Q9

We will prove that the example: $\mathcal{A}_n = (0, 1/n)$ has the property:

$\mathcal{A}_n, n = 1,2,... : \mathcal{A}_{n+1} \subset \mathcal{A}_n \wedge \bigcap_{n=1}^{\infty} \mathcal{A}_n = \emptyset$

Proof:

$\mathcal{A}_{n+1} = (0, 1/(n+1))$

Clearly, $1/(n+1) < 1/n$. And the lower bound is the same, so the first part $\mathcal{A}_{n+1} \subset \mathcal{A}_n$ is TRUE.

Also, be $n \in \mathbb{N}, a_n = (1/n) \to 0 \; as \; n \to \infty$ (1)

Proof of (1): By definition $\forall \varepsilon > 0 \; \exists n \in \mathbb{N} \; \forall m \geq n \; (|a_n - 0| < \varepsilon)$

Be $\varepsilon > 0$ arbitrary, we need to find an n such that $m \geq n$ and $|a_n| < \varepsilon$
Pick n so large that $1/n < \varepsilon$

$|a_m| = |1/m| = (1/m) \leq (1/n) < \varepsilon$

Hence, we proved that $n \in \mathbb{N}, a_n = (1/n) \to 0 \; as \; n \to \infty$

By (1) $\mathcal{A}_n = (0, 1/n) \to (0,0) \; as \; n \to \infty$

And $(0,0)$ don't include any value, so $(0,0) = \emptyset$

And, by intersection, $\emptyset \cap B = \emptyset$

So the second part $\bigcap_{n=1}^{\infty} \mathcal{A}_n = \emptyset$ is also TRUE.

Hence, the example has the property.