Q3

The statement is TRUE.

Proof:

Any odd number can be expressed as follows:
$a \in \mathbb{Z}, (2a+1)$
So we can express the statement to prove as:
$\forall\, n \in \mathbb{Z}\ \exists\, a \in \mathbb{Z}\, (n^2+n+1=2a+1)$
and by subtracting 1 to both sides, we have the equivalent statement (1):
$\forall\, n \in \mathbb{Z}\ \exists\, a \in \mathbb{Z}\, (n^2+n=2a)$

n can be even or odd:

If n is even, there exists an integer m such that $n=2m$
Replacing in n^2+n:

= $(2m)^2+2m$
= $4m^2+2m$
= $2(2m^2+m)$

And by the definition of an even number, the result is even.

If n is odd, there exists an integer s such that $n=2s+1$
Replacing in n^2+n:

$=(2s+1)^2+2s+1$
$=4s^2+4s+1+2s+1$

$=2(2s^2+3t+1)$ [By algebra]

And by the definition of an even number, the result is even.

So for any n, (1) is TRUE. Hence, we proved that the statement is TRUE.