ae^{i\frac{\sqrt{2mE}}{\hbar}x} + Be^{-i\frac{\sqrt{2mE}}{\hbar}x} \ (14)$$

Helaiah hian constant pahnih kan nei a, chungte chu A leh B te an ni a. An tlukpui kan zawn a ngai a ni. Thuziak hnuhnung bera wave function nihphung kan sawite kha kan hman a ngai ta chiah a. Chungte chu a chunga kan sawi probability density function hian 1 a tluk ziah thin a ni tih leh wave equation hi a continuous tih kha a ni a. A dang leh chu wave function derivative hmasa ber kha a continuous tih a ni leh a, tin wave function hi infinity a ni thei lo tih kha a ni leh bawk a ni. Khi lai figure-a ka ziah ang khian region I leh reion II inkarah khian heti hian kan nei ang a:

$$\psi_I(x) = region(I) = 0$$

Chuti ang bawkin,

$$\psi_{II}(x) = region(II) = A + B$$

A nih chuan region I leh II hi a intluk a ngai si a, heti ang hian:

$$\psi_I(x) = \psi_{II}(x) = A + B = 0$$

Chumi awmzia chu,

$$A = -B$$

tihna a lo ni dawn a ni.

A nih chuan region II chhungah chuan heti ang wave function hi kan lo nei reng tihna a lo ni a, heti ang hian:

$$\psi_{II} = A\left(e^{i\frac{\sqrt{2mE}}{\hbar}x} - e^{-i\frac{\sqrt{2mE}}{\hbar}x}\right)$$

Emaw heti ang hian kan ziak thei bawk a,

$$\psi_{II}(x = L) = A\left(e^{i\frac{\sqrt{2mE}}{\hbar}L} - e^{-i\frac{\sqrt{2mE}}{\hbar}L}\right) = 2A\ i\ Sin\left(\frac{\sqrt{2mE}}{\hbar}L\right) \text{ (15)}$$

Mahse region III ah khan heti ang hian kan nei leh bawk a:

$$\psi_{III}(x = L) = 0 \text{ (16)}$$

Equation (15) leh (16) hi a inan avangin heti ang hian kan nei leh thei a:

$$2A\ i\ Sin(kL) = 0 \text{ (17)}$$

Hetah hian $\frac{\sqrt{2mE}}{\hbar} = k$ a tluk angah kan ngai a. Equation (17) ah khian A hi zero a tluk thei lo a, a tluk a nih vaih chuan region II ah khian particle khi a awm lo tihna a ni dawn a. A nih chuan Trigonometric equation pawl 11 a kan zir thin ang hian hetah hian relation kan neih a ngai tihna a ni a:

$$kL = n\pi\ (n = 1,2,3\ \ldots)\ \ (18)$$

A nih chuan Schrodinger wave equation chu heti hian kan nei dawn tihna a lo ni a:

$$\psi_{II}(x) = 2A\ i\ Sin(kx), where\ k = \frac{n\pi}{L} \quad (19)$$

$$\psi_I(x) = \psi_{II}(x) = 0$$

A value khi zawn leh a ngai ta a. Heti hian kan zawng thei ang a:

$$\int_0^L \psi_{II}(x)^* \psi_{II}(x) dx = 1 (20)$$

Equation (19) hi equation (20) ah kan dah chuan heti ang hian kan nei ang a:

$$\psi_{II}(x) = \sqrt{\frac{2}{L}} Sin(kx) \ (21)$$

Helaia equation (21) hi a awlsam thei ang bera schrodinger equation chawhchhuahna chu a ni ta a ni…

A quantum-na lai taka chu mawms…

Kan chawh thlak mawp mawp lai khan substitution pawimawh lutuk ka siam a. Chutah chuan $\frac{\sqrt{2mE}}{\hbar}$ aiah k dah thleng a nih kha. A nih leh leh lam ve ve han square chhin teh ang:

$$E = \frac{k^2 \hbar^2}{2m} \ (22)$$

hi kan nei dawn tihna a lo ni. Ngun takin han en la: k hi wave number a nih chuan de Broglie condition hmangin heti hian kan nei dawn tihna a lo ni a:

$$E = \frac{p^2}{2m} \quad (23)$$

Hei hi kinetic energy kha a ni ta chiah a ni. A nih chuan kan thil chawh chhuah hi a dik leh zual em em a ni tih a lo finfiah ta chiah a ni. A nih chuan $k = \frac{2\pi}{\lambda}$ tih kan hriatna atangin heti hian kan nei thei a:

$$L = n\frac{\lambda}{2} \ (24)$$

Equation (24) hi higher secondary Physics lamah kan zir fo thin standing wave (stationary) waves kha a lo ni ta chiah a. Chumi awmzi chu matter wave function kha region II chhungah khan standing wave angina a awm dawn tihna a lo nih chu… Hetah hian a duh ang angin a awm thei bik hauh lo a. Equation (18) lama condition kan neih ang khian wavelength chanve chauh hmangin emaw wavelength pakhat chauh hmangin emaw wavelength chanve pathum hmangine maw chutiang zelin $n = 1,2,3,4,5$ ... kan dah dan hmangin wave function chuan nihphung a lo nei dawn tihna a lo ni. A hnuaiah hian diagram lo dah tel ta ila:

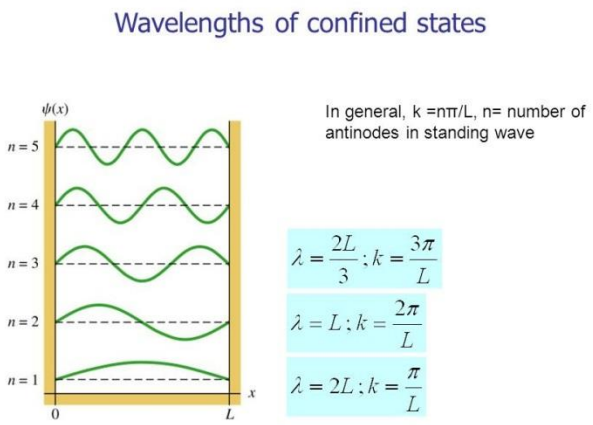

Figure 2: Potential well chhunga wave function awm theih dan
(http://slideplayer.com/slide/3269518/)

A chunga graph-a kan entir ang khian n value hrang hrang hmangin wave awm dan tur kan entir thei a. A hnuai ber khi standing wave tana fundamental mode an tih thin kha a ni a. Node pahnih leh antinode pakhat chauh a nei dawn a ni. A chung chunglam amite khi a indawt danin first harmonics, second harmonics…tiin a hming an vuah a; n nihna a danglamin wave function chuan nihphung a nei dawn tihna a ni a, mode hrang hrang lem kan ziak ang khian heng standing wave hian schrodinger equation solution chu a pe ta a ni. Mahse hriat reng tur chu region I leh II ah te hian wave function hi a bo dawn a ni. Tin, n hian hming a nei bawk a, chu chu quantum number an ti bawk a nih kha… *Tin, n hian value neih theih bik a neih avangin quantized kan ti thei ta bawk a, hei hi quantum mechanics awmzia ber chu a lo ni ta a ni.*

Quantization of energy…

Khilaia kan tarlan angina wave function chuan mode hran hran a nei thei a – grapha kan entir ang khian. Mahse wave function tinte khian a hran theuhin energy an nei bawk a. Equation (18) leh (22) hmang hian hei hi kan nei thei a:

$$E = \frac{\hbar^2}{2m}\frac{n^2\pi^2}{L} = n^2\left(\frac{\hbar^2}{2m}\frac{\pi^2}{L}\right) \quad (25)$$

Wave function kha n value neih theih chin chauh a nei ang chiahin helaiah pawh hian energy hian neih theih chin chauh a nei veleh ta a, hei hi energy quantization an tih a kha a ni ta chiah a ni. Hei Bohnr's energy quantization ang chiah kha a ni a. A nih leh khawi atangin nge he quantization hi a lo chhuah tak ang le? A chhanna chu eng dang ni loin quantization hi confinement avanga lo awm a ni. *Khi laiah pawh khian particle khi potential well chhungah chauh awm tura kan tih avangin quantization a lo awm ta a ni*. Equation (25) ah khian n aiah

1,2,3,4… kan dah chuan particle tana energy zat chu kan nei thei dawn a ni. Hetah hian energy neih theih zat hi a inang lo a, a inzawm lo a, a hrang vek tihna a ni. Energy level hran hran kan lo nei tihna a lo ni. Electron chu energy level khat atanga a dangah a kal chuan a chhuk emaw a chho emaw a nihin energy a la lute maw a pe chhuak emaw a ni thin a. Chu chu eng pek avangin emaw eng a lo pek chhuah avangin emaw a thleng thin a ni.

A thui em mai a…

Tun tum chu physics bupui lo intanna – quantum mechanics innghahna equation kan chawk bawk a equation a tam bawk si a, a thui hle mai! Titawp tawh dawn ila. Amaherawhchu sawi duh dang ka la nei a. Khi laia energy state khi en ta ila enge a energy neih thei tlem ber (lowest energy state)? A neitheih tlem ber chu $\frac{\hbar^2}{2m}\frac{\pi^2}{L}$ a ni tiraw… Classical mechanics-ah kha chuan particle eng pawh hian zero energy an nei thei a mahse quantum mechanics-ah chuan zero energy an nei thei lo tiin a chawk chhuak thung a ni hi... Hei hi zero point energy emaw vacuum fluctuation emaw tih a ni thin a. Chumi awmzia chu khawi hmunah pawh hian energy chambang (residual) a awm zel tihna a ni. Hei hi uncertainty principle hmang pawh khan a finfiah theih a mahse titel tawh lo mai ila.

A pwimawh ber probability density function kan la sawi lo tlat mai. Figure 1 khi ngun taka kan en chuan electron chu state hniam ber emaw a dawttu emaw a chung leh chiahah emaw a awm thei vek a. A nih chuan a probability density enge ni ta ang? Probability density zawn dan kha kan la hre theuh ang a – wave function modulus a nih kha… A hnuai ami ang hian kan nei dawn tihna a ni a.

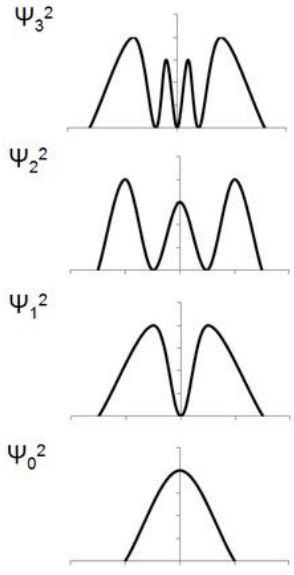

Figure 3: Probability density states
(https://en.wikiversity.org/wiki/Quantum_mechanics/Harmonic_oscillator)

A nih leh electron chu va man tum ta ila emaw experiment emaw ti ta ila engtin nge ni ang? Sawi tawh angin wave function hi fields ang an ni a, electron kan va man rual rualin an bo (collapse) anga eletron-ah an lo chang dawn tihna a lo ni. Entirnan: microscope hmangin electron awmna chu zawng ta ila, tin first excited state, $n = 2$ ah khian awm ta bawk se. Pakhat kan zawn hmuh zawh apiangin wave function a bo zel anga mahse system inang chiah nei ta ila experiment chu ti ta zel ila a chunga probability wave function a chung ami ang khi kan lo nei dawn a lo ni.

Engpawhnise, a sei em mai! Duh tawk tawh ang. In lo hlawkpui ngei ka beisei.