Chaffey College

Math Success Center Activity Validation Form

Student: Sean Granados Class: 65B

Activity: 65B Integrals

☒ Workshop ○ Learning Group ○ Tutoring Session

Instructor/Tutor Signature:

Date: 3/12/20 Stamp: CHAFFEY COLLEGE MATH SUCCESS CENTER RANCHO CUCAMONGA CAMPUS

Sean Granados

1. $\int_0^\infty 11e^{-11x}\,dx \Rightarrow \lim_{b\to\infty} \int_0^b 11e^{-11x}\,dx$ $\quad u=-11x \quad du=-11dx$

$\Rightarrow \lim_{b\to\infty} \int_0^b \cancel{11}e^u \frac{du}{-\cancel{11}} \Rightarrow \lim_{b\to\infty} \left[-e^u\right]_0^b$

$\Rightarrow \lim_{b\to\infty} \left[-e^{-11x}\right]_0^b \Rightarrow \lim_{b\to\infty} \left[-\frac{1}{e^\infty} - (-e^0)\right]$

$0 + 1 = \boxed{1}$

2. $\int 5\cos^3 4x\,dx \Rightarrow 5\int \cos^3 4x \Rightarrow 5\int (1-\sin^2 4x)\cos 4x\,dx$

$\Rightarrow 5\int (\cos 4x - \cos 4x \sin^2 4x)\,dx$

$5\int \cos 4x\,dx - 5\int \cancel{\cos 4x}\sin^2 4x\,dx$ $\quad u=\sin 4x \quad \frac{du}{dx}=4\cancel{\cos 4x}$

$\frac{5\sin 4x}{4} - \frac{5}{4}\cdot\frac{\sin^3 4x}{3} + C$

$$\boxed{\frac{5\sin 4x}{4} - \frac{5\sin^3 4x}{12} + C}$$

Partial fractions

3. $\int \frac{11x+48}{x^3+8x^2+16x}\,dx \Rightarrow \int \frac{11x+48}{x(x+4)^2}\,dx$

$$\left[\frac{11x+48}{x(x+4)^2} = \frac{A}{x} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}\right] \cancel{x}(x+4)^2$$

$11x+48 = A(x+4)^2 + B(x)(x+4) + Cx$

Let $x=0$

$48 = 16A$

$\boxed{3 = A}$

Let $x=-4$

$4 = -4C$

$\boxed{-1 = C}$

$4B + 8A + C = 11$

$4B + 24 - 1 = 11$

$4B + 24 = 12$

$4B = -12$

$\boxed{B = -3}$

$\int \left[\frac{3}{x} - \frac{3}{(x+4)} - \frac{1}{(x+4)^2}\right] dx$

$u = x+4$
$du = dx$

$= -u^{-2}$

$\frac{-u^{-1}}{-1} \Rightarrow +\frac{1}{u}$

$\boxed{3\ln|x| - 3\ln|x+4| + \frac{1}{(x+4)} + C}$

$\frac{d}{dx}$

$\csc \to \csc\cot$

$\cot \to \csc^2$

	D	I
+	y^4	e^{3x}
−	$4y^3$	$e^{3x}/3$
+	$12y^2$	$e^{3x}/9$
−	$24y$	$e^{3x}/27$
+	24	$e^{3x}/81$
−	0	$e^{3x}/243$

4. $\int y^4 e^{3x}\,dx$

$$\boxed{= \frac{y^4e^{3x}}{3} - \frac{4y^3e^{3x}}{9} + \frac{12y^2e^{3x}}{27} - \frac{24ye^{3x}}{81} + \frac{24e^{3x}}{243} + C}$$

5. $\int \frac{\sqrt{4-x^2}}{x^4}\,dx$ $\quad x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$$\int \frac{\sqrt{4-4\sin^2\theta}}{16\sin^4\theta} \cdot 2\cos\theta\, d\theta$$

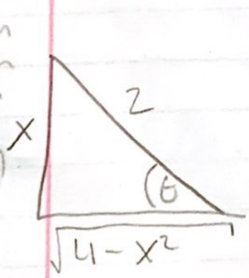
$$\int \frac{\cancel{2}\cos\theta}{\underset{4\ 8}{\cancel{16}}\sin^4\theta} \cdot \cancel{2}\cos\theta\, d\theta$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^4\theta} \cdot \cos\theta\, d\theta \Rightarrow \frac{1}{4}\int \frac{\cos^2\theta}{\sin^4\theta}\, d\theta \Rightarrow \frac{1}{4}\int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\sin^2\theta}\, d\theta$$

$$\Rightarrow \frac{1}{4}\int \cot^2\theta\csc^2\theta\, d\theta \quad u = \cot\theta \quad \frac{du}{-\csc^2\theta} = d\theta \quad \Rightarrow \frac{1}{4}\int u^2\cancel{\csc^2\theta} \cdot \frac{du}{-\cancel{\csc^2\theta}}$$

$$\Rightarrow -\frac{1}{4} \cdot \frac{\cot^3\theta}{3} \Rightarrow -\frac{\cot^3\theta}{12} + C$$

Soh
Cah
Toa



$$\cot^3\theta = \left(\frac{\sqrt{4-x^2}}{x}\right)^3 \Rightarrow \boxed{-\frac{\left(\sqrt{4-x^2}\right)^3}{12x^3} + C}$$

$\sin^2 5x = \frac{1}{2}(1-\cos 2x)$

6. $\int 4\sin^2 5x\,dx \Rightarrow 4\int \sin^2 5x\,dx$ $\quad u = 5x$, $\frac{du}{5} = dx$

$\Rightarrow \frac{4}{5}\int \sin^2 u\,du \Rightarrow \frac{4}{5}\int \frac{1}{2}(1-\cos 2u)\,du$

$\frac{4}{10}\int (1-\cos 2u)\,du \Rightarrow \frac{4u}{10} - \frac{4\sin 2u}{20} + C$

$\frac{2\cancel{0}x}{\cancel{1}0} - \frac{\cancel{4}\sin(10x)}{5\,\cancel{2}\cancel{0}} + C$

$$\boxed{2x - \frac{\sin(10x)}{5} + C}$$

7. $\int \frac{8x^2+x+63}{x^3+9x}\,dx \Rightarrow \int \frac{8x^2+x+63}{x(x^2+9)}\,dx$

$\left[\frac{8x^2+x+63}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+D}{(x^2+9)}\right] x(x^2+9)$

$8x^2+x+63 = A(x^2+9) + (Bx+D)(x)$

Let $x=0$ $\quad Ax^2+9A+Bx^2+Dx$

$63 = 9A$ $\quad A+B=8$

$\boxed{7 = A}$ $\quad 7+B=8$

$\boxed{1 = D}$ $\quad \boxed{B=1}$

$\int\left[\frac{7}{x} + \frac{x+1}{(x^2+9)}\right]dx$ $\quad u = x^2+9 \quad du = 2x\,dx$

$7\ln|x|$

7, continued

$$\int \left[\frac{7}{x} + \frac{x+1}{(x^2+9)} \right] dx$$

$$\int \left[\frac{7}{x} + \frac{x}{(x^2+9)} + \frac{1}{(x^2+9)} \right] dx$$

$$\boxed{7\ln|x| + \frac{1}{2}\ln|x^2+9| + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$$

$\frac{3}{3}$

$x^{-2/3}$ $\frac{x^{1/3}}{\frac{1}{3}}$

8. $$\int_{-27}^{8} \frac{dx}{x^{2/3}} \Rightarrow \lim_{b \to 0} \int_{-27}^{b} \frac{dx}{x^{2/3}} + \lim_{a \to 0} \int_{a}^{8} \frac{dx}{x^{2/3}}$$

$$\lim_{b \to 0} \left[3x^{1/3} \right]_{-27}^{b} + \lim_{a \to 0} \left[3x^{1/3} \right]_{a}^{8}$$

$$\lim_{b \to 0} \left[0 - (3(-3)) \right] + \lim_{a \to 0} \left[6 - 0 \right]$$

$$+9 + 6$$

$$= \boxed{15}$$

csc $-\csc\cot$

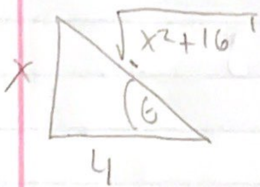
9. $\displaystyle\int \frac{12}{x^2\sqrt{x^2+16}}\,dx \Rightarrow 12\int \frac{1}{x^2\sqrt{x^2+16}}\,dx$ $x = 4\tan\theta$, $dx = 4\sec^2\theta$

$\displaystyle\Rightarrow 12\int \frac{\cancel{4\sec^2\theta}}{16\tan^2\theta\,\cancel{4\sec\theta}}\,d\theta \Rightarrow \frac{12}{16}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$

$\displaystyle\Rightarrow \frac{3}{4}\int \frac{\frac{1}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\cdot\cot\theta\,d\theta \Rightarrow \frac{3}{4}\int \csc\theta\cot\theta\,d\theta$

$\displaystyle\Rightarrow -\frac{3}{4}\csc\theta + C$

Soh
Cah
Toa



$\displaystyle\csc\theta = \frac{\sqrt{x^2+16}}{x}$

$\displaystyle\Rightarrow \boxed{-\frac{3\sqrt{x^2+16}}{4x} + C}$

10. $\displaystyle\int e^{3x}\cos 2x\,dx$

	D	I
+	e^{3x}	$\cos 2x$
−	$3e^{3x}$	$\frac{\sin 2x}{2}$
+	$9e^{3x}$	$\frac{-\cos 2x}{4}$

$\int$

$\displaystyle\frac{4}{4}\int e^{3x}\cos 2x\,dx = \frac{e^{3x}\sin 2x}{2} + \frac{3e^{3x}\cos 2x}{4} - \frac{9}{4}\int e^{3x}\cos 2x\,dx$

$\displaystyle\frac{13}{4}\int e^{3x}\cos 2x\,dx = \frac{\frac{e^{3x}\sin 2x}{\cancel{2}} + \frac{3e^{3x}\cos 2x}{\cancel{4}}}{\frac{13}{\cancel{4}}} \Rightarrow \boxed{\frac{2e^{3x}\sin 2x}{13} + \frac{3e^{3x}\cos 2x}{13} + C}$

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11. $\int \frac{2x^3 + x^2 - 21x + 24}{x^2 + 2x - 8}\,dx$

$$2x - 3 + \frac{x}{x^2+2x-8}$$

$$x^2 + 2x - 8 \,\overline{\big)\, 2x^3 + x^2 - 21x + 24}$$

$$-(2x^3 + 4x^2 - 16x)$$

$$-3x^2 - 5x + 24$$

$$-(-3x^2 - 6x + 24)$$

$$x$$

$$\int \left(2x - 3 + \frac{x}{x^2+2x-8}\right)dx \qquad \left[\frac{x}{(x+4)(x-2)} \Rightarrow \frac{A}{(x+4)} + \frac{B}{(x-2)}\right](x+4)(x-2)$$

$$\int 2x\,dx - \int 3\,dx + \int \frac{2/3\,dx}{(x+4)} + \int \frac{1/3\,dx}{(x-2)}$$

$$\boxed{x^2 - 3x + \frac{2}{3}\ln|x+4| + \frac{1}{3}\ln|x-2| + C}$$

$x = A(x-2) + B(x+4)$

Let $x = 2$

$2 = 6B$

$\boxed{\frac{1}{3} = B}$

Let $x = -4$

$-4 = -6A$

$\boxed{\frac{2}{3} = A}$